INTERACTION OF PLASMA AND SPIN WAVES IN FERROMAGNETIC SEMICONDUCTORS AND METALS

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Submitted to JETP editor October 7, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) **50**, 576–588 (March, 1966)

We consider coupled plasma, electromagnetic, and spin waves in ferromagnetic semiconductors and in metals with magnetic anisotropy of the "easy axis" and "easy plane" type. We investigate the region of wave vectors **k** in which the spatial dispersion of the magnetic permeability tensor may be significant, but the spatial dispersion of the dielectric constant is weak (the wavelength is much greater than the Larmor electron radius and the phase velocity of the waves is much greater than the thermal or Fermi velocity of the electrons). The refractive indices of the waves are determined and the transparency regions are determined. We show that coupled waves may possess anomalous dispersion (uncoupled spin waves and electromagnetic waves in a plasma possess normal dispersion in the absence of space dispersion of the tensor ϵ). The spectra of coupled cyclotron and spin waves moving perpendicular to the magnetic field are also determined in the case when the wavelength is of the order of the Larmor electron radius.

1. INTRODUCTION

As is well known, spin, electromagnetic and plasma oscillations in ferromagnetic semiconductors and metals can be strongly coupled. Coupled spin waves (magnons) and helical waves (helicons) propagating in a ferromagnetic metal in the direction of an external magnetic field were first considered by Stern and Callen.[1] Blank[2] considered the connection between spin and helicoidal waves in the case of an arbitrary direction with

$$\omega \ll \omega_B, \quad kr_L \ll 1, \quad \omega / k \ll v_F \tag{1.1}$$

($\omega_B = eB/m^*c$—gyrofrequency, e and m^* the charge and the effective mass, r_L the Larmor radius, and v_F the limiting Fermi velocity of the conduction electrons). Blank[2] also considered the connection between spin waves and magnetohydrodynamic waves, existing when conditions (1.1) are satisfied in metals with identical number of electrons and holes. The authors of [3] considered the propagation of coupled spin and electromagnetic waves under conditions of anomalous skin effect ($kr_L \gg 1$).

In metals, the plasma frequency ω_p is much larger than the gyrofrequency of the conduction electrons and the frequency of the ferromagnetic resonance. The coupling between plasma oscillations and electromagnetic waves with frequencies $\omega \gtrsim \omega_p$, on one side, and the spin waves on the other, is therefore negligibly small in metals. However, in ferromagnetic semiconductors with a small number of carriers, the plasma frequency can be comparable to the ferromagnetic resonance frequency. The interaction of electromagnetic waves with spin waves propagating perpendicular to an external magnetic field in such semiconductors was considered by Mendelson and Spector[4] for the case of weak spatial dispersion ($kr_L \ll 1$, $\omega/k \gg v_T$, $r_L = v_T/\omega_B$, $v_T = \sqrt{T/m^*}$—thermal velocity of electrons in the semiconductors).

We consider in this paper coupled spin, plasma, and electromagnetic waves in ferromagnetic semiconductors and metals in a magnetic field, for arbitrary direction of field propagation, in the case of long-wave oscillations ($kr_L \ll 1$) having in the direction of the magnetic field a phase velocity ($v_{ph\parallel} = \omega/k \cos\theta$) considerably in excess of the electron thermal velocity (in semiconductors) and of the limiting Fermi velocity (in metals). We find the refractive indices of the waves and determine the transparency regions.

Near plasma (electrostatic) and ferromagnetic (magneto-acoustic) resonance, when the refractive index becomes very large, we took into account the spatial dispersion due to the thermal motion of the conduction electrons and to exchange magnetic interaction. We also considered the propagation of coupled spin and electron-cyclotron waves with a finite Larmor radius ($kr_L \sim 1$) in semiconductors and metals.

2. WAVES IN A MEDIUM WITHOUT SPATIAL DISPERSION

The dielectric tensor (ϵ_{ik}) and the permeability tensor (μ_{ik}) for uniaxial ferromagnetic semiconductors and metals are

$$(\varepsilon_{ik}) = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}, \quad (\mu_{ik}) = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1' & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}. \tag{2.1}$$

The 3-axis is directed here along the external magnetic field **H**.

The components of the dielectric tensor are[5]

$$\varepsilon_1 = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2 - \omega_{B\alpha}^2}, \tag{2.2}$$

$$\varepsilon_2 = -\sum_\alpha \frac{\eta_\alpha \omega_{p\alpha}^2 \omega_{B\alpha}}{\omega(\omega^2 - \omega_{B\alpha}^2)}, \qquad \varepsilon_3 = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2},$$

where

$$\omega_{p\alpha} = (4\pi n_{0\alpha} e_\alpha^2 / m_\alpha^*)^{1/2}, \qquad \omega_{B\alpha} = |e_\alpha| B_0 / m_\alpha^* c$$

are the Langmuir frequency and the gyrofrequency, $n_{0\alpha}$ is the equilibrium density of the carrier with charge e_α and effective mass m_α^*, and $\eta_\alpha = e_\alpha / |e_\alpha|$.

Expressions (2.2) are applicable in the case of weak spatial dispersion of an electron gas, when

$$kr_L \ll 1, \quad |\omega - n\omega_{B\alpha}| \gg kv\cos\theta \quad (n = 0, 1, 2)$$

(θ is the angle between the wave vector **k** and the magnetic field **H**, $v = v_F$ for a degenerate electron gas and $v = v_T$ for a Maxwellian electron velocity distribution).

In the case of ferromagnets with "easy axis" magnetic anisotropy the components of the tensor μ_{ik} are[6]

$$\mu_1 = 1 - \frac{\Omega\Omega_M}{\omega^2 - \Omega^2}, \qquad \mu_2 = \frac{\Omega_M \omega}{\omega^2 - \Omega^2}, \qquad \mu_3 = 1,$$

$$\Omega = gM(\beta + H/M), \quad \Omega_M = 4\pi g M, \quad g = e/2mc > 0, \tag{2.3}$$

H is the constant magnetic field in the ferromagnet, directed along the easy-magnetization axis, M is the saturation magnetic moment, and β is the magnetic anisotropy constant.

For ferromagnets with anisotropy of the "easy plane" type we have[7]

$$\mu_1 = \frac{\omega^2 - \Omega\Omega_B}{\omega^2 - \Omega\Omega_H}, \quad \mu_1' = \frac{\omega^2 - \Omega_H(\Omega + \Omega_M)}{\omega^2 - \Omega\Omega_H}, \tag{2.4}$$

$$\mu_2 = -\frac{\Omega_M \omega}{\omega^2 - \Omega\Omega_H},$$

$$\mu_3 = 1, \quad \Omega_H = gH, \quad \Omega_B = \Omega_M + \Omega_H.$$

Formulas (2.4) are given for the case when a constant magnetic field H and the magnetic moment M lie in the basal plane. The anisotropy axis is directed along the y axis.

Using (2.1) we obtain for Maxwell's equations a dispersion equation for the electromagnetic waves:

$$An^4 + Bn^2 + C = 0, \tag{2.5}$$

where $A = A_e A_m$,

$$A_e = \varepsilon_1 \sin^2\theta + \varepsilon_3 \cos^2\theta,$$

$$A_m = \mu_1 \sin^2\theta\cos^2\varphi + \mu_1' \sin^2\theta \sin^2\varphi + \mu_3 \cos^2\theta,$$

$$B = -\{(\varepsilon_1^2 - \varepsilon_2^2)(\mu_1 \sin^2\theta\cos^2\varphi + \mu_1'\sin^2\theta\sin^2\varphi)\mu_3 + 2\varepsilon_2\varepsilon_3\mu_2\mu_3\cos^2\theta + \varepsilon_1\varepsilon_3[\mu_3(\mu_1 + \mu_1')\cos^2\theta + (\mu_1\mu_1' - \mu_2^2)\sin^2\theta]\},$$

$$C = \varepsilon_3(\varepsilon_1^2 - \varepsilon_2^2)\mu_3(\mu_1\mu_1' - \mu_2^2). \tag{2.6}$$

Here $n = kc/\omega$ is the refractive index, and φ the azimuthal angle in the wave-vector space, measured from the x axis.

From the dispersion equation (2.6) it follows that in the ferromagnet in question there can propagate two waves with given frequency ω and refractive index defined by the relation

$$n^2 = [-B \pm \sqrt{B^2 - 4AC}]/2A. \tag{2.7}$$

Let us investigate the behavior of the refractive index (2.7) as a function of the frequency. For simplicity, we confine ourselves here to the case of carriers of one species and omit the index α for the quantities $\omega_{p\alpha}$ and $\omega_{B\alpha}$.

Ferromagnets with Magnetic Anisotropy of the "Easy Axis" Type

1. We consider first waves in a ferromagnet with magnetic anisotropy of the "easy axis" type for $\theta \neq 0, \pi/2$. In this case the refractive index vanishes at points where $C = 0$. Using (2.2), (2.3), and (2.6), we find that $n^2 = 0$ when $\omega = \omega_p, \omega_\pm, \omega$ where

$$\omega_\pm = \pm\tfrac{1}{2}\omega_B + \sqrt{\tfrac{1}{4}\omega_B^2 + \omega_p^2}, \quad \tilde{\omega} = \Omega + \Omega_M. \tag{2.8}$$

The refractive index becomes infinite at $A = 0$. From the equations $A_e = 0$ and $A_m = 0$, using (2.2) and (2.3), we find that the resonant frequencies are respectively equal to $\omega = \omega_{1,2}$ and $\omega = \omega_m$, where

$$\omega_{1,2}^2 = \tfrac{1}{2}\{\omega_p^2 + \omega_B^2 \pm [(\omega_p^2 + \omega_B^2)^2 - 4\omega_p^2\omega_B^2\cos^2\theta]^{1/2}\},$$

$$\omega_m^2 = \Omega(\Omega + \Omega_M \sin^2\theta). \tag{2.9}$$

The oscillations with frequencies $\omega \approx \omega_{1,2}$ are almost longitudinal: their electric-field component parallel to the wave vector $\mathbf{e}_{\parallel} = \mathbf{k}(\mathbf{e}\cdot\mathbf{k})/k^2$ greatly exceeds the component $\mathbf{e}_{\perp} = \mathbf{k}\times[\mathbf{e}\times\mathbf{k}]/k^2$, so that the oscillations can be regarded as potential, $\mathrm{e} \approx -\nabla\varphi$.

Oscillations with frequency $\omega \approx \omega_m$ (spin wave) are also almost longitudinal, for in this case $\mathbf{h}_{\parallel} = \mathbf{k}(\mathbf{h}\cdot\mathbf{k})/k^2$ is much larger than $\mathbf{h}_{\perp} = \mathbf{k}\times[\mathbf{h}\times\mathbf{k}]/k^2$ and therefore such oscillations can be regarded as magnetostatic, $\mathbf{h} \approx -\nabla\varphi$.

It is obvious that the inequalities $\omega_m < \tilde{\omega}$ and $\omega_- < \omega_p < \omega_1 < \omega_+$ apply. The frequency of the plasma oscillations, ω_2, which is always smaller than the Langmuir frequency ω_p, can be either larger or smaller than the frequency ω_-. We note that the sign of the product AC in (2.7) coincides with the sign of the quantity

$$(\omega-\omega_1)(\omega-\omega_2)(\omega-\omega_m)(\omega-\omega_p)(\omega-\omega_-)\times(\omega-\omega_+)(\omega-\tilde{\omega}).$$

Since the electron effective mass m* does not coincide, generally speaking, with the mass of the free electron, the frequency ω_B can in the general case be either larger or smaller than ω_m or $\tilde{\omega}$.

When $\omega \to 0$ the expressions (2.7) for the refractive index simplify to

$$n^2 = \pm\frac{\omega_p{}^2}{\omega\omega_B\cos\theta}\frac{\mu_0}{(\cos^2\theta+\mu_0\sin^2\theta)^{1/2}}, \qquad (2.10)$$

where $\mu_0 = 1 + \Omega_M/\Omega$ is the static magnetic permeability (we have used here formulas (2.2) and (2.3)). Thus, in the region of low frequency ($\omega \ll \omega_B$, ω_m) only one wave (helicon) can propagate.[1)]

The solid line and the dashed lines in Fig. 1, a show schematically the refractive index as a function of the frequency for those cases (which differ in the possible arrangement of the zeroes of n^2 relative to the poles of n^2) when there is at least one zero to the left of the second pole of n^2.

If the zeroes of the refractive index lie to the right of the second pole, then the behavior of n^2 is that shown schematically in Fig. 1, b. Two cases are then possible: in the first case, there is only one zero of n^2 to the right of the largest pole of n^2; in the second case, two zeroes are possible. In the first case the plots of n^2 are drawn solid,

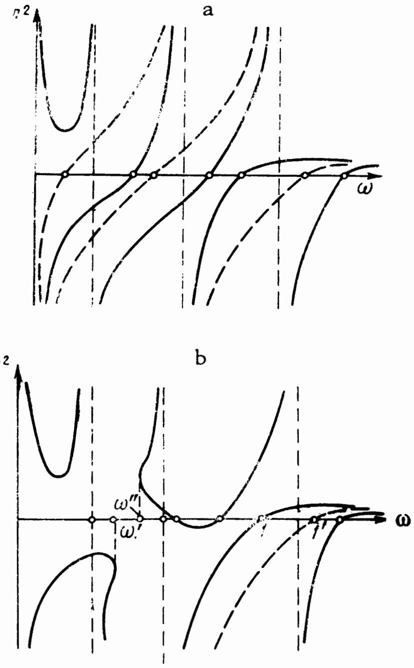

FIG. 1

and in the second the plot of the branch of the oscillations passing through the point 1′ is drawn dashed.

In the frequency interval $\omega' < \omega < \omega''$ wave propagation is impossible, since the square of the refractive index is complex. The transparency regions where $n^2 > 0$ and the number of propagating waves can be easily determined by using Figs. 1, a and b, with account taken of the inequalities presented above for the characteristic frequencies.

Corresponding to the given wave vector k are five different values of the frequency $\omega = \omega^i(k, \theta)$ (we recall that when $\theta \neq 0$, $\pi/2$ only four oscillation branches exist in a plasma with $\mu_1 = \mu_1' = 1$, $\mu_2 = 0$). The behavior of the natural frequencies $\omega^{(i)}$ as functions of the wave vector is shown in Figs. 2, a and b, which correspond to Figs. 1, a and b. The frequencies $\omega^{(1)}$ and $\omega^{(2)}$ approach

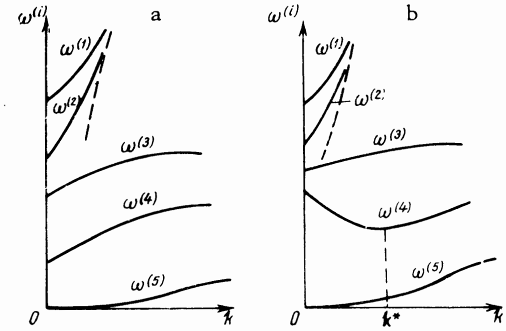

FIG. 2

1)Formula (2.10) can be obtained from expression (10) of[2], if we neglect there the value of ω compared with $\Omega \sim \omega_m$. For waves with frequencies $\omega \sim \Omega$ the expression (10) from[2] can be used only for $m^* \ll m$, when $\Omega \ll \omega_B$, since this expression was obtained under the assumption that $\omega \ll \omega_B$.

the values of $\omega^{(1),(2)} = kc$ with increasing k; the frequency $\omega^{(3)}$ tends with increasing k to the larger of the frequencies ω_1 and ω_m; the frequency $\omega^{(5)}$ approaches with increasing k the smaller of the frequencies ω_2 and ω_m; finally, $\omega^{(4)}$ tends with increasing k to the mean value of the three resonant frequencies ω_1, ω_2, and ω_m.

We see from Fig. 2, b that the frequency $\omega = \omega^{(4)}$ decreases with increasing k in the region $k < k^*$. These oscillations have an anomalous dispersion, the angle between the phase and group velocities being larger than $\pi/2$.

2. We now consider the propagation of waves along the magnetic field H ($\theta = 0$). In this case the dispersion equation (2.5) breaks up into three equations:

$$\varepsilon_3 = 0, \quad n^2 = (\varepsilon_1 + \varepsilon_2)(\mu_1 + \mu_2),$$
$$n^2 = (\varepsilon_1 - \varepsilon_2)(\mu_1 - \mu_2). \tag{2.11}$$

The first of these equations determines the frequency of the plasma oscillations $\omega = \omega_p$ (see (2.2)). The second equation of (2.11) can be rewritten by using formulas (2.2) and (2.3) in the form

$$n^2 = \frac{\Omega + \Omega_M + \omega}{\Omega + \omega}\left[1 - \frac{\omega_p^2}{\omega(\omega + \omega_B)}\right].$$

This equation determines the originary electromagnetic wave with frequency $\omega = \omega(k) > \omega_m$.

Substituting in lieu of ϵ_1, ϵ_2, and μ_1, μ_2 the values in accord with formulas (2.2) and (2.3), we represent the third equation in (2.11) in the form

$$n^2 = \left[1 - \frac{\omega_p^2}{\omega(\omega - \omega_B)}\right]\frac{\omega - (\Omega + \Omega_M)}{\omega - \Omega}. \tag{2.12}$$

The possible behavior of the refractive index, defined by this equation as a function of the frequency, is shown schematically in Fig. 3, a and b.

From (2.12) we can obtain also the frequency ω as a function of the wave vector k. We can show

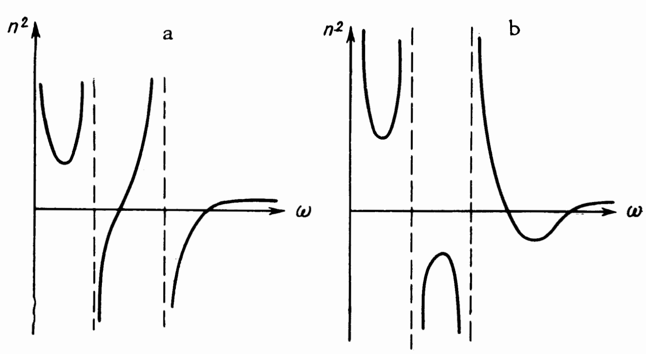

FIG. 3

that for a given k there exist three different branches of oscillations with frequencies $\omega = \omega^i(k)$, the dependence of which on the wave vector is shown schematically in Figs. 4, a and b. The refractive index (2.12) vanishes when $\omega = \omega_+$ and $\tilde{\omega} = \Omega + \Omega_M$; $n^2 = \infty$ when $\omega = \omega_B$ and $\omega = \Omega$. With increasing wave vector k, the frequencies $\omega^i(k)$ tend respectively to

$$\omega^{(1)} \to kc, \quad \omega^{(2)} \to \max(\omega_B, \Omega), \quad \omega^{(3)} \to \min(\omega_m, \Omega).$$

The frequency $\omega^{(2)}(k)$ at $\max(\omega_+, \Omega + \Omega_M) > \min(\omega_B, \Omega)$ (see Fig. 4, b) decreases with increasing k; the phase and group velocities for this wave are directed opposite each other.

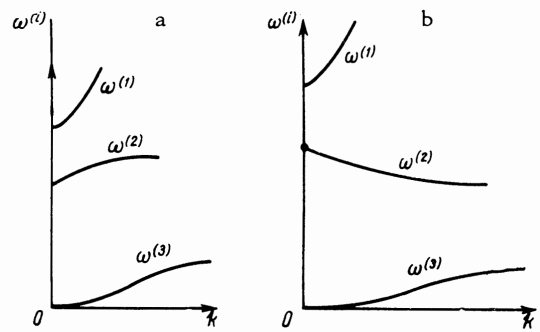

FIG. 4

Thus, when $\theta = 0$ there are five branches of oscillation, the first of which corresponds to the plasma wave, the second to the ordinary wave, and the remaining three to the extraordinary and spin waves.

3. When $\theta = \pi/2$, Eq. (2.5) breaks up into two equations:

$$n^2 = (\varepsilon_1^2 - \varepsilon_2^2) / \varepsilon_1, \quad n^2 = \varepsilon_3(\mu_1^2 - \mu_2^2) / \mu_1. \tag{2.13}$$

The first of these equations determines the refractive index of the extraordinary electromagnetic wave in a plasma, and does not depend on the magnetic properties of the medium: for a given k this equation has two different solutions $\omega(k)$.

The second equation of (2.13), corresponding when $\mu_1 = 1$ and $\mu_2 = 0$ to the ordinary wave in the plasma, is

$$n^2 \frac{\Omega^2 + \Omega_M\Omega - \omega^2}{(\Omega + \Omega_M)^2 - \omega^2} = 1 - \frac{\omega_p^2}{\omega^2}. \tag{2.14}$$

Equation (2.14), which was considered by Mendelson and Spector,[4] determines the frequencies of two oscillation modes.

Thus, when $\theta = \pi/2$ there are four different oscillation modes.

Ferromagnets with Magnetic Anisotropy of the "Easy Plane" Type

1. We now proceed to study electromagnetic waves in ferromagnetic media with anisotropy of the "easy plane" type.

Let us consider the propagation of electromagnetic waves at $\theta \neq 0$ and $\theta \neq \pi/2$. Using formulas (2.7), (2.2), and (2.4), we can readily show that in this case the expressions for the refractive index vanish at the points $\omega = \omega_p$, $\omega = \omega_\pm$, and

$$\omega = \tilde{\omega} \equiv [(\Omega + \Omega_M)(\Omega_H + \Omega_M)]^{1/2}, \qquad (2.15)$$

while the poles of n^2 are located at the point $\omega = \omega_{1,2}$ and

$$\omega^2 = \omega_m^2 = \Omega\Omega_H + \Omega_M(\Omega \cos^2\varphi + \Omega_H \sin^2\varphi)\sin^2\theta. \qquad (2.16)$$

As $\omega \to 0$ (region of helicon waves) we obtain from (2.7), using (2.2) and (2.4), the following expression for the refractive index

$$n^2 = \pm \frac{\omega_p^2}{\omega\omega_B \cos\theta}\left[\frac{\Omega_B(\Omega + \Omega_M)}{\omega_m^2}\right]^{1/2}. \qquad (2.17)$$

Since the frequency $\omega_m(k)$ of the magnetostatic oscillations (2.16) is always smaller than the frequency $\tilde{\omega}$ (2.15), the case under consideration, that of ferromagnets with anisotropy of the "easy plane" type, is analogous to the case of ferromagnets with "easy axis" anisotropy. The behavior of the refractive indices as a function of the frequency and of the natural frequencies as functions of the wave vector are illustrated by the plots in Figs. 1 and 2.

2. When $\theta = 0$, Eq. (2.5) breaks up into two equations: $\epsilon_3 = 0$ and

$$n^4 - [\varepsilon_1(\mu_1 + \mu_1') + 2\varepsilon_2\mu_2]n^2 + (\mu_1\mu_1' - \mu_2^2)(\varepsilon_1^2 - \varepsilon_2^2) = 0. \qquad (2.18)$$

The zeroes of n^2 are located at the points $\omega = \omega_\pm$ and $\omega = \tilde{\omega}$, where $\tilde{\omega}$ is determined by (2.15); the poles of n^2 correspond to the points $\omega = 0$, $\omega = \omega_B$, and $\tilde{\omega} = \omega_m = \Omega\Omega_H < \tilde{\omega}$.

3. If $\theta = \pi/2$ and $\varphi = \pi/2$, then (2.5) breaks up into two equations:

$$n^2 \frac{\omega^2 - \Omega_H(\Omega + \Omega_M)}{\omega^2 - (\Omega + \Omega_H)\Omega_B} = 1 - \frac{\omega_p^2}{\omega^2} \qquad (2.19)$$

and the first equation of (2.13). Equation (2.19) is similar to (2.14), provided all the following substitutions are made in (2.14):

$$(\Omega + \Omega_M)^2 \to (\Omega + \Omega_M)\Omega_B, \quad \Omega(\Omega + \Omega_M) \to \Omega_H(\Omega + \Omega_M).$$

Thus, when $\theta = \pi/2$ and $\varphi = \pi/2$ there are four oscillation modes.

4. If $\theta = \pi/2$ and $\varphi = 0$, then (2.5) again breaks up into the first equation of (2.13) and the equation

$$n^2 \frac{\omega^2 - \Omega\Omega_B}{\omega^2 - (\Omega + \Omega_H)\Omega_B} = 1 - \frac{\omega_p^2}{\omega^2}, \qquad (2.20)$$

which is also analogous to (2.14).

3. COUPLED ELECTROSTATIC AND SPIN OSCILLATIONS

Near the frequencies $\omega = \omega_{1,2}$ and $\omega = \omega_m$ the refractive index, as shown in the preceding section, becomes very large, so that in this case the spatial dispersion of the medium can become significant. Allowance for the spatial dispersion of the dielectric constant, due to the thermal motion of electrons, leads[5] to the replacement of the coefficient A_e in the dispersion equation (2.5) by A_e':

$$A_e' = A_e - (kv_T/\omega)^2 b, \qquad (3.1)$$

where v_T is the thermal velocity of electrons and

$$b = \frac{\omega_p^2}{\omega^2}\left[3\cos^4\theta + \cos^2\theta\sin^2\theta\frac{6\omega^6 - 3\omega^4\omega_B^2 + \omega^2\omega_B^2}{(\omega^2 - \omega_B^2)^3} + 3\sin^4\theta\frac{\omega^4}{(\omega^2 - \omega_B^2)(\omega^2 - 4\omega_B^2)}\right].$$

Expression (3.1) is valid when $kr_L \ll 1$, $\omega/k \gg v_T$, and ω is not close to ω_B or $2\omega_B$.

When account is taken of the spatial dispersion, the permeability of a ferromagnet is again determined by formulas (2.3) and (2.4) but Ω, Ω_H must be replaced by Ω_k, Ω_{Hk}:

$$\Omega_k = \Omega + Ik^2, \quad \Omega_{Hk} = \Omega_H + Ik^2, \qquad (3.2)$$

where I is the exchange integral, whose order of magnitude is

$$I \approx T_K a^2/\hbar \approx 10^{-2}\ \text{sec}^{-1}\cdot\text{cm}^2$$

(T_C is the Curie temperature of the ferromagnet, $T_C \sim 10^3$ °K; a is the lattice constant, $a \approx 10^{-8}$ cm, and $\hbar$ is Planck's constant). Consequently, the additions to the frequency Ω, due to spatial dispersion, are of the order of Ω_M when $k \sim 10^6\ \text{cm}^{-1}$.

The dispersion equation near the frequency $\omega = \omega_{1,2}$ or $\omega = \omega_m$ is

$$[A_e - (kv_T/\omega)^2 b]A_m(\omega, k) + B(\omega, k)n^{-2} = 0. \qquad (3.3)$$

The values of $A_e(\omega)$ and $A_m(\omega, k)$ in accord with (2.6) and (2.2)–(2.4), are

$$A_e(\omega) = (\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)/\omega^2(\omega^2 - \omega_B^2),$$
$$A_m(\omega, k) = [\omega^2 - \omega_m^2(k)]/[\omega^2 - \Omega_1(k)],$$

where the frequencies $\omega_{1,2}$ are determined by formulas (2.9), and the frequencies $\omega_m(k)$ by formulas (2.9) and (2.16). $\Omega_1 = \Omega_k$ in the case of ferromagnets with magnetic anisotropy of the "easy axis" type, and $\Omega_1 = (\Omega_k \Omega_{Hk})^{1/2}$ in the case of anisotropy of the "light axis" type. In (3.3) we have left out the small term C/n^4, since we are interested in a solution of the dispersion equation in the region where

$$n^2 = c^2k^2/\omega^2 \gg 1.$$

The frequencies of electrostatic (plasma) oscillations, determined from $A_e(\omega) - (kv_T/\omega)^2 b = 0$, are equal to $\omega = \tilde{\omega}_{1,2}$, where

$$\tilde{\omega}_j = \omega_j \left[1 + \left(\frac{kv_T}{\omega_j}\right)^2 \alpha_e(\omega_j)\, b(\omega_j)\right],$$

$$\alpha_e(\omega_j) = \frac{|\omega_j^2 - \omega_B^2|}{2(\omega_1^2 - \omega_2^2)} \tag{3.4}$$

($j = 1, 2$) and frequencies ω_j are determined by formula (2.9). With increasing wave vector, the resonance frequencies $\tilde{\omega}_j$ increase if $b(\omega_j) > 0$, and decrease if $b(\omega_j) < 0$.

The frequencies of the magnetostatic oscillations (spin waves) determined from the equation $A_m(\omega, k) = 0$, are determined as before by formulas (2.9) and (2.16), in which Ω, Ω_H must be replaced by Ω_k, Ω_{Hk}. $\omega_m(k)$ increases with increasing wave vector; in the region $Ik^2 \gg \Omega_M$ the frequency ω_m is proportional to k^2.

If $\omega_{1,2}$ and ω_m are not close to each other, then we can readily obtain from (3.3) with $\omega \approx \omega_{1,2}$ or $\omega \approx \omega_m$ the following expressions for the oscillation frequencies:

$$\omega = \tilde{\omega}_{1,2}\left[1 - \frac{B(\omega_{1,2})}{\alpha_e n^2 A_m(\omega_{1,2})}\right], \tag{3.5}$$

$$\omega = \omega_m(k)\left[1 - \frac{B(\omega_m)}{\alpha_m n^2 A_e(\omega_m)}\right], \tag{3.6}$$

where

$$\alpha_m = \omega_m (dA_m/d\omega)_{\omega=\omega_m}.$$

For ferromagnets with "easy axis" and "easy plane" anisotropy, the coefficients α_m are respectively

$$\alpha_m = \frac{2\omega_m^2}{\omega_m^2 - \Omega_k^2}, \quad \alpha_m = \frac{2\omega_m^2}{\omega_m^2 - \Omega_k \Omega_{Hk}}$$

From (3.5) and (3.6) it follows that the corrections to the frequencies $\sim B/n^2$, which take into account the coupling of the oscillations, become significant when $\tilde{\omega}_{1,2} \approx \omega_m$. However, when $\tilde{\omega}_{1,2} \to \omega_m$ the formulas (3.5) and (3.6) can no longer be used.

For $\tilde{\omega}_{1,2} \approx \omega_m$ we seek a solution of (3.3) in the form

$$\omega = \tilde{\omega}_j(1 + \delta) \quad (j = 1, 2), \tag{3.7}$$

where $|\delta| \ll 1$. We then obtain for δ the expression

$$\delta = \frac{1}{2}\left\{\left(\frac{\omega_m}{\tilde{\omega}_j} - 1\right) \pm \left[\left(\frac{\omega_m}{\tilde{\omega}_j} - 1\right)^2 - \frac{4B(\tilde{\omega}_j)}{\alpha_m \alpha_e n^2}\right]^{1/2}\right\}. \tag{3.8}$$

Formulas (3.7) and (3.8) determine the frequencies of the coupled electrostatic and magnetostatic oscillations. With increasing difference $|\omega_m - \tilde{\omega}_j|$ the expression (3.7) goes over into (3.5) and (3.6). When $\tilde{\omega}_j = \omega_m$ and $\omega_p \sim \omega_B \sim \omega_m$ we obtain $\delta \sim \omega/kc \ll 1$. The behavior of the frequencies of the coupled electrostatic and magnetostatic oscillations is shown schematically in Fig. 5.

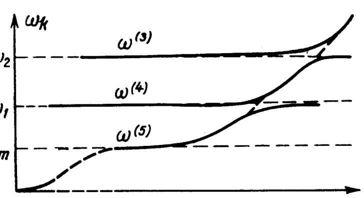

FIG. 5

4. INTERACTION OF SPIN AND CYCLOTRON WAVES

It is well known (see, for example, [8]), that when $\theta = \pi/2$ there can propagate in the plasma, taking into account the finite magnitude of the Larmor radius of the electrons, electromagnetic waves with frequencies that come close when $kr_L \ll 1$ or $kr_L \gg 1$ to values that are multiples of the electron gyrofrequencies (electron cyclotron waves). Such waves exist in ordinary metals.

In ferromagnetic semiconductors and metals, the frequency of the spin waves $\omega_m(k)$ can be close to the frequencies of the cyclotron waves. In this case the spectra of the cyclotron and spin waves change significantly.

We confine ourselves to an examination of ferromagnets with magnetic anisotropy of the "easy axis" type, when the constant magnetic field is parallel to the anisotropy axis. The magnetic permeability tensor, with account of spatial dispersion, is determined in this case by formulas (2.3), in which Ω should be replaced by Ω_k (see (3.2)).

If the equilibrium velocity distribution function is isotropic, then the dielectric tensor takes the form[5]

$$\hat{\varepsilon}(\omega, k) = \begin{pmatrix} \varepsilon_1 & i\varepsilon_2 & 0 \\ -i\varepsilon_2 & \varepsilon_1' & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix} \tag{4.1}$$

(we recall that we are considering waves propagating perpendicular to the constant magnetic field

H, that is, $\theta = \pi/2$). Using formulas (4.1), (2.3) and Maxwell's equations, we can readily obtain the following dispersion equations:

$$\varepsilon_1\varepsilon_1' - \varepsilon_2^2 - \varepsilon_1 n^2 = 0, \qquad (4.2)$$

$$\varepsilon_3 - n^2\frac{\omega_m^2(k) - \omega^2}{(\Omega_k + \Omega_M)^2 - \omega^2} = 0. \qquad (4.3)$$

The first of these equations describes electromagnetic waves that are not coupled to spin waves, while the second describes coupled spin and cyclotron waves. Let us examine the latter in greater detail.

In accordance with [5], the quantity $\epsilon_3(\omega, k)$ is defined by the formula

$$\varepsilon_3(\omega, k) = 1 - \frac{\omega_p^2}{\omega^2}a_0 - 2\omega_p^2\sum_{l=1}^{\infty}\frac{a_l}{\omega^2 - (l\omega_B)^2}. \qquad (4.4)$$

For semiconductors with Maxwellian particle velocity distribution, the values of a_l are[5]

$$a_l = e^{-\xi}I_l(\xi), \quad \xi = (kv_T/\omega_B)^2, \qquad (4.5)$$

where $I_l(\xi)$ is a modified Bessel function.

For metals we have

$$a_l = \int_0^{\pi/2} d\vartheta \sin\vartheta\cos^2\vartheta\Big\{J_l'^2(\beta\sin\vartheta) + \Big(3 - \frac{l^2}{\beta^2\sin^2\vartheta}\Big)J_l^2(\beta\sin\vartheta)\Big\}, \qquad (4.6)$$

where $\beta = kv_F/\omega_B$, and J_l are Bessel functions.

Knowing the frequency dependence of $\epsilon_3(\omega, k)$, which is defined by (4.4), we can readily obtain by a graphic method a solution of (4.3) for the general case. Figure 6 shows schematically a plot of the functions $\epsilon_3(\omega)$ and

$$f \equiv \frac{c^2k^2}{\omega^2}\,\frac{\omega_m^2(k) - \omega^2}{(\Omega_k + \Omega_M)^2 - \omega^2}.$$

The points of intersection 1, 2, ... correspond to solutions of equation (4.3). From Fig. 5 we see that in the frequency interval $\omega_B < \Omega_k + \Omega_M < (l+1)\omega_B$ there exist two solutions lying in the intervals

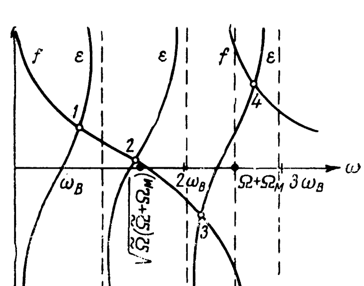

FIG. 6

$$l\omega_B < \omega_{\rm I} < \Omega_k + \Omega_M, \quad \Omega_k + \Omega_M < \omega_{\rm II} < (l+1)\omega_B;$$

in the intervals $l'\omega_B < \omega < (l'+1)\omega_B$, where $l' \neq l$ (l, l' are integers[2)]), there is one solution each.

Explicit expressions for the frequency $\omega(k)$, which are solutions of the dispersion equation (4.3), can be obtained only in certain particular cases. Let us consider first ferromagnetic metals. In this case Eq. (4.3) is conveniently rewritten in the form

$$\Big[a_0 + 2\sum_{l=1}^{\infty}\frac{a_l\omega^2}{\omega^2 - (l\omega)^2} - \frac{\omega^2}{\omega_p^2} + \beta^2\Big(\frac{v_A}{v_F}\Big)^2\Big] \times [(\Omega_k + \Omega_M)^2 - \omega^2] - \Omega_M(\Omega + \Omega_M)\beta^2\Big(\frac{v_A}{v_F}\Big)^2 = 0, \qquad (4.7)$$

where $v_A^2 = B^2/4\pi n_0 m$ is the Alfven velocity. Since the electron velocity in metals is large ($n_0 \sim 10^{22}$ cm^{-3}), at not too large fields B($B \lesssim 10^6$ G) we have the inequality $v_A \ll v_F$. In other words, the coupling between magnetic and cyclotron waves in metals is proportional to the small parameter $(v_A/v_F)^2$.

If, in addition, the following inequality is satisfied

$$\beta^2 = (v_F k/\omega_B)^2 \ll 1,$$

then the solutions of the dispersion equation (4.7) are of the form

$$\omega_l = l\omega_B(1 - a_l(k)), \quad \omega_s = \Omega_k + \Omega_M \qquad (4.8)$$

(we take into account here the fact that $a_0 \approx 1$ and $a_l \ll 1$).

If the spin-wave frequency $\omega_S = \Omega_K + \Omega_M$ approaches the cyclotron-wave frequency $\omega_l(k)$, then account must be taken of the interaction of these waves, and we have in this case

$$\omega = \omega_l(k)(1 - \Delta), \quad \omega_l \approx \omega_s,$$

where

$$\Delta = \frac{1}{2}\Big\{\Big(\frac{\omega_s}{\omega_l} - 1\Big) \pm \Big[\Big(\frac{\omega_s}{\omega_l} - 1\Big)^2 + 2\Big(\frac{kv_F}{\omega_B}\Big)\Big(\frac{v_A}{v_F}\Big)^2\frac{\Omega_M}{\omega_l^2 f'(\omega_l)}\Big]^{1/2}\Big\},$$

$$f'(\omega_l) = -\frac{d}{d\omega}\Big(\frac{\omega^2}{\omega_p^2}\varepsilon_3(\omega, k)\Big). \qquad (4.9)$$

We now consider ferromagnetic conductors with a small number of carriers when $v_A^2 \gg v_T^2$ ($B^2/4\pi n_0 m \gg T$). The dispersion equation (4.3) is represented in this case in the form

[2)]For nonmagnetic media when $\mu_1 = 1$ and $\mu_2 = 0$ in all intervals $l\omega_B < \omega < (l+1)\omega_B$ there is only one solution each.

$$a_0 + 2\sum_{l=1}^{\infty}\frac{a_l\omega^2}{\omega^2 - l^2\omega_B^2} - \frac{\omega^2}{\omega_p^2}$$
$$+ \left(\frac{v_A}{v_T}\right)^2 \xi \frac{\Omega_k(\Omega_k + \Omega_M) - \omega^2}{(\Omega_k + \Omega_M)^2 - \omega^2} = 0,$$
$$\xi = (v_T k/\omega_B)^2. \tag{4.10}$$

If the wave vector k is not too small, so that $(v_A/v_T)^2\xi \gg 1$, then one of the solutions of the dispersion equation will be

$$\omega = \omega_m(k) = \sqrt{\Omega_k(\Omega_k + \Omega_M)}.$$

Cyclotron waves correspond to solutions of the dispersion equation (4.10) in the form

$$\omega = l\omega_B(1 + \Delta_1), \tag{4.11}$$

where

$$\Delta_1 = -\left(\frac{v_T}{v_A}\right)^2 \frac{1}{\xi} e^{-\xi} I_n(\xi) \frac{(\Omega_k + \Omega_M)^2 - (l\omega_B)^2}{\omega_m^2 - (l\omega_B)^2},$$
$$\xi \gg (v_T/v_A)^2;$$

$$\Delta_1 = -\left(\frac{v_T}{v_A}\right)^2 \frac{\xi^n}{n!2^n}\left[\left(\frac{v_T}{v_A}\right)^2\right.$$
$$\left. + \xi\frac{\omega_m^2 - (l\omega_B)^2}{(\Omega_k + \Omega_M)^2 - (l\omega_B)^2}\right]^{-1}, \quad \xi \ll 1.$$

These expressions can be used when $\omega_m(k)$ are not too close to $l\omega_B$. If ω_m is sufficiently close to $l\omega_B$, then the root of (4.10) takes the form

$$\omega = l\omega_B(1 + \delta_1), \tag{4.12}$$

where

$$\delta_1 = \frac{1}{2}\left\{\left(\frac{\omega_m}{l\omega_B} - 1\right) \pm \left[\left(\frac{\omega_m}{l\omega_B} - 1\right)^2 + \left(\frac{v_T}{v_A}\right)^2\gamma\right]^{1/2}\right\},$$
$$\left(\frac{v_T}{v_A}\right)^2 \ll \xi;$$

$$\delta_1 = \frac{1}{2}\left[\frac{\omega_m}{l\omega_B} - \left(\frac{v_T}{v_A}\right)^2 \frac{(\Omega_k + \Omega_M)^2 - \omega_m^2}{2\xi\omega_m^2} - 1\right]$$
$$\pm\left[\frac{\omega_m}{l\omega_B} - \frac{(\Omega_k + \Omega_M)^2 - \omega_m^2}{2\xi\omega_m^2} - 1\right]^2 + \left(\frac{v_T}{v_A}\right)^2\gamma\bigg]^{1/2},$$

$$\gamma = \frac{2e^{-\xi}I_n(\xi)}{\xi}\left[\frac{(\Omega_k + \Omega_M)^2}{\omega_m^2} - 1\right], \quad \left(\frac{v_T}{v_A}\right)^2 \ll \xi \ll 1.$$

Formulas (4.11) and (4.12) determine the frequencies of coupled spin and cyclotron waves in semiconductors with low electron density.

In conclusion we note that the analysis presented above can be readily generalized for the case of carriers of several species.

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Translated by J. G. Adashko
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