CHANGE OF RESONANCE NUCLEAR PARAMETERS DURING SCATTERING BY REGULAR SYSTEMS

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Scattering by nuclear systems with a low-lying resonance level is considered. It is found that if the nuclei form a regular one-dimensional chain or a two-dimensional lattice, the elastic width Γ_1' and the position of the resonance level change significantly. It is demonstrated that the change in the width is related to the lifetime of the collective excited state of the nuclear system. The lifetime of such a state and the width may respectively be larger or smaller than the lifetime (width) of an isolated excited nucleus. It is shown that in the case of a three-dimensional crystal the elastic width vanishes in general and the energy dependence of the resonance interaction is defined exclusively by the inelastic width Γ_2. In the presence of spin in the ground state of the nucleus, and if the scattering that is inelastic with respect to the phonons plays a noticeable role, the width assumes a value intermediate between Γ_2 and the total width.

1. INTRODUCTION

THE amplitude of resonance scattering by an individual nucleus is determined, when $\lambda >> d$ (λ = wavelength of the incident particles, d = dimension of the nucleus), by the energy E_0 of the resonance level and by the elastic and inelastic widths Γ_1 and Γ_2. At first glance it might appear that on going over to a system of nuclei, at least when λ is smaller than the characteristic distance between the particles, the energy dependence of the elastic and inelastic scattering amplitudes should remain practically unchanged and should be determined by the same parameters. However, as will be made clear in this paper, in the case of scattering by regular systems of identical nuclei, the resonance parameters can experience appreciable alterations. Physically this is connected with the fact that in the case when it is impossible to point out the individual nucleus responsible for the scattering, the excitation of the resonance level or the formation of a compound nucleus will have a collective character. On the other hand, the decay of such a collective state can differ noticeably from the case of an isolated nucleus [1], and this is what causes the change in the resonance parameters.

We confine ourselves to an examination of scattering by extremely low-lying resonance levels and by sufficiently heavy nuclei, and we neglect the influence of the oscillations. In addition, we assume for simplicity that the ground state of the nucleus has zero spin. The question of the influence of the spin incoherence and oscillations of the nuclei in the crystal (in other words, the phonon spectrum and the temperature) will be considered in a special paper.

2. ONE-DIMENSIONAL CHAIN

We consider the scattering of particles by a one-dimensional chain of identical nuclei with period equal to a, under the condition that $d \ll \lambda$; by the same token only s-scattering by an individual nucleus is significant. Assuming the nuclei to be identical, we write the wave function of the particles in the form

$$\psi(\mathbf{r}) = e^{i\mathbf{kr}} + \sum_m A_m \frac{\exp(ik|\mathbf{r} - \mathbf{r}_m|)}{|\mathbf{r} - \mathbf{r}_m|}, \qquad (2.1)$$

and we seek the coefficients A_m from the condition that the logarithmic derivative of the function $\chi = |\mathbf{r} - \mathbf{r}_m|\psi(\mathbf{r})$, as $|\mathbf{r} - \mathbf{r}_m| \to 0$ and for arbitrary m, be equal to

$$\gamma_0 = ik\frac{S+1}{S-1} = \frac{1+ikf}{f}, \qquad (2.2)$$

where S is the usual scattering matrix and f the amplitude of scattering by an individual nucleus (see, for example, the paper by Brueckner [2], where a method similar to ours is used to analyze the scattering by two centers). As a result we arrive at the following system of algebraic equations

$$\sum_m g_{pm}A_m = -e^{i\mathbf{k}\mathbf{r}_p},$$

$$g_{pm} = \frac{\exp(ik|\mathbf{r}_p - \mathbf{r}_m|)}{|\mathbf{r}_p - \mathbf{r}_m|} \quad (m \neq p), \qquad g_{mm} = ik - \gamma_0. \tag{2.3}$$

For a sufficiently long chain, when the end effects can be neglected, the solution of the system (2.3) is written in the form (the z axis is directed along the chain)

$$A_m = Ae^{ik_z z_m}, \qquad z_m = am. \tag{2.4}$$

In the case of pure resonance scattering

$$f = -\frac{1}{2k}\frac{\Gamma_1}{E - E_0 + i\Gamma/2}.$$

Then, substituting (2.4) in (2.3), we obtain

$$A = -\frac{1}{2k}\frac{\Gamma_1}{E - E_0 + i\Gamma/2 + R},$$

$$R = \frac{\Gamma_1}{2k}\sum_{m\neq p} g_{mp}e^{ik_z(z_p - z_m)}. \tag{2.5}$$

The series in (2.5) can be summed (see [3]), and as a result we arrive at the following expressions for the change in the elastic width of the level $\Delta\Gamma$ and for the level shift ΔE:

$$\Delta\Gamma = 2\,\mathrm{Im}\,R = \frac{\pi\Gamma_1}{ak}\left(1 - \left\{\frac{a(k+k_z)}{2\pi}\right\} - \left\{\frac{a(k-k_z)}{2\pi}\right\}\right),$$

$$\Delta E_0 = -\,\mathrm{Re}\,R$$

$$= \frac{\Gamma_1}{ak}\ln\left[2\left|\sin\left(\frac{a(k+k_z)}{2}\right)\sin\left(\frac{a(k-k_z)}{2}\right)\right|\right]. \tag{2.6}$$

Here $\{x\}$ is the fractional part of x.

We call attention first to the fact that when $ak \ll 1$ the elastic part of the width increases rapidly, roughly in proportion to the number of nuclei in a segment equal to the wavelength. When $ak \gtrsim 1$, the quantity $\Delta\Gamma$ reverses sign and then begins to execute discontinuous oscillations about its zero value, both when ak increases and the angle of inclination of the incident beam to the chain axis is fixed, and when ak is fixed and the angle increases. This can be clearly seen in Figs. 1 and 2, where for purposes of illustration we present a plot of $\Delta\Gamma/\Gamma_1$ against ak with $k_z = 0$ (Fig. 1) and against ak_z with ka = const.

Thus, the elastic part of the width $\Gamma_1' = \Gamma_1 + \Delta\Gamma$ can be either larger or smaller than the value Γ_1 corresponding to the isolated nucleus. This is the consequence of the fact that the excited state which is not localized along the chain decays with a probability that can either exceed or be smaller than $W_0 = \Gamma_1/\hbar$ (see Sec. 3).

Appreciable changes take place also in the position of the resonance level. As seen from

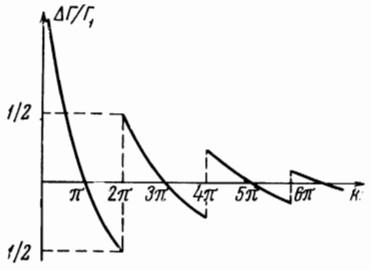

FIG. 1

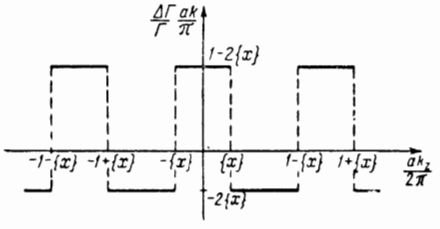

FIG. 2

(2.6), when $ak \ll 1$ the shift of level (towards lower energies) can be very large. With increasing ak, the shift decreases, and when $ak \gtrsim 1$ it can even reverse sign. It is interesting that at large values of ak the level will shift rapidly towards lower energies whenever the parameters are such that

$$(k \pm k_z)a \approx 2\pi p. \tag{2.7}$$

Formally, when (2.7) is exactly satisfied, the shift even becomes infinite. Really, however, any disturbances of the periodicity will limit the value of the shift (and will smooth the curves near the discontinuities of $\Delta\Gamma$—see Figs. 1 and 2).

It is easy to conclude from (2.1), (2.4), and (2.5) that the energy dependence of the scattering intensity is determined entirely by the value of $|A|^2$. It follows therefore from our results that in scattering by such a system the resonance parameters pertaining to each individual nucleus, and with them also the energy dependence of the scattering intensity as a whole, can really experience noticeable changes compared with the case of scattering by an isolated nucleus.

It is interesting that in this case the relation between the intensities of the inelastic and elastic channels changes, a change in either direction being possible. These results can be readily traced by considering a finite chain and obtaining the corresponding expressions for the cross sections.

At large distances from such a chain, expression (2.1) takes the form

$$\psi(\mathbf{r}) \approx e^{i\mathbf{k}\mathbf{r}} + A\left(\sum_m \exp[i(k_z - k_z')z_m]\right)\frac{e^{ikr}}{r},$$

where k_z' is the component of the wave vector of the scattered particles along the chain. For the differential scattering cross section per nucleus we have here, taking (2.5) into account,

$$d\sigma_1 = \frac{1}{4k^2} \times \frac{\Gamma_1^2}{(E-E_0')^2+\Gamma'^2/4}\frac{1}{N}\left|\sum_m \exp[i(k_z-k_z')z_m]\right|^2 d\Omega_{k'}$$
$$E_0' = E_0 + \Delta E_0, \quad \Gamma' = \Gamma_2 + \Gamma_1 + \Delta\Gamma \qquad (2.8)$$

(N is the total number of particles in the system).

We shall use the fact that $d\Omega_{k'} = 2\pi k^{-1} dk_z'$, and integrate (2.8) in explicit form. Recognizing that the contribution to the integral is made only by narrow intervals with respect to k_z' near the points

$$k_z' = k_z + 2\pi p / a \qquad (2.9)$$

(including p = 0, $k_z' \leq k$), we obtain after suitable analysis

$$\sigma_1 = \frac{\pi}{k^2}\frac{\Gamma_1\Gamma_1'}{(E-E_0')^2+(\Gamma'/2)^2}. \qquad (2.10)$$

The cross section of the inelastic process will be determined by the formula (2.10) with replacement of Γ_1' by Γ_2. Thus,

$$\sigma_1 / \sigma_2 = \Gamma_1' / \Gamma_2 \qquad (2.11)$$

and this ratio can actually be either larger or smaller than the corresponding value for the individual nucleus.

We note that for large wavelengths of the incident particles the periodicity is practically insignificant. However, when ka > 1 all the singularities of the behavior of $\Delta\Gamma$ and ΔE are due entirely to translational symmetry, and this is most clearly manifest in the occurrence of the condition (2.7).

3. TWO-DIMENSIONAL LATTICE

In analyzing the scattering by a two-dimensional lattice, we have for the wave function of the particles the same general expression (2.1), but now only the index m numbers the points of the two-dimensional lattice. Taking the symmetry of the problem into account, we can represent A_m in the form

$$A_m = A \exp(i\mathbf{k}_\parallel \boldsymbol{\rho}_m), \qquad (3.1)$$

where $\boldsymbol{\rho}_m$ is the radius vector of the point, defined in the lattice plane, and $\mathbf{k}_\parallel$ is the projection of k on this plane. We could determine A by the same procedure as in the preceding section, but then the resultant two-dimensional series would be too complicated to analyze. We therefore use here a different method.

Let us consider the value of the wave function (2.1) on a plane coinciding with the crystal plane. Taking (3.1) into account, we have

$$\psi(\boldsymbol{\rho}) = e^{i\mathbf{k}_\parallel\boldsymbol{\rho}} + Ae^{i\mathbf{k}_\parallel\boldsymbol{\rho}}\sum_m e^{i\mathbf{k}_\parallel(\boldsymbol{\rho}_m-\boldsymbol{\rho})}\frac{\exp(ik|\boldsymbol{\rho}-\boldsymbol{\rho}_m|)}{|\boldsymbol{\rho}-\boldsymbol{\rho}_m|}$$
$$\equiv e^{i\mathbf{k}_\parallel\boldsymbol{\rho}} + Ae^{i\mathbf{k}_\parallel\boldsymbol{\rho}}\Phi(\boldsymbol{\rho}). \qquad (3.2)$$

It is easy to see that the function $\Phi(\rho)$ is periodic and, as a result, it can be expanded in a two-dimensional Fourier series in terms of the vectors of the plane reciprocal lattice **b**:

$$\Phi(\boldsymbol{\rho}) = \sum_{\mathbf{K}} C_{\mathbf{K}} e^{i\mathbf{K}\boldsymbol{\rho}}, \qquad \mathbf{K} = 2\pi\mathbf{b}. \qquad (3.3)$$

Direct calculation yields for C_K the following expression

$$C_{\mathbf{K}} = \frac{2\pi i}{S_0}(k^2 - (\mathbf{k}_\parallel + \mathbf{K})^2)^{-1/2}, \qquad (3.4)$$

where S_0 is the area of the unit cell. (The right side of (3.4) is defined for all **K** in such a way that each coefficient C_K is either pure imaginary or pure real.)

The series (3.3), when account is taken of (3.4), diverges when $\boldsymbol{\rho} \to \boldsymbol{\rho}_m$, this being a direct consequence of the singularities of the function (3.2). It is natural therefore that a finite difference remains:

$$\left(\psi(\boldsymbol{\rho}) - A_m\frac{1}{|\boldsymbol{\rho}-\boldsymbol{\rho}_m|}\right)_{\boldsymbol{\rho}\to\boldsymbol{\rho}_m} = e^{i\mathbf{k}_\parallel\boldsymbol{\rho}_m}(1+A\xi),$$
$$\xi = \frac{2\pi}{S_0}\left[i\sum_{\mathbf{K}}(k^2-(\mathbf{k}_\parallel+\mathbf{K})^2)^{-1/2} - \frac{S_0}{(2\pi)^2}\int\frac{d^2\varkappa}{\varkappa}\right]. \qquad (3.5)$$

The second term in the square brackets is the result of the expansion of $1/|\rho - \rho_m|$ in a two-dimensional Fourier integral.

We know that as $\boldsymbol{\rho} \to \boldsymbol{\rho}_m$ we get in the general case $\psi = b_0 + b_1/|\boldsymbol{\rho} - \boldsymbol{\rho}_m|$, and consequently the right side of (3.5) simply coincides with b_0. Then, recognizing that $\gamma_0 = b_0/b_1$, we get

$$A = (\gamma_0 - \xi)^{-1}. \qquad (3.6)$$

Substituting in (3.6) the relation (2.2), we arrive at (2.5) with the following expression for R:

$$R = {}^1/_2\Gamma_1(\xi/k - i).$$

For the change in the width and for the shift we get, with account of (3.5), respectively,

$$\Delta\Gamma = \Gamma_1\left[\frac{2\pi}{kS_0}\sum_{\mathbf{K}}{}' \frac{1}{\sqrt{k^2-(\mathbf{k}_\parallel+\mathbf{K})^2}} - 1\right], \qquad (3.7')$$

$$\Delta E = -\frac{\Gamma_1\pi}{kS_0}\left\{\sum_{\mathbf{K}}{}' \frac{1}{\sqrt{(\mathbf{k}_\parallel+\mathbf{K})^2-k^2}} - \frac{S_0}{(2\pi)^2}\int\frac{d^2\varkappa}{\varkappa}\right\}. \qquad (3.7'')$$

The prime at the summation sign denotes that the summation is only over those **K** which correspond to a positive radicand.

The total elastic width Γ_1' is determined simply by the first term in (3.7′) and again, as in the case of the one-dimensional chain, Γ_1' can be either larger or smaller than Γ_1. When $ka \ll 1$, the elastic width behaves like $\Gamma_1' \approx 2\pi(ka)^{-2}\Gamma_1$, that is, it increases in proportion to the number of nuclei an a surface element whose linear dimension is of the order of the wavelength. When $ka > \pi$, the value of $\Delta\Gamma$ begins to oscillate as a function of ka for a fixed direction of the vector k, and as a function of the direction of this vector for a fixed value of ka. It is interesting that when $k > 2\pi/a$ (we have in mind a simple square lattice) we encounter directions of k which lead to very large values of $\Delta\Gamma$. The corresponding condition is of the form

$$k^2 - (\mathbf{k}_\parallel + \mathbf{K})^2 \approx 0. \tag{3.8}$$

It will be shown in the next section that the described behavior of Γ_1' is in complete correlation with the probability of the decay of the collective excitation in the plane lattice. Thus, the ratio of the intensity of the elastic and inelastic channels again changes, and this ratio can again be either larger or smaller than the corresponding ratio for the isolated nucleus. All the changes become particularly strong when the condition (3.8) is approached; if the left side of (3.8) is larger than zero, then the width increases rapidly, and if it is smaller than zero, then a strong shift takes place towards lower energies. Although an account of the coherent factors does smear out the singularity itself, in all probability the sharp change of the resonance parameters still remains.

We see from (3.7″) that the shift itself also behaves in a puzzling manner with change in **k** and, in particular, reverses sign. We shall not analyze this expression in detail, merely noting that the divergence of the first term in (3.7′) at large values of **K** is strongly compensated for by the second term.

Let us consider a bounded plane lattice and let us determine the scattering cross section in analogy with the procedure used for the one-dimensional chain. It is easy to show that

$$d\sigma_1 = |A|^2 \frac{1}{N}\left|\sum_m \exp[i(\mathbf{k}_\parallel - \mathbf{k}_\parallel')\rho_m]\right|^2 d\Omega_{\mathbf{k}'}.$$

Integration of this expression yields for the cross section, a value that coincides with (2.10), where the elastic part of the width $\Gamma_1' = \Gamma_1 + \Gamma_1$ is determined by formula (3.7′). Obviously, relation (2.11) likewise remains in force here, and as a consequence the ratio of the elastic and inelastic cross sections can change quite strongly simply with change in the angle of inclination of the incident beam ($ka > 2\pi$).

4. LIFETIME OF COLLECTIVE EXCITED STATES

All the results of the preceding sections, connected with the change in the width and the ratio of the elastic and inelastic scattering cross sections, become quite lucid if one takes into consideration the collective character of the resultant intermediate state. Indeed, if we denote by φ_m the state in which the m-th nucleus is excited and all others are in the normal state, then the wave function of the intermediate state should be written, for either a chain and or a plane, in the form

$$\psi = \frac{1}{\sqrt{N}}\sum_m e^{i\mathbf{k}\mathbf{r}_m}\varphi_m. \tag{4.1}$$

Let us consider the decay of such a state, accompanied by emission of a primary particle. The system of nuclei returns in this case to the initial unexcited state, and we readily obtain for the probability of such a decay

$$W_1 = \frac{2\pi}{\hbar N}\int |M_1^2|\left|\sum_m \exp[i(\mathbf{k}-\mathbf{k}')\mathbf{r}_m]\right|^2 \times \delta(E_0 - E_{\mathbf{k}'})\frac{d^3k'}{(2\pi)^3}. \tag{4.2}$$

Here M_1 is the matrix element corresponding to the transition of the individual nucleus from the excited state into the normal state with emission of a primary particle.

We consider first a one-dimensional chain. We represent the phase volume in the form

$$d^3k' = 2\pi k'\frac{dk'}{dE_{\mathbf{k}'}}dE_{\mathbf{k}'}\,dk_z'.$$

Integration with respect to $dE_{\mathbf{K}'}$ is trivial, and the calculation of (4.2) reduces to the integral

$$\int_{-h}^{h}\frac{\sin^2[(k_z - k_z')aN/2]}{\sin^2[(k_z - k_z')a/2]}dk_z' = \frac{2\pi}{a}NF,$$

where F is the number of different values of p (including p = 0) in (2.9) admitted by the limits of integration:

$$F = \left[\frac{(k+k_z)a}{2\pi}\right] + \left[\frac{(k-k_z)a}{2\pi}\right] + 1 = \frac{ka}{\pi} + 1 - \left\{\frac{(k+k_z)a}{2\pi}\right\} - \left\{\frac{(k-k_z)a}{2\pi}\right\}$$

([x] is the integer part of x). As a result we get

$$W_1 = W_1^0(\pi/ak)F, \tag{4.3}$$

where W_1^0 is the decay probability, corresponding to the isolated nucleus.

Analyzing formula (2.6), we can readily establish that

$$\hbar W_1 = \Gamma_1' = \Gamma_1 + \Delta\Gamma_1 \quad (\hbar W_1^0 = \Gamma_1). \tag{4.4}$$

In the case of a plane lattice the phase-volume element will be transformed into

$$d^3k' = \frac{2}{(k'^2 - k_\parallel'^2)^{1/2}} k' \frac{dk'}{dE_{\mathbf{k}'}} dE_{\mathbf{k}'} d^2k_\parallel'.$$

here

$$\int \Big| \sum_m \exp[i(\mathbf{k}_\parallel - \mathbf{k}_\parallel')\mathbf{r}_m] \Big|^2 \frac{d^2k_\parallel'}{(k'^2 - k_\parallel'^2)^{1/2}}$$
$$= \frac{4\pi^2}{S_0} N \sum_{\mathbf{K}}' \frac{1}{[k^2 - (\mathbf{k}_\parallel + \mathbf{K})^2]^{1/2}}$$

and we obtain for the decay probability

$$W_1 = W_1^0 \frac{2\pi}{S_0 k} \sum_{\mathbf{K}}' \frac{1}{[k^2 - (\mathbf{k}_\parallel + \mathbf{K})^2]^{1/2}}. \tag{4.5}$$

Comparing this result with (3.7′), we again arrive at relation (4.4).

Thus, the change in the width obtained in the analysis of the resonant scattering turns out to be uniquely related with the lifetime of the resultant collective excited state (4.1). This lifetime can be either smaller or larger than the lifetime of the excited state of the individual nucleus, and differs noticeably in magnitude (cf. [1]).

It is easy to verify that the probability of inelastic decay of the state (4.1), in which fixation of the nucleus that experiences the transformation unavoidably takes place, remains exactly the same as in the case of the decay of an isolated excited nucleus. This is what determines the occurrence of relation (2.11).

5. THREE-DIMENSIONAL CRYSTAL

The picture of resonant interaction in a three-dimensional crystal differs essentially from the cases considered above. This is connected decisively with the fact that now the amplitude of the primary wave will have different values at nuclei of different crystal planes, decreasing along $\mathbf{k}$. This raises great difficulties in determining the explicit form of the solution, especially if the crystal dimension l is comparable with the thickness of the transition layer, that is, if

$$l|f| / V_0 k \lesssim 1$$

(V_0 is the volume of the unit cell).

In the opposite limiting case $l\,|\,f\,|/V_0 k \gg 1$, however, the problem simplifies because, on the one hand, we can ignore the finite dimensions of the crystal, and on the other we need take into account the incident wave in fact only in the boundary condition. We confine ourselves to an examination of this last case only.

Let us return to the general expression (2.1). We seek the coefficient in the form

$$A_m = A e^{i\boldsymbol{\varkappa}\mathbf{r}_m}, \tag{5.1}$$

where $\boldsymbol{\varkappa}$ is a complex vector. Then we get, after identical transformations

$$\psi(\mathbf{r}) = e^{i\mathbf{k}\mathbf{r}} + 2iA \sum_m e^{i\boldsymbol{\varkappa}\mathbf{r}_m} \frac{\sin k|\mathbf{r} - \mathbf{r}_m|}{|\mathbf{r} - \mathbf{r}_m|}$$
$$+ A \sum_m e^{i\boldsymbol{\varkappa}\mathbf{r}_m} \frac{\exp(-ik|\mathbf{r} - \mathbf{r}_m|)}{|\mathbf{r} - \mathbf{r}_m|}. \tag{5.2}$$

We shall use the relation

$$\frac{\sin k|\mathbf{r} - \mathbf{r}_m|}{|\mathbf{r} - \mathbf{r}_m|} = \frac{k}{4\pi} \int \exp[ik'(\mathbf{r} - \mathbf{r}_m)]\, d\Omega_{\mathbf{k}'},$$

the validity of which can be readily verified directly. The second term in (5.2) is then written as

$$A \frac{ik}{2\pi} \int e^{i\mathbf{k}'\mathbf{r}} \Big(\sum_m \exp[i(\boldsymbol{\varkappa} - \mathbf{k}')\mathbf{r}_m] \Big) d\Omega_{\mathbf{k}'}. \tag{5.3}$$

We represent the vector $\boldsymbol{\varkappa}$ in the form

$$\boldsymbol{\varkappa} = \mathbf{k} + \mathbf{q} \tag{5.4}$$

and consider for simplicity the case when the flux is normally incident on the surface of the crystal, that is, $\mathbf{q} \parallel \mathbf{k}$. Since the interaction with each crystal plane is assumed to be weak ($|f| \ll a$), we get

$$|q| \ll 2\pi / a.$$

In this case, the sum in (5.3) can be replaced in standard fashion with an integral. As a result we find (it is assumed henceforth that there is no Bragg scattering)

$$\sum_m \exp[i(\boldsymbol{\varkappa} - \mathbf{k}')\mathbf{r}_m] \approx \frac{(2\pi)^2}{V_0} \delta(k_x') \delta(k_y') \frac{i}{k_z - k_z' + q} \tag{5.5}$$

(the c axis is directed along $\mathbf{k}$).

The presence of two δ-functions in (5.5), in conjunction with the condition $k' = k$, leads to the equality $k_z' = k_z$ and to $\mathbf{k}' = \mathbf{k}$. Then, substituting (5.5) in (5.3) and introducing the result in (5.2), we obtain ultimately

$$\psi(\mathbf{r}) = e^{i\mathbf{k}\mathbf{r}} - A \frac{2\pi}{V_0 k q} e^{i\mathbf{k}\mathbf{r}}$$
$$+ A e^{i\boldsymbol{\varkappa}\mathbf{r}} \Big(\sum_m e^{i\boldsymbol{\varkappa}(\mathbf{r}_m - \mathbf{r})} \frac{\exp(-ik|\mathbf{r} - \mathbf{r}_m|)}{|\mathbf{r} - \mathbf{r}_m|} \Big). \tag{5.6}$$

The expression in the brackets in the last term of (5.6) has translational symmetry. Owing

to the factor $\exp(i\kappa)$, the entire term as a whole continuously decreases with increasing crystal thickness ($\mathrm{Im}\, q > 0$). It is clear that in a sufficiently thick crystal the second term should cancel the first, that is,

$$A\frac{2\pi}{V_0 kq} = 1, \tag{5.7}$$

and

$$\psi(\mathbf{r}) = Ae^{i\varkappa\mathbf{r}}\left(\sum_m e^{i\varkappa(\mathbf{r}_m-\mathbf{r})}\frac{\exp(-ik|\mathbf{r}-\mathbf{r}_m|)}{|\mathbf{r}-\mathbf{r}_m|}\right) \equiv Ae^{i\varkappa\mathbf{r}}\Phi(\mathbf{r}). \tag{5.8}$$

Relation (5.7) determines in fact the constant A which figures in the solution (5.1). The condition (2.2) on each nucleus makes it possible to determine the equation for q by using (5.8).

The function $\Phi(\mathbf{r})$ is periodic, and we can expand it in a Fourier series in terms of the vectors of the three-dimensional reciprocal lattice. Direct calculations lead to the expression

$$\Phi(\mathbf{r}) = \frac{4\pi}{V_0}\sum_{\mathbf{K}}\frac{e^{i\mathbf{K}\mathbf{r}}}{(\varkappa+\mathbf{K})^2-k^2}. \tag{5.9}$$

By separating in (5.9), the singularity when $\mathbf{r} \to \mathbf{r}_m$, in exactly the same way as was done in the derivation of (3.5) and (3.6), we arrive at a relation which is in fact an equation for the determination of κ (see a similar analysis in [4]):

$$\frac{4\pi}{V_0}\left[\sum_{\mathbf{K}}\frac{1}{(\varkappa+\mathbf{K})^2-k^2} - \frac{V_0}{(2\pi)^3}\int\frac{d^3q}{q^2}\right] = \gamma_0. \tag{5.10}$$

Let the value of the wave vector of the incident particle be such that there is no Bragg scattering. Then the principal role is played in the left side of (5.10) by the term of the sum with $\mathbf{K} = 0$. Using the explicit form of (2.2), we get

$$\varkappa^2 - k^2 = -\frac{4\pi}{V_0}\frac{1}{2k}\frac{\Gamma_1}{E-E_0'+i\Gamma_2/2} \qquad E_0' = E_0 - \frac{\Gamma_1}{2k}D,$$

$$D = \frac{4\pi}{V_0}\left[\sum_{\mathbf{K}\neq 0}\frac{1}{(\mathbf{k}+\mathbf{K})^2-k^2} - \frac{V_0}{(2\pi)^3}\int\frac{d^3q}{q^2}\right]. \tag{5.11}$$

Assuming that $q/k \ll 1$, we have, using (5.4),

$$\mathrm{Im}\, q = \frac{\pi}{2V_0k^2}\frac{\Gamma_1\Gamma_2}{(E-E_0')^2+\Gamma_2^2/4}. \tag{5.12}$$

It follows from the form of (5.11) and (5.12) that in a crystal the resonance nuclear parameters can change noticeably. Indeed, the energy dependence of the intensity of the transmitted beam ceases to depend on the elastic width Γ_1 completely, and is determined entirely by Γ_2. The position of the resonance level also changes at the same time. As a consequence, in the classical relation

$$I = I_0 e^{-n\sigma_t l}$$

the total cross section σ_t must be replaced for a crystal by

$$\frac{\pi}{k^2}\frac{\Gamma_1\Gamma_2}{(E-E_0')^2+\Gamma_2^2/4}$$

that is, the total width Γ decreases to Γ_2.

Thus, the difference in the character of the resonant interaction in a regular crystal and in an arbitrary irregular system will become manifest already in traditional transmission experiments.

The vanishing of the elastic width can be clearly understood on physical grounds. Indeed, for this purpose it is sufficient to recall that the presence of translational symmetry in a nonabsorbing system causes the momentum (more accurately, quasimomentum) of the particles to be conserved regardless of the magnitude of their interaction with the particles forming the system.

Let us make in conclusion several general remarks concerning the applicability of the results. If the particles in the crystal experience incoherent scattering, then this should entail a certain decrease in the change of the width and the shift of the resonance level, proportional in absolute magnitude to the decrease in the elastic-scattering amplitude. Such incoherent scattering, in particular, appears if the oscillations of the nuclei become significant, that is, processes in which phonons are emitted or absorbed. The elastic-scattering amplitude (see, for example, [5,6]) turns out to be proportional in the case of narrow resonances ($\Gamma \ll \omega_0$, ω_0 = characteristic frequency of the phonon spectrum) to the probability of the Mossbauer effect, and in the case of broad resonances ($\Gamma \gg \omega_0$) it is proportional to the corresponding Debye-Waller factor. It is well known that for low-lying resonant levels of medium and heavy nuclei the probability of the Mossbauer effect and the value of the Debye-Waller factor are close to unity in many cases, not only at low temperatures but also at room temperatures. In all these cases the influence of the oscillations turns out to be weak. On the other hand, the following general tendency is also clear: all the effects should become weaker with rising temperature. In particular, in the three-dimensional case the two widths will have an intermediate value between Γ_2 and Γ, tending in the high-temperature limit to Γ. We note that the presence of incoherent scattering due to the presence of spin in the ground state of the nucleus, also leads, without changing the qualitative picture at all, to a certain increase of the width compared with Γ_2 (see [6]).

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35