RING GENERATOR IN A ROTATING REFERENCE SYSTEM

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An expression for the frequency shift of oppositely moving waves in a ring laser in a non-inertial reference system is derived by taking into account the motion of the dielectric medium filling the resonator.

A traveling-wave laser was used in several investigations[1,2] to study the dependence of the frequency shift of the opposing waves, which occurs in it, on the angular velocity of laser rotation, and also the Fizeau effect[3]. For exact measurements of this type, it is useful to carry out a rigorous analysis of these effects, taking into account the presence of a dielectric medium filling the laser resonator.. A similar analysis was carried out by Heer[4], but his final formulas are incorrect.

A rotating resonator can be naturally regarded in a reference frame in which the resonator is at rest. In such a reference frame there occurs a stationary gravitational field[5]. The nonzero components of the metric tensor, in first order in $\Omega r/c$ (Ω—angular velocity of rotation) are equal to

$$-g_{00}=g_{11}=g_{22}=g_{33}=1, \quad \mathbf{g}=\frac{1}{c}[\mathbf{\Omega r}], \qquad (1)^*$$

where we have introduced the symbol $g_\alpha = -g_{0\alpha}/g_{00}$. The spatial metrix is Euclidean in this approximation. Introducing, as usual[6], the co- and contravariant electromagnetic field tensors F_{ik} and H^{ik}, we write down for the connection between their components in the medium

$$g_{il}g_{km}H^{lm}u^k=\varepsilon F_{ik}u^k, \qquad \varepsilon_{iklm}g^{ks}g^{lt}F_{st}u^m=\mu\varepsilon_{iklm}H^{kl}u^m, \qquad (2)$$

where u^k is the four-velocity of the medium, and ϵ and μ are the dielectric constant and the magnetic permeability of the medium. Taking (1) into account, Eqs. (2) together with Maxwell's equations for a plane wave $\exp(i\mathbf{k}\cdot\mathbf{r}-i\omega t)$ form the following system:

$$\mathbf{D}=\varepsilon\mathbf{E}+[\mathbf{HG}], \qquad \mathbf{B}=\mu\mathbf{H}+[\mathbf{GE}],$$

$$\mathbf{D}=[\mathbf{Hn}], \qquad \mathbf{B}=[\mathbf{nE}],$$
$$\mathbf{G}=\mathbf{g}-\frac{\varepsilon\mu-1}{c}\mathbf{v}, \qquad \mathbf{n}=\frac{c}{\omega}\mathbf{k}. \qquad (3)$$

Eliminating **E**, **H**, and **B** from these equations we obtain an equation for the "refractive index" $n = |\mathbf{n}|$ of the moving medium in the gravitational field

$$\mathbf{D}\left[\left(1+\frac{\mathbf{nG}}{n_0^2}\right)-\frac{n^2}{n_0^2}\left(1+\frac{\mathbf{nG}}{n_0^2}\right)^{-1}\right]=0. \qquad (4)$$

From this we get

$$n=n_0+\mathbf{n}_0\mathbf{G}/n_0, \qquad n_0^2=\varepsilon\mu. \qquad (5)$$

The frequency shift of the opposing waves in the ring laser is obtained by equating the changes in the phase occurring when these waves go around the closed loop of a resonator of length L:

$$\oint_+\left(n_0+\frac{\mathbf{n}_0\mathbf{G}}{n_0}\right)dr=\oint_-\left(n_0+\frac{\mathbf{n}_0\mathbf{G}}{n_0}\right)dr. \qquad (6)$$

Assuming that a section of the resonator of length l is filled with a dielectric moving with velocity **v** and another section of length a is filled with an active medium with refractive index n_a, and taking into account the dispersion of n_0 and n_a, we obtain for the frequency shift the expression

$$\frac{\Delta\omega}{\omega}=\left[4\frac{\mathbf{\Omega S}}{c}-2l\left(n_0-1-\omega n_0\frac{dn_0}{d\omega}\right)\frac{\mathbf{Vv}}{c}\right]$$
$$\times\left[L+l(n_0-1)+\omega l\frac{dn_0}{d\omega}+a\omega\frac{dn_a}{d\omega}\right]^{-1},$$

$$\mathbf{S}=\frac{1}{2}\oint_+[\mathbf{r}\,d\mathbf{r}]; \qquad (7)$$

where **V** is a unit vector in the direction of propagation of the light.

The expression in the denominator of (7) is the total optical length of the resonator. Allowance for the dispersion in dielectrics yields a contribution of the order of 10^{-3}–10^{-4} to the main shift, owing to the smallness of the ratio l/L. In spite of the closeness of the refractive index of the active (gas) medium to unity, the corresponding dispersion term must be retained, owing to the anomalous

*$[\Omega\mathbf{r}]\equiv\Omega\times\mathbf{r}$.

dispersion in the region of laser action. This term has an order of magnitude[7] $\omega a dn_\alpha/d\omega = L\Delta\omega_s/\Delta\omega_{sup}$, where $\Delta\omega_s$ and $\Delta\omega_{sup}$ are the respective widths of the resonator band and of the spectral line of the active transition (Doppler curve). Because of the mode interaction, the magnitude of this term depends on the disposition of the modes relative to the Doppler maximum.

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