DISINTEGRATION OF ONSAGER–FEYNMAN VORTICES AND COLLECTIVIZATION OF VORTEX OSCILLATIONS

G. A. GAMTSEMLIDZE, Sh. A. DZHAPARIDZE, and K. A. TURKADZE

Tbilisi State University

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We investigate the disintegration of vortices following the stopping of rotating helium II in which an oscillating disc is immersed. Two values of the vortex half-lives τ are obtained, depending on the angular velocity of the helium as it slows down after the vessel is stopped. The presence of two values of τ is attributed to collectivization of the vortices.

1. THE purpose of this investigation was to measure the half-life of the vortices produced in rotating helium II after the vessel stopped rotating. By measuring this quantity at different angular velocities ω_0 (up to stopping) we observed that the decay of the vortices occurs at different rates, depending on whether the oscillations of the vortex filaments are collectivized or not. We recall[1,2] that the collectivization of the vortex oscillations occurs when the vortex density exceeds a value at which the effective radius of the vortex-filament oscillation is of the order of half the distance between them. The corresponding angular velocity $\tilde{\omega}_0$ is of the order of $\tilde{\omega}_0 \approx 0.1\,\Omega$, where Ω is the angular frequency of the oscillations of the vortices, which in our case coincides with the frequency of the oscillations of the disc $\Omega = 0.47 \pm 0.01\ \text{sec}^{-1}$.

2. The measurement procedure consisted in the following: a container with liquid helium[1)] was rotated with angular velocity ω_0. After establishing a stationary rotation mode (after 30 min) the container was stopped by switching off the electric motor.

Up to the instant of stopping of the rotating helium II, the number of vortices per square centimeter is given by the well known formula[4]

$$n = m\omega_0 / \pi\hbar, \tag{1}$$

where m is the mass of the helium atom. It is known that these vortices cause additional damping of the oscillations of the disc. It is therefore clear that the logarithmic damping decrement of the oscillating disc will be larger immediately after the liquid has stopped than when the liquid is fully at rest. Naturally, the excess damping will decrease with time, owing to the decay of the vortices. Thus, knowing the time dependence of the additional damping of the oscillating disc, we can determine the half-life of the vortices. The additional damping was calculated as the difference $\Delta\alpha = \alpha - \alpha_0$, the damping of the disc in stationary helium II, and α, the damping at a certain instant of time after stopping of the container ($\alpha \to \alpha_0$ as $t \to \infty$). The value of α was determined from the slope of the $\ln A = f(\nu)$ curve, where A is the total swing of the pointer on the scale, $\nu = t/\theta$ is the number of disc oscillations, θ the period of the disc oscillations, and t the observation time reckoned from the instant of stopping of the container with the helium II.

3. Figure 1 shows the time dependence of the logarithmic decrement of the damping α. These curves were plotted for different angular velocities at 1.46° K. An analysis of these curves shows (see Fig. 2) that they can be described by an experimental formula

$$\Delta\alpha = (\Delta\alpha)_0 e^{-\lambda t}, \tag{2}$$

where $\Delta\alpha$ and $(\Delta\alpha)_0$ are the additional vortex

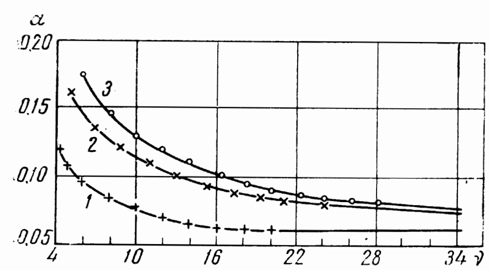

FIG. 1. Time dependence of the damping α at different velocities ω_0($\nu = t/\theta$, $\theta = 13$ sec): curve 1 – $\omega_0 = 0.100 \pm 0.002\ \text{sec}^{-1}$; curve 2 – $\omega_0 = 0.20 \pm 0.01\ \text{sec}^{-1}$; curve 3 – $\omega_0 = 0.48 \pm 0.02\ \text{sec}^{-1}$; T = 1.46°K.

1)The experiment was carried out with the setup described in[3].

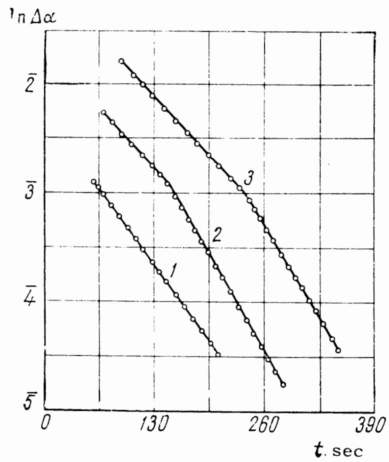

FIG. 2. Dependence of the logarithm of the additional damping on the time for T = 1.46°K. Curve 1 – ω_0 = 0.100 ± 0.002 sec^{-1}; curve 2 – ω_0 = 0.20 ± 0.01 sec^{-1}; curve 3 – ω_0 = 0.48 ± 0.02 sec^{-1}.

damping at the instants of time t and t = 0, while $\lambda = d \ln \alpha/dt$ is the slope of the curves shown in Fig. 2.

If we recognize that the vortex damping at a given temperature depends essentially on the number of vortices, then Eq. (2) describes the decay of the vortices, with half-life $\tau = 0.69/\lambda$.

4. We can separate on curves 2 and 3 of Fig. 2 two straight-line sections with different slopes λ. This indicates that the decay of the vortices after the vessel is stopped at different instants of time occurs with different periods τ. For example, at a velocity 0.24 sec^{-1} (curve 2) the decay has a half-life $\tau_1 = 70 \pm 5$ sec at a time $t \leq 140$ sec after the start of the deceleration of the liquid and $\tau_2 = 55 \pm 5$ sec after 140 sec. A similar picture is observed for the 0.48 sec^{-1} velocity (curve 3 in Fig. 2) when the change in the half-life occurs at t = 250 sec.

Curve 1 corresponds to decay at a velocity $\omega_0 = 0.10 \pm 0.02$ sec^{-1}; in this case the decay has a single half-life 55 ± 5 sec.

The decrease of the half-life after a definite time $t = t_0$ can be attributed only to a cessation of the collectivization of the vortex oscillations. Indeed, after stopping the vessel, the effective speed of rotation of the liquid decreases with time like $\omega_{0t} = \omega_0 e^{-\lambda t}$, and at some instant of time t_0 it should drop to a value $\tilde{\omega}_0$ when the effective radii of the vortex oscillation become smaller than the half-distance between the vortices, that is, the vortices begin to vibrate independently of one another. The experimental data presented show that if we use for t_0 the value of the time corresponding to the kinks on the curves 2 and 3, then $\omega_0 e^{-\lambda t_0}$ yields values of $\tilde{\omega}_0$ corresponding to the relation $\tilde{\omega}_0/\Omega = 0.10 - 0.01$ which is in splendid agreement with theory.

We note that after the container is stopped, the disc oscillates for some time, depending on the initial velocity of the liquid, with an amplitude characteristic of the transcritical mode in which the disc itself can produce vortices.

To exclude the influence of the disc on the decay of the vortices produced during the preceding rotation, we plotted the amplitude dependence of the damping decrement of the disc in the moving liquid. For the critical amplitude φ_c we obtained a value 0.12 ± 0.01 rad. The curves shown in Fig. 1 and 2 were plotted starting with that instant of time, at which the disc started to oscillate with amplitude lower than the critical φ_c.

The lack of a kink on the curve 1 (Fig. 2) indicates that the vortex oscillations were no longer collective at the instant of establishment of the pre-critical disc oscillations. Indeed, an estimate shows that this cessation should occur at t = 70 ± 7 sec which indeed corresponds to the initial point of the curve 1.

Similar measurements, but by a different procedure (second sound) were made by Bablidze.[5] The value $\tau = 30$ sec obtained by him is somewhat lower than ours (55 sec) possibly because of the presence of an oscillating disc in our installation.

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44