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RADIATION FLUCTUATIONS IN A GAS LASER

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Low-frequency fluctuations in the radiation from a gas laser are considered; we introduce a quantity called the depth of modulation, which characterizes the variation in radiation intensity. The dependence of the spectral density of this quantity on frequency and power in the single-mode regime are discussed. We present the results of measurements in a helium-neon laser with a Fabry-Perot resonator and in a ring laser. The ring laser exhibits a negative correlation between the intensity fluctuations in the opposed waves. The depth of modulation associated with optical oscillations at combination frequencies is determined. The experiments described here were carried out at a wavelength of 0.63 microns.

THE fluctuations in the radiation intensity of the helium-neon laser have been discussed by a number of authors[1-8] and measurements of the low-frequency fluctuations in the current from photoelectric detectors of the laser radiation have been described. Nonetheless, the problem still requires further theoretical and experimental development.

In the present work we consider certain fundamental aspects of this problem and present a number of results concerning measurements of the radiation fluctuations (also low-frequency) in a helium-neon laser operating with a conventional Fabry-Perot system and in a ring laser.

1. GENERAL REMARKS CONCERNING RADIATION FLUCTUATIONS IN A LASER

Fluctuations in the intensity of laser radiation can be studied by a demodulation method based on the low-inertia properties of the conversion of light into photoelectric current.[1)] In analyzing photocurrent one usually encounters fluctuations in the photocurrent due to the shot effect as well as those due to the variation in the level of optical radiation. In observation of fluctuations of the photocurrent from light produced by thermal or luminescent sources one usually only observes a shot effect which is characterized by a uniform spectral density (at frequency F) of the photocurrent $\overline{i_F^2} = 2eI$, where I is the constant component of the current and e is the charge of the electron.[2)] In the study of laser radiation primary interest attaches to the so-called excess fluctuations in the photocurrent; the corresponding spectral density $\overline{i_{eF}^2}$ is superimposed on the usual level 2eI, that is to say, $\overline{i_{eF}^2} = \overline{i_F^2} - 2eI$.

To characterize the fluctuations in the radiation intensity of a laser it is convenient to introduce a coefficient that defines the depth of random modulation of the photocurrent; this quantity is defined by the expression $M(t) = \sqrt{2}\, i(t)/I$ where i(t) is the fluctuation in photocurrent at time t. Experimentally one determines the spectral density of this coefficient which is denoted by $\overline{M_F^2} = 2\overline{i_F^2}/I^2$; the mean square of the depth of modulation of the photocurrent is then given by

$$\overline{M^2} = \int_0 \overline{M_F{}^2}\, dF.$$

When a photodetector is illuminated by ordinary light, because of the shot effect we find $\overline{M_{0F}^2} = 2 \cdot 2eI/I^2 = 4e/I$. In the case of laser light, however, the difference between the quantities $\overline{M_F^2}$ and $\overline{M_{0F}^2}$ characterizes the relative level of the fluctuations in the intensity of the radiation. Obviously, the quantity $\overline{M_{eF}^2} = \overline{M_F^2} - \overline{M_{0F}^2}$ can be observed conveniently only if $\overline{M_{eF}^2} \gtrsim \overline{M_{0F}^2}$. In order to simplify the analysis of the fluctuations the

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[1)] The basic ideas concerning the low-inertia properties of photoelectric measurement and demodulation analysis of light were indicated generally by Gorelik in 1947.[9] Somewhat later, independent work was reported by Forrester et al.[10] concerning heterodyning of two optical lines.

[2)] When a photomultiplier is used one discusses the primary photocurrent. The expression given here for $\overline{i_F{}^2}$ holds for a frequency F for which the electron transit time can be neglected.

value $\overline{M_{0F}^2} = 4e/I$ can be regarded as a threshold value for observing the fluctuations in laser radiation. This threshold can be understood as the increase in I that requires the largest possible quantum yield of the photodetector and the complete use of the output power of the laser.[3] The question of the noise developed in the apparatus following the photodetector can be treated easily; in observation of low-frequency fluctuations the noise contributed by this factor can usually be neglected.

It should be noted that in the work cited above[1-6, 8] the fluctuation level in the laser radiation is characterized by the quantity $N_F = \overline{i_{eF}^2}/2eI$. However, N_F does not completely characterize the fluctuations. It is evident that the following relation holds between $\overline{M_{eF}^2}$ and N_F:

$$\overline{M_{eF}}^2 = 4eN_F/I. \tag{1}$$

Consequently, for a given fluctuation level N_F is proportional to the constant component of the photocurrent I which, in particular, depends on the parameters of the photodetector. For this reason N_F is not a reliable quantity for characterizing the fluctuations in laser radiation.

It is well known that gas lasers can operate stably in several modes simultaneously, the frequencies of these modes lying within the limits of a given spectral line. If fluctuations are neglected the photocurrent spectrum will contain discrete components at frequencies equal to the difference frequencies of the individual modes. The presence of fluctuations tends to smear these discrete components, including the component at zero frequency. As noted above, we shall only consider the low-frequency spectrum in the intensity fluctuations.

We describe the laser oscillations by the expression

$$\sum_{k=1}^{n}[A_k + a_k(t)]\cos[\omega_k t + \varphi_k(t)], \tag{2}$$

where $a_k(t)$ and $\varphi_k(t)$ are the fluctuations in amplitude and phase of the k-th mode. It is assumed that these fluctuations vary relatively slowly and that $a_k \ll A_k$. Introducing the quantity $m_k(t) = \sqrt{2}\, a_k(t)/A_k$, which is defined as the depth of random amplitude modulation of the k-th mode,[4] it is easy to find an expression for the depth of random modulation of the photocurrent $M_e(t)$ due to the low-frequency fluctuations in the radiation:

$$M_e(t) = 2\sum_{k=1} m_k(t)A_k^2 \Big/ \sum_{k=1} A_k^2. \tag{3}$$

In single-mode oscillation $M_e(t) = 2m(t)$, $\overline{M_e^2} = 4m^2$, and $\overline{M_{eF}^2} = 4\overline{m_F^2}$; in the general case the quantity $\overline{M_e^2}$ depends on the degree of correlation between the individual $m_k(t)$:

$$\overline{M_e^2} = 4\sum_{k,\,j=1}^{n} \overline{m_k m_j}\, A_k^2 A_j^2 \Big/ \Big[\sum_{k=1}^{n} A_k^2\Big]^2. \tag{4}$$

In particular, in the absence of correlation between fluctuations in the individual modes we find

$$\overline{M_e^2} = 4\sum_{k=1} \overline{m_k^2}\, A_k^4 \Big/ \Big[\sum_{k=1}^{n} A_k^2\Big]^2 \tag{5}$$

and similar relations between $\overline{M_{eF}^2}$ and $\overline{m_{kF}^2}$.

Let us consider the mechanisms which can give rise to fluctuations in the radiation from a gas laser. It is evident that mechanical vibrations of the system can make a contribution; time-varying inhomogeneities in the radiation path in the resonator (in lasers with external mirrors these are due to air bubbles and so on) which lead to the scattering or refraction of the light beam. The method for dealing with these effects is obvious. Another possible mechanism is due to the fluctuation in the gas discharge itself. It is well known that the parameters of a plasma are not usually stable: one frequently encounters low-frequency phenomena in the discharge. The effect of plasma fluctuations on laser radiation can be observed easily when the discharge is excited by direct current. In this case one must examine the correlation between the fluctuations in discharge current and the fluctuations in photocurrent due to the laser radiation. A suitable method has been used in [4] and a correlation of this kind has in fact been observed.

These sources of radiation fluctuations might be called technical causes(we use the terminology adopted in electronics[12]); the analysis of these effects is not of fundamental physical interest. It is usually possible to choose a mode of operation of the discharge in which the correlations between the low-frequency fluctuations in discharge current and photocurrent disappear. In [6] data is reported indicating that even the photocurrent due to the incoherent emission from the discharge (detected perpendicularly to the axis of the discharge tube) exhibits an excess fluctuation level which is many times greater than the standard level 2eI. This

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[3] It is evident that the application of a modulation method of detection with sufficiently long averaging time can be used to find $\overline{M_{eF}} \ll \overline{M_{0F}^2}$.

[4] A similar quantity was introduced by Bershtein[11] to characterize the fluctuations in amplitude of a vacuum-tube oscillator and has received wide usage in other applications.

result can be attributed directly to fluctuations in the discharge. Tests on our own apparatus have shown (in the absence of the correlation noted above) the presence of only the usual spectral density of fluctuations $2eI$ in the spontaneous emission from the discharge.

Another mechanism that can cause the emission to vary arises when more than two modes are excited simultaneously. If the frequencies of the modes are not equally spaced, the nonlinear properties of the active medium give rise to additional fluctuations at combination frequencies;[13] as a result the low-frequency photocurrent spectrum exhibits components at frequencies determined by the deviation from equal spacing of the optical frequencies. Typically the frequencies of these oscillations range from kilocycles to tens of kilocycles. Small uncontrolled changes in resonator geometry (or mode of operation of the discharge) lead to relatively large changes in these frequencies and corresponding oscillations in photocurrent.

Strictly speaking, oscillations in the photocurrent that appear because the modes are not equally spaced cannot be called fluctuations; the usual statistical features do not apply. The level of these oscillations in photocurrent can be characterized by the depth of modulation of the photo current to which they give rise.

The various mechanisms indicated above as being responsible for low-frequency oscillations in the photocurrent are not regarded as fundamental; we shall be interested in the basic cause of fluctuations in laser radiation, which might be called "inherent" mechanisms (again we use the terminology adopted in electronics). The basic mechanism is evidently the spontaneous emission of excited atoms in the active medium.

We shall neglect technical sources of fluctuations and assume that the spectrum of perturbations of the laser oscillations is uniform (at least within the frequency range of interest F); in this case, for single-mode oscillation with amplitude A the dependence of the quantity $\overline{m_F^2}$ on F can be expected to exhibit the same features as in an electronic oscillator:[11]

$$\overline{m_F^2} = B / [(p/2\pi)^2 + F^2]A^2, \tag{6}$$

where B is a quantity that characterizes the level of the perturbing forces while p characterizes the stability of the limit cycle of the oscillator. Under conditions of soft excitation (experimentally it is found that this is always the case in the helium-neon laser) it can be shown that at sufficiently low power levels the quantity p is proportional to A^2, that is to say, the power of the oscillator P.

Data reported in [8], in which investigations have been made of the fluctuations of laser radiation near threshold for single-mode oscillation have shown that this dependence of $\overline{m_F^2}$ on F and p on P is well verified.5)

It is also easy to show that if B is constant the quantity $\overline{m_F^2}$ at frequencies $F \ll p/2\pi$ (and the quantity $\overline{M_{eF}^2}$ which is proportional to it) is inversely proportional to the cube of the laser power.

2. RESULTS OF MEASUREMENTS WITH A FABRY-PEROT LASER

We present below the results obtained in experiments with lasers using dc gas-discharge tubes with Brewster windows at the ends. The mode of operation of the discharge is always chosen so that the radiation fluctuations are not correlated with the correlations in discharge current. The absence of any such correlation is verified by means of a two channel amplifier in conjunction with a narrow-band spectrum analyzer; the fluctuations in photocurrent are applied to the input of one channel of the amplifier while the fluctuations in discharge current are applied to the input of the other. The outputs of the two channels can be added and subtracted; the outputs are then amplified and applied to a spectrum analyzer (in the frequency region of interest F). Identical readings in the add and subtract modes indicate the absence of any correlation. The apparatus can be used to measure either the spectral density of a fluctuation or ρ_F, the correlation between them, in a narrow spectral range near the frequency F. The apparatus is calibrated with either a sinusoid or a noise signal; the latter derives from the fluctuations in the photocurrent that are produced when the photocathode (in a photodiode) is illuminated by white noise.

The experiments were carried out with lasers radiating at 0.63 microns with gas-discharge tubes of various length. Radiation at relatively high power (several mW) and a high number of modes is obtained from a laser 100–120 cm in length; it is difficult to obtain stable operation with 1–3 modes in such tubes. Hence, in the investigations concerned with single-mode operation we have used shorter lasers (resonator length 50 cm). The number of modes is determined by means of a scanning Fabry-Perot interferometer.[14] Using this device single-mode operation of the laser is

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5) In[8] a quantity called the "effective band" $\Delta\nu$ has been introduced; it can be shown that under conditions of soft excitation this quantity is equal to $p/2\pi$.

verified by measuring the frequency at the top of the Doppler peak of the spectral line. In these measurements the radiation power level is estimated from the magnitude of the photocurrent.

A typical dependence of the spectral density of the depth of photocurrent random modulation $\overline{M_F^2}$ on the frequency F in single-mode operation is shown in Fig. 1 by curve 1; in this case $I = 6\,\mu A$. The curve $\overline{M_F^2}$ (starting at F = 2kHz and going towards higher frequencies) is described satisfactorily by Eq. (6) with $p/2\pi \approx 110$ kHz.6) Evidently this part of the spectrum is due to spontaneous emission from the active medium. The rise in the curve below F = 2kHz is attributed to technical causes.

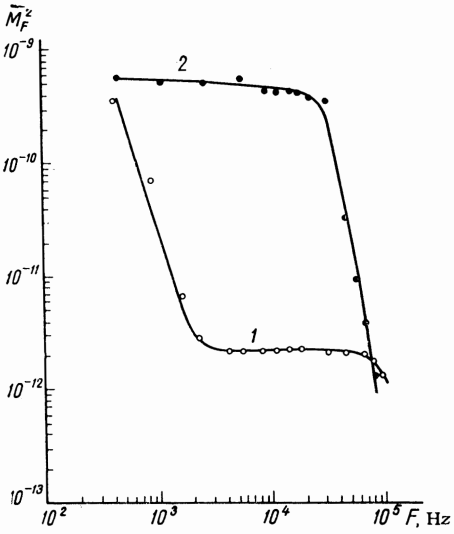

FIG. 1

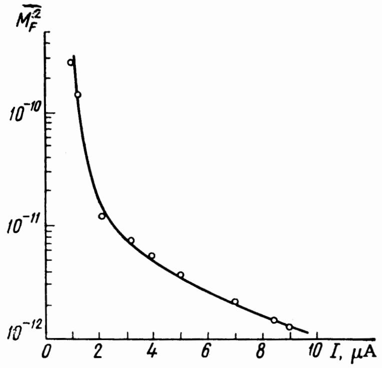

FIG. 2.

When the generated power is varied the quantity $\overline{M_F^2}$ and its dependence on frequency also vary. The results of measurements of $\overline{M_F^2}$ (in the frequency range F = 3–10kHz where it was constant) at various power levels in single-mode operation are shown in Fig. 2. The power level P is regulated (by small changes in the inclination of one of the resonator mirrors) up to some maximum value beyond which the system jumps into three-mode operation. Analysis of Fig. 2 shows that the functional dependence noted earlier $\overline{m_F^2} \sim \overline{M_F^2} \sim P^{-3}$ holds only at low power levels (up to a photocurrent of approximately $2.5-3\,\mu A$). This value of the photocurrent, according to a rough estimate, corresponds to a laser power of 10–30 μW. At higher values of the current the quantity $\overline{M_F^2}$ falls, but more slowly. This deviation from $\overline{M_F^2} \sim P^{-3}$ is evidently due to the fact that the relation noted above $p \sim P$ does not hold at high power levels.

From the data given in [8] it follows that the dependence of $\overline{M_F^2}$ on P for $F \ll p/2\pi$ is in complete agreement with the relation $\overline{M_F^2} \sim P^{-3}$. Estimates show that the highest power in [8] is of the same order as the lowest power in the present work. We also note that both experiments were carried out with lasers of the same length and with similar discharge tubes.

In the laser with a resonator 50 cm in length we have also carried out measurements of the depth of modulation of the photocurrent due to combination tones in multimode operation. Measures were taken to reduce the vibrations of the laser and in this way it was found possible, by smooth adjustment of the resonator length, to produce conditions for relatively long observations of stable low frequency beats in the photocurrent; the beat frequencies (with three or five longitudinal modes excited) vary from unity up to 100–150 kHz. In this case the depth of modulation was measured by means of an oscilloscope which was calibrated with a sinusoidal signal; M reaches values of 0.003–0.015.

In the lasers with resonator length of 120–150 cm the modulation associated with the combination tones (three or five TEM_{00q} modes) is rather intermittent in nature. Variations in the mode frequencies (due to small fluctuations in resonator length) give rise to rapidly varying processes of secondary beats and mode locking which lead to highly nonstationary modulation of the radiation intensity. When the number of modes is increased (> 5–7) under the same conditions the observed modulation assumes a quasistationary nature. Measurements carried out with a large number of tubes show that the basic contribution to $\overline{M^2}$ comes from fluctuations in the frequency

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6)We recall that in single-mode operation $\overline{M_{eF}^2} = 4\overline{m_F^2}$. In the experiments described here $\overline{M_F^2} \gg \overline{M_{0F}^2}$ and consequently $\overline{M_F^2} \approx \overline{M_{eF}^2}$.

range up to 30–40 kHz where the level of $\overline{M_F^2}$ is usually 10^{-10}–5×10^{-9}; it follows that $\overline{M^2}$ is of the order of 10^{-6}–10^{-4}. For purposes of illustration, in Fig. 1 (curve 2) we show the dependence of M_F^2 on frequency F for a laser with a long resonator (approximately 120 cm) in multimode operation.

3. RADIATION FLUCTUATIONS IN A RING LASER

The experiments described below were carried out in a ring laser with a three mirror resonator in a single dc discharge tube, at a wavelength of 0.63 microns (a more complete description of the apparatus is given in [15]). The radiation from two traveling waves propagating in opposite directions is observed separately on two photodiodes, the output signals of which are applied to the apparatus (two-channel amplifier etc.) described in the preceding section. This arrangement allows us to measure $\overline{M_F^2}$ for the opposed radiation beams (without splitting the optical traveling waves) and also allows us to measure ρ_F, the correlation of the fluctuations in a narrow spectral range around the frequency F.

The values of $\overline{M_F^2}$ are found to be the same for the opposed beams. Typical examples of the dependence on frequency for single-mode and multimode operation (the number of TEM_{00q} modes is greater than 7) are shown in Fig. 3 (curves 1 and 2 respectively). Curve 1 satisfies Eq. (6) with $p/2\pi$ approximately equal to 3 kHz. In multimode operation the fluctuations are due primarily to oscillations at combination frequencies; curve 2 falls off sharply starting at approximately 1 kHz; the value of $\overline{M_F^2}$ for $F < 1$ kHz is two orders greater than the level for single-mode operation while $\overline{M^2}$ is approximately 10^{-4}.

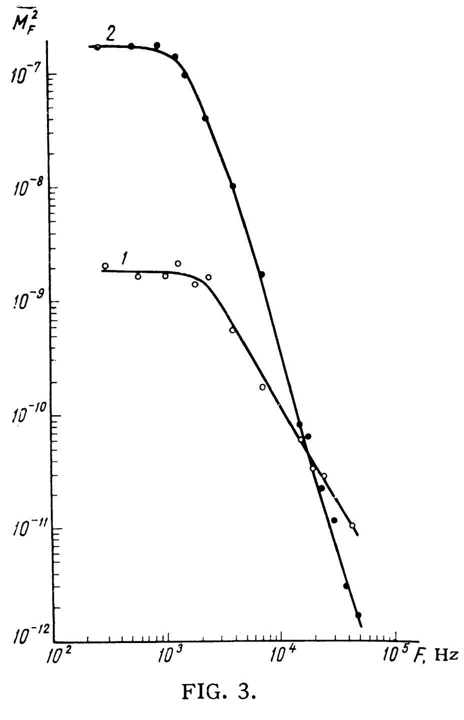

FIG. 3.

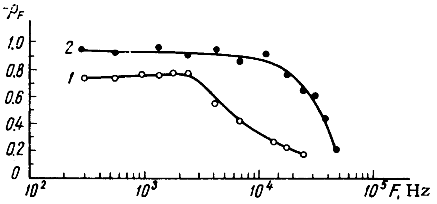

FIG. 4

Curves showing the dependence of the correlation function for the fluctuations in the opposed beams ρ_F for these cases are shown in Fig. 4; curve 1 applies for single-mode operation and curve 2 for multimode operation. In both cases we find $\rho_F < 0$; in the multimode regime the value of ρ_F is approximately -0.9 at least in the frequency range 0.4 to 10 kHz.[7] In the single mode regime ρ_F is approximately -0.7, when F varies from tens of cycles to ~2.5 kHz, and falls off smoothly to a value of 0.1 at a frequency of approximately 20 kHz. The negative correlation of the fluctuations in photocurrent shows that the fluctuations in the intensity of the opposed beams occur in antiphase. It should be emphasized that this "anticorrelation effect" holds only in the absence of correlation between fluctuations in the radiation and the discharge current. If this mode of operation of the discharge plasma is not maintained, then, as in the case of a laser with a two-mirror resonator, the fluctuation level increases appreciably in both beams (for example by one or two orders of magnitude in single-mode operation) and these fluctuations exhibit a positive correlation ρ_F which is close to the value +1 over a wide range of F.

The "anticorrelation" observed here is undoubtedly of physical interest. A qualitative explanation of the effect might be formulated as follows. The energy in the opposed waves arises by virtue of induced emission from excited atoms in

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[7] When 3 – 5 modes are excited in a ring laser the negative correlation can be observed conveniently with a two-beam oscilloscope; combined oscillations in the photocurrent for both diodes arise and stop at the same times but are in time antiphase.

the same active medium. Taking account of the inhomogeneous broadening of the Doppler line of the operating transition and using the notion of "hole burning" as described by Bennett[16] one expects a partial or complete overlapping of the "holes" in the gain curve responsible for the maintenance of the generation of the opposed waves. Under these conditions the same atoms can couple into either one of the opposed waves; the latter (for a given population inversion in the operating level) obviously leads to antiphase variations in the power of the opposed radiation beams. It is quite possible that this explanation is not a complete one; a description of the effect will require appropriate verification in the theory of the ring laser.8)

In addition to the fluctuations described here there can be fluctuations associated with random variations in the differences of the populations in the operating levels which can also lead to a reduction in the degree of anticorrelation as well as to a positive correlation in the radiation fluctuations of the opposed beams in the ring laser.

We would like to thank I. L. Bershteĭn for stimulating discussions of the work described here and the manuscript. We are indebted to V. I. Katsman and L. V. Nikolaev for fabrication of the optical units and the gas-discharge tubes.

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8)Within the framework of the explanation given here one expects, in particular, that the correlation in the intensity fluctuation will depend on the position of the holes with respect to the center of the transition line, that is to say, the degree of overlap. In order to verify this assertion it would be necessary to use a ring laser with high long-term stability; such a laser was not available in the present experiments.

[1] P. T. Bolwijn, C. T. J. Alkemade and G. A. Boschloo, Phys. Letters **4**, 59 (1963).

[2] J. A. Bellisio, C. Freed and H. A. Haus, Appl. Phys. Letters **4**, 5 (1964).

[3] R. L. Bailey and J. H. Sanders, Phys. Letters **10**, 295 (1964).

[4] L. J. Prescott and A. Van der Ziel, Appl. Phys. Letters **5**, 48 (1964).

[5] L. J. Prescott and A. Van der Ziel, Phys. Letters **12**, 317 (1964).

[6] P. T. Bolwijn, Phys. Letters **13**, 311 (1964).

[7] A. Van der Ziel, Proc. Inst. Elec. and Electron. Engrs. **52**, 1738 (1964).

[8] C. Freed and H. A. Haus, Appl. Phys. Letters **6**, 85 (1965).

[9] G. S. Gorelik, DAN SSSR **58**, 45 (1947).

[10] A. T. Forrester, W. E. Parkins and E. Gerjuoy, Phys. Rev. **72**, 728 (1947).

[11] I. L. Bershteĭn, Izv. AN SSSR ser. Fiz. **14**, 145 (1950).

[12] G. S. Gorelik, JETP **20**, 351 (1950).

[13] W. E. Lamb, Jr., Phys. Rev. **134**, A1429 (1964).

[14] R. Fork, D. Herriott and H. Kogelnik, Appl. Optics **3**, 1471 (1964).

[15] I. L. Bershteĭn and Yu. I. Zaĭtsev, JETP **49**, 953 (1965), Soviet Phys. JETP **22**, 663 (1966).

[16] W. R. Bennett Jr., Phys. Rev. **126**, 580 (1962).

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