CONCERNING THE INVESTIGATION OF THE STATISTICAL PROPERTIES OF INCOHERENT LIGHT

V. S. LETOKHOV

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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It is shown that by using the amplified spontaneous emission of radiation ("super-radiation") from negative-temperature media, one can measure the Bose-Einstein distribution function for photons in a quantum state. Two possible kinds of experiments are considered.

1. CONSIDERABLE interest has recently been evinced in the study of the statistical properties of light (see, for example, [1]). The main investigations are carried out in three directions: a study of the statistical properties of the radiation of new light sources—lasers,[2] which are essential in the determination of the spectral width and the noise characteristics of the laser emission, establishing the difference between the statistical properties of laser light and the incoherent light of the conventional (thermal or luminescent) sources,[3, 4] and working out a general coherence theory including a description of laser and incoherent radiation.[5, 6] The purpose of this note is to draw attention to the possibility of an experimental study of the properties of incoherent light which presented itself with the appearance of the methods of generation of coherent light.

2. Let us consider certain statistical properties of equilibrium radiation which is an example of incoherent light. For equilibrium radiation the probability of filling one quantum state of an ensemble of a large number G of quantum states filled with a large number N of photons ($N/G = \langle n\rangle$; $N, G \gg \langle n\rangle$) with n quanta is given by the Bose-Einstein distribution function:[7]

$$p(n) = \langle n\rangle^n / (1 + \langle n\rangle)^{n+1}. \tag{1}$$

In the classical limit $\langle n\rangle \gg 1$ we obtain on observing single quantum states the distribution function:

$$p_{\text{cl}}(n) = \langle n\rangle^{-1} \exp(-n/\langle n\rangle). \tag{2}$$

In the classical limit one can go over to the wave representation: $n \sim A^2$, where A is the amplitude of the light field. In this case the amplitude distribution $w(A) = |dn/dA|\, p(A^2)$ is of the form

$$w(A) = (2A/\langle A^2\rangle) \exp(-A^2/\langle A^2\rangle). \tag{3}$$

Consequently, the amplitude of the light field of the photons with a Bose-Einstein distribution (3) has a Rayleigh distribution.[1)]

As was shown experimentally,[4] the nonequilibrium laser radiation does not comply with distributions (2) and (3).

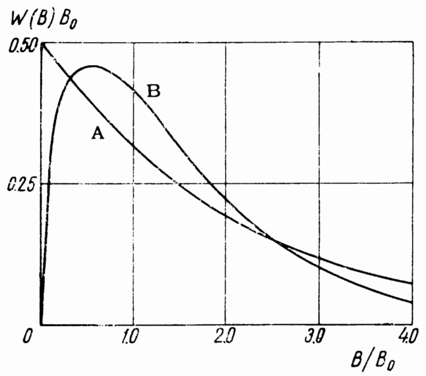
In the quantum limit $\langle n\rangle \ll 1$ the Bose-Einstein distribution function goes over into

$$p_{\text{qu}}(n) = \ln\frac{1}{\langle n\rangle} \exp\left(-n \ln\frac{1}{\langle n\rangle}\right). \tag{4}$$

It is interesting to note that the Bose-Einstein distribution function has, as follows from (2) and (4), the same functional form in the classical and quantum limit, $p(n) = \gamma \exp(-\gamma n)$, the parameter γ being in the classical case $\gamma_{\text{cl}} = 1/\langle n\rangle$ and in the quantum case $\gamma_{\text{qu}} = \ln(1/\langle n\rangle)$.

The Bose-Einstein distribution function for photons has not been verified experimentally either in the classical or in the quantum limit. Classical experiments have until now been unfeasible, since for equilibrium radiation of conventional sources $\langle n\rangle \ll 1$. Experiments in the quantum limit are difficult for another reason. In order to measure the distribution function p(n) one must count photons in single quantum states. In particular this means that the time constant $\Delta\tau$ of the circuit detecting the photons should not exceed $1/\Delta\nu$ where $\Delta\nu$ is the spectral range of the investigated radiation.

[1)]The Rayleigh distribution of the amplitude of the equilibrium radiation field in a narrow spectral range can be obtained from the following considerations. Inasmuch as the radiation is emitted by a large number of atoms independently, the summary field is a random process, and according to the limit theorem of Lyapunov has a gaussian distribution function. The amplitude of a gaussian, narrow-band, random process has a Rayleigh distribution.[8]



Distribution function of the amplitude W(B) of the beat signal in two experiments: A – the amplitude fluctuations of the light oscillations are matched, B – the amplitude fluctuations of the light oscillations are independent.

For the narrowest spectral lines of conventional sources $\Delta\nu \approx 10^9$ cps, and one requires therefore the difficult to attain value of the time constant $\Delta\tau \approx 10^{-9}$ sec.

The development of methods which utilize stimulated radiation made it possible to produce new sources of incoherent radiation in the optical region which possess a very high brightness temperature. They differ from the laser by the absence of feedback,[9] and indeed radiate a considerably intensified spontaneous radiation into a large number of quantum states. This type of radiation is often referred to as "super-radiation" in order to differentiate it from coherent radiation generated when there is feedback. For example, Röss[10] describes a source of super-radiation in the form of an optically excited, cooled ruby crystal. The radiation of this source according to data cited in [10] should have an average occupation number of the quantum state $\langle n\rangle \approx 10^5$, and should fill simultaneously about 10^7 quantum states. Intense super-radiation has also been obtained in gases.

The statistics of super-radiation in sources without saturation (for instance, in the source of [10]) should be close to the statistics of equilibrium radiation, since in such sources there occurs essentially a linear amplification of the spontaneous emission of a great number of atoms in a large number of quantum states. The setting up of experiments for measuring the amplitude distribution function of incoherent light in the classical limit with the aid of super-radiation sources appears realistic. We restrict ourselves to a discussion of precisely this case.

The main difficulty in carrying out experiments in the quantum limit ($\Delta\nu \gtrsim 10^9$ cps) can also be eliminated by the use of long Fabry-Perot interferometers in conjunction with conventional radiation sources.

3. Experiments in the classical limit ($\langle n\rangle \gg 1$). In this case it is most convenient to apply the method of heterodyning two light oscillations of different frequencies. The method consists of the following. Two light oscillations with different mean frequencies ω_1 and ω_2 are heterodyned with the aid of a photo mixer. The amplitude of the beat signal B(t) at the frequency $\Delta\omega = \omega_2 - \omega_1$ is proportional to the product of the amplitudes $A_1(t)$ and $A_2(t)$ of the light oscillations $E_1(t) = A_1(t)\cos[\omega_1 t + \varphi_1(t)]$ and $E_2(t) = A_2(t)\cos[\omega_2 t + \varphi_2(t)]$. The amplitude distribution of the beat signal W(B) is related to the combined distribution of the amplitudes of the light oscillations:[8] 2)

$$W(B) = \int_0^\infty w_{12}(A_1, A_2) A_1^{-1}\, dA_1; \qquad B = A_1 A_2. \quad (5)$$

(The constant coefficient depending on the heterodyning efficiency is included in B.)

The form of the function w_{12} depends on the experimental conditions. Let us consider two types of experiments.

A. The amplitude fluctuations of the light oscillations are completely matched, i.e., $A_2(t) = cA_1(t)$, where c is a constant coefficient (for brevity, let c = 1). Then

$$w_{12}(A_1, A_2) = w(A_1)\,\delta(A_1 - A_2),$$

where w(A) is the amplitude distribution of the light oscillation. According to (5) and the equation $w(A) = |dB/dA|\,p(B)$ we then have

$$W(B) = {}^1\!/_2 B^{-1/2} w(\sqrt{B}) = p_{\text{cl}}(B). \quad (6)$$

Thus the measured pulse-height distribution of the beat signal W(B) coincides with the distribution of the number of photons $p_{\text{cl}}(n)$. If the investigated light obeys a Bose-Einstein distribution function then W(B) should coincide with the distribution (2).

To carry out such an experiment, it is sufficient to select a narrow spectral range $\Delta\nu \approx 1$ Mcs

2)Strictly speaking, the distribution of the amplitude of the beat signal can change owing to a certain stochastic nature of the emission of a photoelectron in the absorption of a photon. It was shown by Mandel[7] and Ghielmetti[11] that the photoelectron distribution P(N) is related to the photon distribution p(n) by the Poisson transformation:

$$P(N) = \int_0^\infty \frac{(qn)^N}{N!} e^{-qN} p(n)\, dn, \qquad n = \int_0^T m(t')\, dt',$$

where T is the time constant of the observation, q is the detector efficiency, and m(t) is the intensity of the photon flux. One can readily show that if p(n) obeys the distribution (2), then P(N) = p(n) and therefore the stochastic nature of the relation between the photon and the photoelectron does not change the distribution function p(n).

with a long (meter) Fabry-Perot interferometer, split the ray at the interferometer exit into two rays, and shift the frequency of one of the rays by means of the Doppler effect by $\nu_D = 10-15$ Mcs before heterodyning. Then one must separate the beat signal in the entire frequency band $\sim 2\Delta\nu$ with its center at ν_D and measure the distribution density of its amplitude with a pulse-height analyzer.

B. The amplitude fluctuations of the light oscillations are completely independent. In this case

$$w_{12}(A_1, A_2) = w(A_1)\,w(A_2).$$

If w(A) is the Rayleigh distribution (3), then according to (5)

$$W(B) = BB_0^{-2}K_0(B/B_0), \qquad (7)$$

where $B_0 = 2\pi^{-1}\langle B\rangle$, $K_0(x)$ is the cylinder function, and it is assumed that $\langle A_1^2\rangle = \langle A_2^2\rangle$. The form of W(B) for experiments A and B with photons having a Bose-Einstein distribution is shown for comparison in the figure.

To carry out experiment B, one can employ spectrally narrow rays ($\Delta\nu \approx 1-10$ Mcs) from two super-radiation sources, or two neighboring spectral lines appearing in the transmission of the super-radiation line of one source through a long Fabry-Perot interferometer (transmission "modes").

Finally it can be shown that the ratio ξ of the power of the beat signal to the power of the shot noise in observing the radiation in single quantum states is given by the expression

$$\xi = q\langle n\rangle, \qquad (8)$$

where q is the efficiency of the photomixer (in electrons per photon), and $\langle n\rangle$ is the mean occupation number of the quantum state. If any type of absorbing element (for instance, an interferometer) is placed between the radiation source and the photomixer, then ξ is somewhat smaller (see [12]).

It follows from (8) that for photomixers with $q \approx 0.01$ one can attain with available sources[10] $\xi \approx 10^3$.

There exists thus a real possibility of measuring the amplitude distribution of incoherent light directly.

In conclusion I take this opportunity to express my deep gratitude to N. G. Basov for a discussion of the problem and critical remarks.

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Translated by Z. Barnea
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