SINGULARITIES OF THE SPIN WAVE SPECTRUM OF FERROMAGNETIC AND ANTIFERROMAGNETIC METALS

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The nature of the singularities of the spin wave spectrum of a ferromagnetic metal, due to interaction with conductivity electrons, is investigated. It is shown that two new spin wave branches arise in antiferromagnetic metals. They are caused by the interaction between spin waves corresponding to oscillations of the sublattice magnetic moments and zero-sound waves.

1. THE author has shown in an earlier article[1] that in addition to Fermi excitations, there occur in ferromagnetic metals also spin waves which, as in ferrodielectrics, possess a quadratic dispersion law. According to [1], an interaction exists between the spin waves and the Fermi excitations; this interaction is determined by the characteristics of the Fermi spectrum. We shall consider here the influence of this interaction on the spin-wave spectrum in ferromagnetic metals. We shall show that the spectrum obtained in [1] remains valid so long as the momentum k of the spin wave is small compared with the separation $\Delta p_0 = p_+ - p_-$ between the Fermi surfaces of the conduction electrons (p_+, p_- —characteristic values of the Fermi momenta of electrons with positive and negative spin orientations, respectively). In this region of values of k, no decay of the spin wave into an electron and a hole from the conduction band takes place, owing to the impossibility of satisfying the energy and momentum conservation laws simultaneously. At values of k close to Δp_0, a threshold occurs for the decay of the spin wave into a pair of Fermi excitations, and this leads to the appearance of a singularity in the spin-wave spectrum of a metallic ferromagnet.

Kaganov and Tsukernik[2] and Turov[3] investigated spin waves in antiferromagnets. Since the spins comprising the magnetic sublattices of the investigated ferromagnets were assumed to be stationary, the results of these investigations actually pertain to dielectrics. We consider an antiferromagnetic metal whose lattice ions form two magnetic sublattices shifted by a half-period relative to each other, with magnetic moments that cancel each other. It will be shown below that in such a metal, if it is invariant with respect to the space inversion, the Fermi surfaces of the conduction electrons for different signs of spin polarization coincide. This uncovers a possibility for the existence of spin excitations of the zero-sound type.[4] As a result of the interaction of the oscillations of the magnetic sublattices with the conduction electrons, the spectrum of the magnetic excitations of the metallic antiferromagnet differs essentially from the spectrum of the spin waves in an antiferromagnetic dielectric.

2. Before we investigate the singularities of the spin-wave spectrum in ferromagnetic metals, let us obtain certain approximate relations for the quantities that characterize the interactions of the Fermi excitations and the spin waves.

We assume for simplicity that in the metal in question there are only two incompletely filled bands of Fermi excitations: the d-band, characterized by a large energy-level density, and the s-band (or conduction-electron band) with low level density. In [1] we obtained the condition for the occurrence of ferromagnetism, which constitutes a relation for one of the limits of the exchange vertex part of the interaction of two Fermi excitations in the paramagnetic states:

$$\Gamma^{\omega}(p, p') = \lim_{\mathbf{k}=0,\ \omega\to 0} \Gamma(p, p'; k).$$

The quantities Γ^{ω} are characteristics of the given metal and depend little on how close its state is to the ferromagnetic transition. In the ferromagnetic state they correspond to the regular part of the vertex Γ. If we denote the characteristics of the exchange interaction of the s-excitations with one another by Γ_{ss}, those of the s- and d-excitations with one another by Γ_{dd}, and if we denote by ϑ_s and ϑ_d the level densities for the s- and d-band, respectively, then in the presence of ferromagnetism, as follows from formula (10) in [1], they are related in order of magnitude by the following equation:

$$(1 + \vartheta_s\Gamma_{ss})(1 + \vartheta_d\Gamma_{dd}) \sim \vartheta_s\vartheta_d\Gamma_{sd}^2. \qquad (1)$$

If none of the quantities Γ is anomalously small, as is apparently the case, for example in metals of the iron group, then $\Gamma_{ss}\Gamma_{dd} \sim \Gamma_{sd}^2$, that is, the ferromagnetism exists as a result of the self-consistent interaction of the s- and d-electrons. Since the characteristic energies which determine the exchange interaction of the d-electrons with one another are of the order of Θ, we have $\Gamma_{sd} \sim \sqrt{\Theta/\epsilon_F}\,\Gamma_{ss}$ (Θ is the Curie temperature and ϵ_F the Fermi energy of the conduction electrons). If we denote by M_s the contribution of the conduction electrons to the total magnetic moment of the system M_0, then it follows from the resultant relation for the interactions that $M_s/M_0 \sim \sqrt{\Theta/\epsilon_F}$.

In the case when the d-electrons interact directly with one another very weakly ($\vartheta_d\Gamma_{dd} \ll 1$), the following relation is obtained: $\Gamma_{sd} \sim \sqrt{\Gamma_{ss}/\vartheta_d}$. This situation is apparently realized in metals of the rare-earth elements and is called the indirect exchange interaction. Since in this case $\vartheta_s/\vartheta_d \sim \sqrt{\Theta/\epsilon_F}$ we have also in this case $M_s/M_0 \sim \sqrt{\Theta/\epsilon_F}$.

A case is also possible in which the interaction between the s- and d-excitations is very small ($\Gamma_{sd} \ll \sqrt{\Theta/\epsilon_F\Gamma_{ss}}$). In this case the ferromagnetism is the result of exchange interaction of d-electrons only; from (1) we get $\vartheta_d\Gamma_{dd} \sim -1$, the conduction electrons are, so to speak, an independent subsystem of the ferromagnetic metal, and $M_s/M_0 \ll \sqrt{\Theta/\epsilon_F}$.

It follows from the obtained relations that if the interaction between the s- and d-excitations is not anomalously small, the ratio of the characteristic value of the separation $\Delta p_0 = p_+ - p_-$ of the Fermi surfaces of the conduction electrons (p_+, p_- — Fermi momenta for the excitations with positive and negative spin orientations) to the Fermi momentum itself is

$$\Delta p_0 / p_0 \sim \sqrt{\Theta / \varepsilon_F}.$$

3. As shown in [1], $\Gamma_{n_+m_-l_-s_+}(p_1, p_2; k)$, the component of the two-particle vertex part transverse to the spin (the notation here is the same as in [1]), whose poles with respect to the momentum transfer $k(\omega, \mathbf{k})$ determine the spectrum of the spin waves of the system, has for $\mathbf{k} = 0$ the form

$$\Gamma_{n_+m_-l_-s_+}(p_1, p_2; k) = \Delta_n^+(p_1)\frac{1}{\omega - 2\mu_0 H}\Delta_m^-(p_2)\,\delta_{nl}\delta_{ms},$$

$$\Delta_n^\pm(p) = \sqrt{\frac{\mu_0}{M_0}}\,[\Sigma_n^+(p) - \Sigma_n^-(p)], \qquad (2)$$

where $\Sigma_n^\pm(p)$ is the self-energy part of the n-th band of excitations, and μ_0 is the Bohr magneton. It follows from the reasoning presented above that if the interaction between the s- and d-excitations is not anomalously small (only such cases will be considered), then the amplitude of the interaction of the spin waves with the conduction electrons for $p \sim p_0$ turns out to be of the order of $\Delta_s^\pm \sim (\mu_0\Theta\epsilon_F/M_0)^{1/2}$. The possibility of the spin wave decaying into an electron and a hole from the s-band when $k > \Delta p_0$ leads to a situation wherein the diagram of Fig. 1, which makes a contribution to Γ, becomes singular, owing to the fact that when $k > \Delta p_0$ there appear points where the poles of the Green's functions, corresponding to the internal electron lines, coalesce.

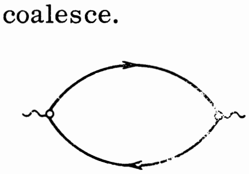

FIG. 1

In order to find the corresponding singularity in the spin-wave spectrum, it is necessary to sum all the diagrams which contain an arbitrary number of repetitions of the G-function loops. This procedure is expressed by an equation that relates the vertex part Γ with the irreducible vertex $\Gamma^{(1)}$. (The function $\Gamma^{(1)}$ does not contain the singularity which is separated in the equation.) We have

$$\Gamma_{n_+m_-n_-m_+}(p_1, p_2; -k) = \Gamma^{(1)}_{n_+m_-n_-m_+}(p_1, p_2; -k)$$

$$- i\sum_{m'}\int\frac{d^4p'}{(2\pi)^4}\Gamma^{(1)}_{n_+m'_-n_-m'_+}(p_1, p'; -k)$$

$$\times G_{m'}^-(p' - k)\,G_{m'}^+(p')\,\Gamma_{m'_+m_-m'_-m_+}(p', p_2; -k). \qquad (3)$$

The fact that $\Gamma^{(1)}(p_1, p_2; -k)$ is regular when $k \to 0$ gives us the right to replace it by $\Gamma^{(1)}(p_1, p_2; -\tilde{k})$ ($\tilde{k} = (\omega, 0)$) accurate to the quantity $\sim |\mathbf{k}|/p_0$, and allows to obtain from (3) the following equation:

$$\Gamma_{n_+m_-n_-m_+}(p_1, p_2; -k) = \Gamma_{n_+m_-n_-m_+}(p_1, p_2; -\tilde{k})$$

$$- i\sum_{m'}\int\frac{d^4p'}{(2\pi)^4}\Gamma_{n_+m'_-n_-m'_+}(p_1, p', -\tilde{k})$$

$$\times [G_{m'}^-(p' - k) - G_{m'}^-(p' - \tilde{k})]$$

$$\times G_{m'}^+(p')\,\Gamma_{m'_+m_-m'_-m_+}(p', p_2; -k). \qquad (4)$$

Since $\Gamma(p_1, p_2; -\tilde{k})$ is given by (2) when $\omega \ll \Theta$, the solution of (4) is

$$\Gamma_{n_+m_-n_-m_+}(p_1, p_2; -k) = \Delta_n^+(p_1)D(k)\Delta_m^-(p_2), \quad (5)$$

where

$$D(k) = 1/[\omega - 2\mu_0 H - K(\omega, \mathbf{k})], \quad (6)$$

$$K(\omega, \mathbf{k}) = -i\sum_r \int \frac{d^4p}{(2\pi)^4}\Delta_r^+(p)\Delta_r^-(p)$$

$$\times [G_r^-(p-k) - G_r^-(p-\tilde{k})]G_r^+(p). \quad (7)$$

The function $K(\omega, \mathbf{k})$ consists of an analytic part and a term containing a singularity. The singularity is manifest in the term with $r = s$ from the regions of integration near the Fermi surfaces. The remaining components, and also the contribution of the regions of integration, are far from the Fermi surface, have in the term with $r = s$ an analytic form, and are of the order $\Theta(k/p_0)^2$.

For simplicity in the calculations we assume that the Fermi surfaces of the conduction band are spheres of radii p_+ or p_-, respectively, for excitations with positive or negative spin orientation. It will be seen below that the result does not change significantly on going over to the case of an arbitrary form of the Fermi surfaces. In the case in question, the Green's function of the s-electrons near the Fermi boundary has the following form:

$$G_s^\pm(\varepsilon, \mathbf{p}) = \frac{a_\pm}{\varepsilon - v_\pm(|\mathbf{p}| - p_\pm) + i\delta \operatorname{sign} \varepsilon} \quad (8)$$

(v_+ and v_- are the velocities of excitations on the corresponding Fermi surfaces, and a_+ and a_- are renormalization constants). It is obvious that the relations

$$\frac{p_+ - p_-}{p_+} \qquad \frac{v_+ - v_-}{v_+} \qquad \frac{a_+ - a_-}{a_+}$$

are of the order of $\sqrt{\Theta/\epsilon_F}$. Using (8), we can rewrite the function $K(\omega, \mathbf{k})$ in the form

$$K(\omega, \mathbf{k}) = \alpha k^2 + \Pi(\omega, \mathbf{k}), \quad \alpha \sim \Theta/p_0^2, \quad (9)$$

$$\Pi(\omega, \mathbf{k}) = -i\int \frac{d^4p}{(2\pi)^4} a_+a_-\Delta_s^2(p)$$

$$\times [\varepsilon - v_+(|\mathbf{p}| - p_+) + i\delta \operatorname{sign}\varepsilon]^{-1}$$

$$\times \{[\varepsilon - \omega - v_-(|\mathbf{p} - \mathbf{k}| - p_-) + i\delta \operatorname{sign}(\varepsilon - \omega)]^{-1}$$

$$- [\varepsilon - \omega - v_-(|\mathbf{p}| - p_-) + i\delta \operatorname{sign}(\varepsilon - \omega)]^{-1}\}. \quad (10)$$

The spin-wave spectrum is now determined from the equation

$$\omega - \alpha k^2 - \Pi(\omega, \mathbf{k}) = 0. \quad (11)$$

We note that the integrals with respect to the energies that enter in (10) converge in the interval

$$\varepsilon \sim v\Delta p_0 \sim \sqrt{\Theta\varepsilon_F} \ll \varepsilon_F$$

(we recall that $\Delta p_0 = p_+ - p_-$). The quantity $\Delta_S^2(p)$, unlike the pole expressions for the Green's functions under the integral sign, is slowly varying, and we shall therefore replace it by its value at $p \sim p_0$.

Carrying out the integration in (10), we arrive at the following expression for $\Pi(\omega, \mathbf{k})$:

$$\Pi(\omega, \mathbf{k}) = \gamma\sqrt{\Theta\varepsilon_F}\left\{\frac{\omega - v_+\delta}{v_+k}\ln\left|\frac{\omega - v_+\delta}{2v_+k}\right| - \frac{\omega - v_-\delta}{2v_-k}\ln\left|\frac{\omega - v_-\delta}{2v_-k}\right| - 2\left(1 + \frac{\omega - v_+\delta}{2v_+k}\right)\right.$$

$$\times \ln\left|1 + \frac{\omega - v_+\delta}{2v_+k}\right| + 2\left(1 + \frac{\omega - v_-\delta}{2v_-k}\right)$$

$$\times \ln\left|1 + \frac{\omega - v_-\delta}{2v_-k}\right| + 2\ln\left|\frac{\omega + v_+\Delta p_0}{\omega + v_-\Delta p_0}\right|$$

$$\left. - i\pi\left[\frac{v_+\delta - \omega}{v_+k}\theta(v_+\delta - \omega) - \frac{v_-\delta - \omega}{v_-k}\theta(v_-\delta - \omega)\right]\right\}, \quad (12)$$

where we used the notation

$$\delta = k - \Delta p_0, \quad \gamma = \frac{a_+a_-\Delta_s^2p_0^2}{4\pi^2 v\Delta p_0(v_+ - v_-)} \sim 1,$$

$$\theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}.$$

We now proceed to investigate Eq. (11) for different values of k.

When $k < \Delta p_0$, if $\omega \ll v(\Delta p_0 - k)$, we can expand the expression for $\Pi(\omega, \mathbf{k})$ in terms of the small parameter $\omega/v\delta$. As a result we get

$$\Pi(\omega, \mathbf{k}) = -\gamma\sqrt{\frac{\Theta}{\varepsilon_F}}\,\omega\left[2 + \frac{\Delta p_0}{k}\ln\left|\frac{k - \Delta p_0}{k + \Delta p_0}\right|\right]. \quad (13)$$

Substituting (13) into (11), we obtain an expression for the frequency of the spin waves:

$$\omega(\mathbf{k}) = \left[1 - \gamma\sqrt{\frac{\Theta}{\varepsilon_F}}\left(2 + \frac{\Delta p_0}{k}\ln\left|\frac{k - \Delta p_0}{k + \Delta p_0}\right|\right)\right]\alpha k^2. \quad (14)$$

The region of applicability of this formula, as can be seen from the formula itself, is determined by the inequality

$$\rho \equiv (\Delta p_0 - k)/\Delta p_0 \gg (\Theta/\varepsilon_F)^{3/2}.$$

In this region there is no damping connected with the decay into Fermi excitations.

When $k \ll \Delta p_0$ the dispersion law of the spin waves becomes quadratic, $\omega = \alpha k^2$.

Since $\omega/v\Delta p_0 \ll 1$, the function $\Pi(\omega, \mathbf{k})$ can be expanded for values of k close to Δp_0, when $|\rho|$

$\ll 1$, in terms of the quantities $(\omega - v_+\delta)/v_+k$ and $(\omega - v_-\delta)/v_-k$:

$$\Pi(\omega, k) = \gamma\sqrt{\Theta\varepsilon_F}\left\{\frac{\omega - v_+\delta}{v_+k}\left(1 + \ln\left|\frac{\omega - v_+\delta}{2v_+k}\right|\right) - \frac{\omega - v_-\delta}{v_-k}\left(1 + \ln\left|\frac{\omega - v_-\delta}{2v_-k}\right|\right) - i\pi\left[\frac{v_+\delta - \omega}{v_+k}\theta(v_+\delta - \omega) - \frac{v_-\delta - \omega}{v_-k}\theta(v_-\delta - \omega)\right]\right\}. \quad (15)$$

It is easy to verify that this formula is valid, accurate to quantities of the order $\sqrt{\Theta/\epsilon_F}\,\omega\Delta p_0\delta/k^2$. The region of applicability of (15) thus overlaps the region of applicability of formula (13). The velocity of the spin waves for $|\rho| \ll 1$ is determined by the expression

$$\frac{\partial\omega}{\partial k} = \frac{2\alpha k + \partial\Pi/\partial k}{1 - \partial\Pi/\partial\omega} = \left[\eta v_+\left(\ln\left|\frac{\omega - v_+\delta}{v_+\Delta p_0}\right| - \ln\left|\frac{\omega - v_-\delta}{v_-\Delta p_0}\right|\right) - 2\alpha k\right] \times \left[\eta\left(\ln\left|\frac{\omega - v_+\delta}{v_+\Delta p_0}\right| - \frac{v_+}{v_-}\ln\left|\frac{\omega - v_-\delta}{v_-\Delta p_0}\right|\right) - 1\right]^{-1} \quad (16)$$

(here $\eta = \gamma\sqrt{\Theta\epsilon_F}/v_+\Delta p_0 \sim 1$). We see from (16) that with increasing k the quantity $\partial\omega/\partial k$ increases continuously. When $k < \Delta p_0$ and $\omega \ll v(\Delta p_0 - k)$ the velocity of the spin waves is equal to

$$\frac{\partial\omega}{\partial k} \cong 2\alpha k \Big/ \left[1 + \eta\sqrt{\frac{\Theta}{\varepsilon_F}}\ln\left|\frac{\Delta p_0 - k}{\Delta p_0}\right|\right]. \quad (17)$$

When $v|k - \Delta p_0| \ll \omega$ the function $\Pi(\omega, \mathbf{k})$ takes the form

$$\Pi(\omega, k) = \eta\sqrt{\frac{\Theta}{\varepsilon_F}}\left\{\omega\ln\frac{2v\Delta p_0}{\omega} - 2\omega + v\delta + \frac{1}{2}\frac{v^2\delta^2}{\omega}\right\}. \quad (18)$$

The velocity of the spin waves in this interval (which obviously corresponds to $|\rho| \ll (\Theta/\epsilon_F)^{3/2}$ reaches a value of the order of $v\sqrt{\Theta/\epsilon_F}$, and for $k = \Delta p_0$ the spin-wave frequency itself is determined by the equation

$$\omega\left[1 - \eta\sqrt{\frac{\Theta}{\varepsilon_F}}\left(\ln\frac{2v\Delta p_0}{\omega} - 2\right)\right] = \alpha(\Delta p_0)^2. \quad (19)$$

With further increase in k the quantity $\partial\omega/\partial k$ continues to grow rapidly and finally reaches its maximum value v_+ when $\omega = v_+\delta$. In the vicinity of this point, at $|\omega - v_+\delta| \ll \omega - v_-\delta$, the velocity of the spin waves is

$$\frac{\partial\omega}{\partial k} = v_+\left(1 - a\Big/\ln\left|\frac{v\Delta p_0}{\omega - v_+\delta}\right|\right), \quad (20)$$

$$a = \frac{1}{\eta} - \frac{v_+ - v_-}{v_+}\ln\frac{v\Delta p_0}{(v_+ - v_-)\delta_0}$$

(here δ_0 is the root of the equation $\omega = v_+\delta$). Formula (20) is obviously valid when

$$|\delta - \delta_0| \ll {}^1\!/_2\sqrt{\Theta/\varepsilon_F}\,\delta_0.$$

The second derivative becomes infinite at the point $k = \Delta p_0 + \delta_0$, reversing sign:

$$\frac{\partial^2\omega}{\partial k^2} = \frac{a^2 v_+^2}{\omega - v_+\delta}\ln^{-3}\left|\frac{v\Delta p_0}{\omega - v_+\delta}\right| \quad (21)$$

The frequency ω_0 of the spin waves at the point $k = \Delta p_0 + \delta_0$ is the root of the equation

$$\omega_0\left[1 - \eta\sqrt{\frac{\Theta}{\varepsilon_F}}\left(\ln\frac{2v\Delta p_0}{\sqrt{\Theta/\varepsilon_F}\,\omega_0} - 1\right)\right] = \alpha(\Delta p_0)^2. \quad (22)$$

Owing to the possibility of decay into a pair of Fermi excitations, damping of the spin waves appears, and is determined for $\delta - \delta_0 \ll {}^1\!/_2\sqrt{\Theta/\epsilon_F}\,\delta_0$ by the equation

$$\operatorname{Im}\omega(\mathbf{k}) = -\pi(v_+\delta - \omega)\Big/\ln\left|\frac{v\Delta p_0}{v_+\delta - \omega}\right|. \quad (23)$$

In the region $\delta \gtrsim \delta_0(1 + {}^1\!/_2\sqrt{\Theta/\epsilon_F})$ the damping becomes comparable with $\omega - \omega_0$, and therefore, as a result of Heisenberg's uncertainty principle, the frequency remains approximately equal to ω_0 up to values of $k - \Delta p_0$ of the order of several times δ_0. As seen from (16), the spectral curve exhibits a sharp decrease. The spin waves attenuate in this case like

$$\operatorname{Im}\omega(\mathbf{k}) = -\pi\eta\sqrt{\Theta/\varepsilon_F}\,\Delta p_0\omega/k. \quad (24)$$

The decrease in the frequency of the spin wave continues until the momentum becomes equal to

$$k \cong \Delta p_0 + {}^1\!/_2\eta\sqrt{\Theta/\varepsilon_F}\,\Delta p_0.$$

At this point the velocity of the spin waves vanishes and then $\partial\omega/\partial k > 0$. In this region the frequency of the spin waves is considerably lower than $v\delta$. Therefore, as in the case of $k < \Delta p_0$, we can expand the function $\Pi(\omega, \mathbf{k})$ in powers of $\omega/v_+\delta$ and $\omega/v_-\delta$. As a result we arrive at an expression for $\operatorname{Re}\Pi(\omega, \mathbf{k})$ which coincides exactly with (13), and the spin-wave frequency is given by (14). However, unlike the case $k < \Delta p_0$, we have now $(k > \Delta p_0$, $\omega \ll v\delta)$ the spin-wave damping as given by formula (24). When $k \gg \Delta p_0$ the spin waves again have a quadratic dispersion law, but now with a different renormalized coefficient of proportionality in front of k^2:

$$\omega(\mathbf{k}) = (1 - 2\gamma\sqrt{\Theta/\varepsilon_F})\,\alpha k^2. \quad (25)$$

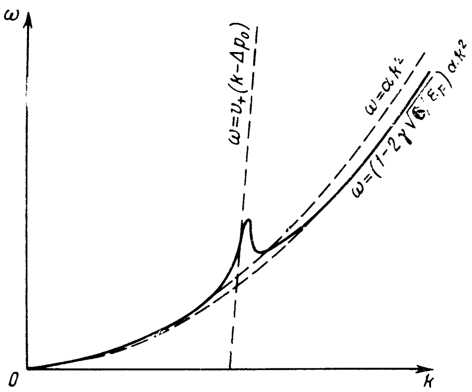

FIG. 2

Thus, the spin-wave spectrum (Fig. 2) is quadratic both when $k \ll \Delta p_0$ and when $k \gg \Delta p_0$. In the intermediate region the dependence of ω on k differs slightly from quadratic, with the exception of the vicinity of the point $k = \Delta p_0$. In the region of wave-vector values close to the separation of the Fermi surfaces of the conduction band, the spectral curve shows a sharp peak. In real metals, its magnitude can reach several dozen percent of the value of ω at these values of k. The width of the peak is of the order of $\sqrt{\Theta/\epsilon_F}\,\Delta p_0$. The maximum spin-wave velocity is attained at the point where the spectral curve crosses the line $\omega = v_+(k - \Delta p_0)$, and is equal to the electron Fermi velocity v_+.

It is easy to verify that in going over to a conduction-electron Fermi surface of arbitrary shape the spin-wave spectrum remains essentially the same. The deviation from the case of isotropic dispersion of the conduction electrons is expressed in the fact that for each direction **k** there is a distinct value of Δp_0 (which is the minimum distance between the Fermi surfaces with opposite spin directions in the direction of the vector **k**). The maximum velocity and the height of the peak at $k \sim \Delta p_0$ depend on the direction of **k**.

We see thus that the singularities of the Fermi spectrum of the conduction electrons are reflected in the spin-wave spectrum. Together with the Kohn anomalies,[5,6] which occur when the spin-wave vector has values equivalent to the external half-sum of the s-band Fermi-surface diameters, the singularity obtained in the spectrum of the spin wave can serve as a means of reconstructing the shape of the Fermi surfaces of the conduction electrons.

4. In order to investigate the spin excitations of an antiferromagnetic metal, let us consider the properties of the single-particle Green's function $G_{\alpha\beta}(\mathbf{r}, t; \mathbf{r}', t')$ of an antiferromagnet, which determines the spectrum of the Fermi excitations. To this end we make use of the symmetry properties of the system. Since the magnetic moment reverses sign upon time reversal, and since the antiferromagnetic metal in question consists of two mirror-symmetry magnetic sublattices which are shifted relative to each other by one half-period of the complete lattice, the symmetry elements of the system include an invariance with respect to the combination of time reversal and translation by one half-period of the lattice. Abrikosov[7] has shown that if $G_{\alpha\beta}(\mathbf{r}, t; \mathbf{r}', t')$ is the Green's function of electrons in the metal, then the G-function of the system into which the metal goes over following time reversal is equal to

$$[\hat{g}\hat{G}(\mathbf{r}', -t'; \mathbf{r}, -t)\,\hat{g}^{-1}]_{\beta\alpha},$$

where $\hat{g}$ is a matrix in spin space:

$$\hat{g} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

It follows therefore that the Green's function of the antiferromagnetic metal under consideration satisfies the relation

$$G_{\alpha\beta}(\mathbf{r}, t; \mathbf{r}', t') = [\hat{g}G(\mathbf{r}' + \mathbf{a}/2, -t'; \mathbf{r} + \mathbf{a}/2, -t)\,\hat{g}^{-1}]_{\beta\alpha}. \qquad (26)$$

We use the translational-symmetry property of the system and go over to the Green's function in the quasimomentum representation

$$G_{\alpha\beta}(\mathbf{r}, t; \mathbf{r}', t') = \int \frac{d\varepsilon d\mathbf{p}}{(2\pi)^4} \exp[i\mathbf{p}(\mathbf{r} - \mathbf{r}') - i\varepsilon(t - t')] \times G_{\alpha\beta}(\varepsilon, \mathbf{p}; \mathbf{r}, \mathbf{r}')$$

(the function $G_{\alpha\beta}(\epsilon, \mathbf{p}; \mathbf{r}, \mathbf{r}')$ has the periodicity of the crystal lattice of the system both with respect to **r** and **r**′; the integration with respect to **p** is over the unit-cell volume of the reciprocal lattice). From (26) we get for $G_{\alpha\beta}(\epsilon, \mathbf{p}; \mathbf{r}, \mathbf{r}')$ the following relation:

$$G_{\alpha\beta}(\varepsilon, \mathbf{p}; \mathbf{r}, \mathbf{r}') = [\hat{g}\hat{G}(\varepsilon, -\mathbf{p}; \mathbf{r}' + \mathbf{a}/2, \mathbf{r} + \mathbf{a}/2)\,g^{-1}]_{\beta\alpha}. \qquad (27)$$

Since the poles of the Green's function determine the spectrum of the Fermi excitations, and since $[\hat{g}\hat{s}\hat{g}^{-1}]_{\alpha\beta} = -\hat{s}_{\beta\alpha}$ ($\hat{s}$ is the spin matrix), we get from (27) that the Fermi excitation energies are equal, $\epsilon^+(\mathbf{p}) = \epsilon^-(-\mathbf{p})$ (the symbols + and − correspond to the spin polarization). We assume that the investigated system is invariant against space inversion. In this case, obviously, $\epsilon(\mathbf{p}) = \epsilon(-\mathbf{p})$ and, consequently, $\epsilon^+(\mathbf{p}) = \epsilon^-(\mathbf{p})$. This means that the spectrum of the Fermi excitations is degenerate with respect to the spin direction, and the Fermi surfaces corresponding to opposite spin orientations coincide.

5. Let us consider the vertex part of the interaction of the conduction electrons $\Gamma(p_1, p_2; k)$. It is obvious that the spin-wave spectrum of an antiferromagnetic metal is determined by the singularities in the momentum transfer k of the components of Γ that are transverse relative to the spin. Inasmuch as the Fermi surfaces of the system in question coincide for opposite spin directions, the possibility that the excitations can go over from one of them to the other creates conditions under which spin waves of zero-sound type can be produced in nonmagnetic metals.[4] On the other hand, the presence of volume interactions between the spins, of which the ionic magnetic sublattices consist, makes possible the existence of the spin waves that arise in antiferromagnetic dielectrics.

The elements that introduce the singularity with respect to k into $\Gamma(p_1, p_2; k)$ are thus the loops of the electronic Green's functions $G(p)G(p+k)$, whose poles come closer together near the Fermi surface when $k \to 0$, and also the Green's function of the spin waves $D(k)$, corresponding to the density oscillations of the magnetic moment of the ionic sublattices (see the Appendix). We denote by $\tilde{\Gamma}^{(1)}(p_1, p_2; k)$ the aggregate of all the diagrams for Γ which do not contain any of the indicated singular elements. Then the irreducible vertex part $\Gamma^{(1)}$ which represents the sum of all the diagrams for Γ, which do not contain loops of electronic Green's functions, will be

$$\Gamma^{(1)}_{l_+m_-n_-r_+}(p_1, p_2; k) = \tilde{\Gamma}^{(1)}_{l_+m_-n_-r_+}(p_1, p_2; k) + g_0^2 \frac{\omega_s^2(\mathbf{k})}{2\delta\mu_0 M_0} \frac{1}{\omega^2 - \omega_s^2(\mathbf{k})} \quad . \tag{28}$$

The second term of this formula corresponds to the interaction of electrons via oscillations of the magnetic moment of the ionic sublattices.

The complete vertex part of Γ is obviously the sum of all the ladder-type diagrams with all possible repetitions of $\Gamma^{(1)}$, joined by loops $G(p+k)G(p)$. The result of this summation is

$$\Gamma_{l_+m_-n_-r_+}(p_1, p_2; k) = \tilde{\Gamma}_{l_+m_-n_-r_+}(p_1, p_2; k) + g_{ln}(p_1, k) \times \frac{\omega_s^2(\mathbf{k})/2\delta\mu_0 M_0}{\omega^2 - \omega_s^2(\mathbf{k})\left(1 + \Pi(k)/2\delta\mu_0 M_0\right)} g_{rm}(p_2, k). \tag{29}$$

Here $\tilde{\Gamma}$ is the sum of all the diagrams for Γ in which there is no D-function with argument k. The values of $\tilde{\Gamma}$ and $\tilde{\Gamma}^{(1)}$ are obviously related by the equation:

$$\tilde{\Gamma}_{l+m-n-r+}(p_1, p_2; k) = \tilde{\Gamma}^{(1)}_{l+m-n-r+}(p_1, p_2; k) - i \sum_{m'r'} \int \frac{d^4p'}{(2\pi)^4} \tilde{\Gamma}^{(1)}_{l+m-'n-r+'}(p_1, p'; k) G_{r'}(p') G_{m'}(p'+k) \times \Gamma_{r+'m-m-'r+}(p', p_2; k). \tag{30}$$

The quantity $g_{nm}(p, k)$ which enters in (29) is the complete vertex of the interaction of electrons with the ion-sublattice spin oscillations. It is connected with the bare vertex g_0 and Γ by the relation

$$g_{nm}(p, k) = g_0 - ig_0 \sum_{lr} \int \frac{d^4p'}{(2\pi)^4} G_r(p'+k) \times G_l(p') \Gamma_{l_+n_-r_-m_+}(p', p; k). \tag{31}$$

The function $\Pi(k)$ is the spin-wave polarization operator. It is expressed in terms of g and g_0 as follows:

$$\Pi(k) = -i \sum_{n,m} g_0 \int \frac{d^4p}{(2\pi)^4} G_m(p) G_n(p+k) g_{nm}(p, k). \tag{32}$$

In connection with the fact that the diagrams corresponding to $\tilde{\Gamma}$, g, and Π contain the singular element $G(p+k)G(p)$, these quantities have singularities. We rewrite the equations (29), (31) and (32) for these singularities in a form wherein the integration is only over the Fermi surfaces. We use for this purpose the device by which Landau[4] made a similar transformation. We assume henceforth for simplicity that the metal in question has only one incompletely filled band of conduction-electron excitations. We shall leave out the band-number indices of the quantities Γ and g, assuming them to correspond to the indicated excitation band. Bearing in mind that near the Fermi surface the Green's function of the conduction electrons becomes, as $\epsilon \to 0$,

$$G(\varepsilon, \mathbf{p}) \cong \frac{a(p)}{\varepsilon + \mu - \varepsilon(\mathbf{p}) + i\delta \operatorname{sign} \varepsilon},$$

we obtain the following group of equations for Γ, g, and Π when $\epsilon \to \mu$ and $|\mathbf{k}| \ll p_0$ (p_0 is the order of magnitude of the Fermi momentum and μ is the chemical potential):

$$\tilde{\Gamma}(p_1, p_2; k) = \tilde{\Gamma}^{\omega}(p_1, p_2) + \int \frac{d^4p'}{(2\pi)^4} \tilde{\Gamma}^{\omega}(p_1, p') \times 2\pi a^2(\mathbf{p}') \delta(\varepsilon') \delta(\varepsilon(\mathbf{p}') - \mu) \frac{\mathbf{vk}}{\omega - \mathbf{vk}} \tilde{\Gamma}(p', p_2; k), \tag{33}$$

$$g(p, k) = g^{\omega}(p) + \int \frac{d^4p'}{(2\pi)^4} \tilde{\Gamma}^{\omega}(p, p') \times 2\pi a^2(\mathbf{p}') \delta(\varepsilon') \delta(\varepsilon(\mathbf{p}') - \mu) \frac{\mathbf{vk}}{\omega - \mathbf{vk}} g(p', k), \tag{34}$$

$$\Pi(k) = \Pi^{\omega} + \int \frac{d^4p'}{(2\pi)^4} g^{\omega}(p') \cdot 2\pi a^2(\mathbf{p}') \delta(\varepsilon') \delta(\varepsilon(\mathbf{p}') - \mu) \times \frac{\mathbf{vk}}{\omega - \mathbf{vk}} g(p', k), \tag{35}$$

$$\tilde{\Gamma}(p_1, p_2; k) = \tilde{\Gamma}^k(p_1, p_2) + \int \frac{d^4p'}{(2\pi)^4} \Gamma^k(p_1, p')$$
$$\times 2\pi a^2(\mathbf{p}')\delta(\varepsilon')\delta(\varepsilon(\mathbf{p}') - \mu)\frac{\omega}{\omega - \mathbf{vk}}\Gamma(p', p_2; k), \quad (36)$$

$$g(p, k) = g^k(p) + \int \frac{d^4p'}{(2\pi)^4}\Gamma^k(p, p')$$
$$\times 2\pi a^2(\mathbf{p}')\delta(\varepsilon')\delta(\varepsilon(\mathbf{p}') - \mu)\frac{\omega}{\omega - \mathbf{vk}} g(p', k), \quad (37)$$

$$\Pi(k) = \Pi^k + \int \frac{d^4p'}{(2\pi)^4} g^k(p')\cdot 2\pi a^2(\mathbf{p}')\delta(\varepsilon')\delta(\varepsilon(\mathbf{p}') - \mu)$$
$$\times \frac{\omega}{\omega - \mathbf{vk}} g(p', k). \quad (38)$$

Here $\tilde{\Gamma}^\omega$, g^ω, and Π^ω are the limits of the functions Γ, g, and Π, respectively, for k = 0 and $\omega \to 0$, while $\tilde{\Gamma}^K$, g^K, and Π^K are the limits of the same quantities for $\omega = 0$ and $k \to 0$; $v = \partial\epsilon/\partial p$ is the excitation velocity on the Fermi surface.

Let us proceed to investigate the singularities of Γ. When $\omega \ll \omega_s(0)$ the term corresponding to the interaction via the spin wave (28) is regular, so that we can introduce

$$\Gamma^\omega(p_1, p_2) = \lim_{k=0,\ \omega\to 0} \Gamma(p_1, p_2; k) = \tilde{\Gamma}^\omega(p_1, p_2)$$
$$- g^\omega(p_1)\frac{\omega_s^2(0)/2\delta\mu_0 M_0}{\omega_s^2(0)(1 + \Pi^\omega/2\delta\mu_0 M_0)} g^\omega(p_2). \quad (39)$$

The equation for Γ in this region of ω is

$$\Gamma(p_1, p_2; k) = \Gamma^\omega(p_1, p_2) + \int \frac{d^4p'}{(2\pi)^4}\Gamma^\omega(p_1, p')$$
$$\times 2\pi a^2(\mathbf{p}')\delta(\varepsilon')\delta(\varepsilon(\mathbf{p}') - \mu)\frac{\mathbf{vk}}{\omega - \mathbf{vk}}\Gamma(p', p_2; k). \quad (40)$$

The poles of Γ, determined from this equation, give the spectrum of the spin waves for $\omega \ll \omega_s(0)$. In analogy with the procedure in [4], we obtain the following dispersion relation for the spectrum of the spin excitations:

$$(\omega - \mathbf{vk})\vartheta(\mathbf{p}) = \mathbf{vk}\int dS' f(\mathbf{p}, \mathbf{p}')\vartheta(\mathbf{p}'). \quad (41)$$

Here

$$f(\mathbf{p}, \mathbf{p}') = a(\mathbf{p})\Gamma^\omega(p, p')a(\mathbf{p}')\,|_{\varepsilon=\varepsilon'=0}$$

is the Landau function of the conduction electrons, and dS is the Fermi-surface element:

$$\int dS' f(\mathbf{p}, \mathbf{p}')\vartheta(\mathbf{p}') \equiv \int \frac{d\mathbf{p}'}{(2\pi)^3}\delta(\varepsilon(\mathbf{p}') - \mu)f(\mathbf{p}, \mathbf{p}')\vartheta(\mathbf{p}').$$

It follows from (40) that the function $f(\mathbf{p}, \mathbf{p}')$ is connected with the amplitude for the zero-angle scattering of two Fermi excitations

$$F(\mathbf{p}, \mathbf{p}') \equiv a(\mathbf{p})\Gamma^k(p, p')a(\mathbf{p}')\,|_{\varepsilon=\varepsilon'=0}$$

(see [4]) by means of the relation

$$f(\mathbf{p}_1, \mathbf{p}_2) = F(\mathbf{p}_1, \mathbf{p}_2) + \int dS' f(\mathbf{p}_1, \mathbf{p}')F(\mathbf{p}', \mathbf{p}_2). \quad (42)$$

In the region $kv \gg \omega_0$, where

$$\omega_0 \equiv \omega_s(0)\,[1 + \Pi^k/2\delta\mu_0 M_0]^{1/2},$$

Γ has a singularity as a result of the presence of a pole in the second term of the right side of (29). Inasmuch as $\Pi(k) \cong \Pi^k$ when $kv \gg \omega$, the indicated pole corresponds to the frequency

$$\omega = [\omega_0^2 + \xi^2 k^2]^{1/2},$$

where

$$\xi^2 = 8\delta\mu_0^2 M_0^2\left(1 + \frac{\Pi^k}{2\delta\mu_0 M_0}\right)(\alpha - \alpha_{12}) \sim \Theta^2 a^2$$

(see the Appendix).

In addition to the singularities just obtained, the vertex part Γ has poles in another region of values of ω and k. It is seen from (29) that when **k** = 0 the function Γ has a pole at frequency

$$\omega = \tilde{\omega}_0 \equiv \omega_s(0)\,[1 + \Pi^\omega/2\delta\mu_0 M_0]^{1/2}.$$

Using (34) and (35) we can show that when $\omega \sim \tilde{\omega}_0$ and when $vk \ll \tilde{\omega}_0$ we have, in the lowest order in the quantities ω/ϵ_F and vk/ω,

$$\Pi(k) - \Pi^\omega \sim \Theta v^2k^2/\omega^2.$$

Therefore when $vk \ll \tilde{\omega}_0$ and $\omega \approx \tilde{\omega}_0$ the Green's function has a pole at a frequency

$$\omega \cong \tilde{\omega}_0 + \gamma(\mathbf{k}/k)v^2k^2/\tilde{\omega}_0 \quad (\gamma(\mathbf{k}/k) \sim 1).$$

Inasmuch as the second term of the right side of (29) becomes small when $\omega \gg \tilde{\omega}_0$, the singularities of the vertex part of Γ coincide in this case with the poles of $\tilde{\Gamma}(p_1, p_2; k)$. Near the poles the equation for $\tilde{\Gamma}$ (33) can be transformed in the same manner as we used in the derivation of (41). As a result we obtain the following dispersion equation for the spin-wave spectrum in the vicinity of the frequencies $\omega \gg \tilde{\omega}_0$:

$$(\omega - \mathbf{vk})\vartheta(\mathbf{p}) = \mathbf{vk}\int dS'\tilde{f}(\mathbf{p}, \mathbf{p}')\vartheta(\mathbf{p}'). \quad (43)$$

The function $\tilde{f}(\mathbf{p}, \mathbf{p}') = a(\mathbf{p})\tilde{\Gamma}^\omega(\mathbf{p}, \mathbf{p}')\,|_{\epsilon=\epsilon'=0}$ is the Landau function, renormalized by the interaction of electrons with the spin waves. We do not concern ourselves here with the existence of a solution of (41) and (43), which correspond to undamped excitations with higher than Fermi velocity, since the condition for the existence of such a solution (we assume that these conditions are satisfied in our case) were discussed in detail for similar equations in the paper of Gor'kov and Dzyaloshinskiĭ.[8]

Thus, the spin waves in an antiferromagnetic metal consist of two branches (see Fig. 3). The

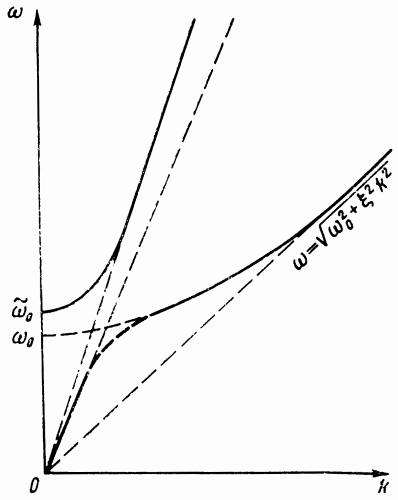

FIG. 3

first, low-frequency branch has for small $k (vk << \omega_0)$ the character of zero-sound spin waves, is described by (41) and has a linear dispersion. In the region $vk \gg \omega_0$ it goes over into excitations with the properties of the spin waves that occur in an antiferromagnetic dielectric:

$$\omega(\mathbf{k}) = (\omega_0^2 + \xi^2 k^2)^{1/2}, \quad \xi \sim \Theta a,$$

where a is the crystal lattice constant. In the intermediate interval $vk \sim \omega_0$ (this section is indicated in the figure by the heavy dashed line) there is strong damping, connected with the possibility of the decay of the spin wave into a pair of Fermi excitations.

The second, high-frequency branch starts with a frequency $\omega(0) = \tilde{\omega}_0$. In the region $vk \ll \tilde{\omega}_0$ the frequency of these excitations is equal to

$$\omega(\mathbf{k}) = \tilde{\omega}_0 + \gamma\left(\frac{\mathbf{k}}{k}\right)\frac{v^2k^2}{\tilde{\omega}_0}$$

$(\gamma \sim 1)$. When ω, $vk \gg \tilde{\omega}_0$ this branch goes over into excitations of the zero-sound type described by (43).

We shall now show that all the quantities characterizing the spectrum of both spin-wave branches can be expressed in terms of two constants and two functions that have a simple physical meaning. We introduce for this purpose the amplitude $\lambda(\mathbf{p}, \mathbf{k})$ for the emission of a spin wave by a Fermi excitation, which obviously is equal to the product of g(p, k) by the square root of the product of the residues of the Green's functions of the final lines of the three-point vertex g(p, k). This process is possible only for the lower branch of the spin waves when $\omega < vk$. The corresponding Green's function of the spin wave in the case when $vk \gg \omega_0$ is equal to

$$D(k) = \frac{\omega_s^2(\mathbf{k})/2\delta\mu_0 M_0}{\omega^2 - (\omega_0^2 + \xi^2k^2)},$$

and the residue at $\xi^2k^2 \ll \omega_0^2$ is equal to $\omega_s^2(0)/4\delta\mu_0 M_0\omega_0$. Therefore

$$\lambda(\mathbf{p}) \equiv \lambda(\mathbf{p}, 0) = a(\mathbf{p})\frac{\omega_s(0)}{2(\delta\mu_0 M_0\omega_0)^{1/2}} g^k(p)\,|_{\varepsilon=0}.$$

The second independent function will be the already-mentioned amplitude $F(\mathbf{p}, \mathbf{p}')$ for zero-angle scattering of two Fermi excitations, with which the function $f(\mathbf{p}, \mathbf{p}')$ is connected by Eq. (42). As the independent constants we take ω_0 and ξ, which characterize the asymptotic behavior of the lower branch of the spectrum.

As follows from (29), the function $\tilde{\Gamma}^k$ is expressed in terms of

$$F(\mathbf{p}, \mathbf{p}') = a(\mathbf{p})\Gamma^k(p, p')a(\mathbf{p}')\,|_{\varepsilon=\varepsilon'=0}$$

by means of the relation

$$a(\mathbf{p})\tilde{\Gamma}^k(p, p')a(\mathbf{p}')\,|_{\varepsilon=\varepsilon'=0} = F(\mathbf{p}, \mathbf{p}') + 2\lambda(\mathbf{p})\lambda(\mathbf{p}')\,/\,\omega_0.$$

On the other hand, $a(\mathbf{p})\tilde{\Gamma}^k(p, p')a(p')\,|_{\epsilon=\epsilon'=0}$ and $\tilde{f}(\mathbf{p}, \mathbf{p}')$ are two different limiting values of the same function as $k \to 0$. Therefore, using (36), we obtain the sought-for connection between $\tilde{f}(\mathbf{p}, \mathbf{p}')$ and the physical quantities $F(\mathbf{p}, \mathbf{p}')$ and $\lambda(\mathbf{p})$:

$$\tilde{f}(\mathbf{p}_1, \mathbf{p}_2) = F(\mathbf{p}_1, \mathbf{p}_2) + \frac{2\lambda(\mathbf{p}_1)\lambda(\mathbf{p}_2)}{\omega_0} + \int dS' F(\mathbf{p}_1, \mathbf{p}')\tilde{f}(\mathbf{p}', \mathbf{p}_2) + \frac{2\lambda(\mathbf{p}_1)}{\omega_0}\int dS'\lambda(\mathbf{p}')\tilde{f}(\mathbf{p}', \mathbf{p}_2). \quad (44)$$

In order to express $\tilde{\omega}_0$ finally in terms of the remaining quantities, we multiply the limit of Eq. (29) at $\mathbf{k} = 0$, $\omega \to 0$ by $a(\mathbf{p}_1)\lambda(\mathbf{p}_1)a(\mathbf{p}_2)\lambda(\mathbf{p}_2)$, integrating them over the Fermi surfaces. As a result we obtain

$$\int dS\,dS'\,\lambda(\mathbf{p})[\tilde{f}(\mathbf{p}, \mathbf{p}') - f(\mathbf{p}, \mathbf{p}')]\,\lambda(\mathbf{p}') = \frac{1}{2\omega_0\tilde{\omega}_0^2}\left(\frac{\omega_s^2(0)}{2\delta\mu_0 M_0}\right)^2 (\Pi^k - \Pi^\omega)^2. \quad (45)$$

Since

$$\tilde{\omega}_0^2 = \omega_0^2 - \frac{\omega_s^2(0)}{2\delta\mu_0 M_0}(\Pi^k - \Pi^\omega),$$

we get from (45) ultimately

$$\frac{(\omega_0^2 - \tilde{\omega}_0^2)^2}{2\omega_0\tilde{\omega}_0^2} = \int dS\,dS'\,\lambda(\mathbf{p})[\tilde{f}(\mathbf{p}, \mathbf{p}') - f(\mathbf{p}, \mathbf{p}')]\,\lambda(\mathbf{p}'). \quad (46)$$

In conclusion I consider it my pleasant duty to express deep gratitude to I. E. Dzyaloshinskiĭ for valuable indications and continuous interest in the work.

APPENDIX

Let us determine the singular element which is contained in the vertex $\Gamma(p_1, p_2; k)$ and which is due to the exchange interaction of electrons with magnetic sublattices of the antiferromagnet. The Hamiltonian of this interaction is[9]

$$\mathcal{H}_{int} = \frac{g_0}{\mu_0 M_0} \int dr\, \psi_\alpha^+(x) S_{\alpha\beta} \psi_\beta(x) \mathbf{M}(x),$$

where ψ and ψ^+ are electronic operators, $\mathbf{M}(x) = \mathbf{M}_1(x) + \mathbf{M}_2(x)$ the operator of the total magnetic moment of the ionic sublattices (the absolute value of the magnetic moment of each of them in the ground state is equal to M_0), $g_0 \sim (\mu_0 \Theta \epsilon_F / M_0)^{1/2}$, μ_0 is the Bohr magneton, Θ the Curie-Néel temperature, and ϵ_F the Fermi energy of the conduction electrons. It is clear that a singularity in Γ is brought about by expressions describing the interactions of two Fermi excitations in terms of the vibrations of the magnetic sublattices. It is contained in the Green's functions of the spin waves corresponding to these oscillations:

$$D(x, x') = -\frac{i}{\mu_0 M_0} \langle T(M^+(x), M^-(x)) \rangle$$

($M^\pm(x) = M_x(x) \pm iM_y(x)$ and T is the chronological ordering operator).

According to [2], the Hamiltonian of the system of two mirror-symmetry magnetic sublattices of a uniaxial antiferromagnet is

$$\mathcal{H} = \int dr \left\{ \frac{1}{2} \alpha \frac{\partial \mathbf{M}_1}{\partial x_i} \frac{\partial \mathbf{M}_1}{\partial x_i} + \frac{1}{2} \alpha \frac{\partial \mathbf{M}_2}{\partial x_i} \frac{\partial \mathbf{M}_2}{\partial x_i} + \alpha_{12} \frac{\partial \mathbf{M}_1}{\partial x_i} \frac{\partial \mathbf{M}_2}{\partial x_i} + \delta \mathbf{M}_1 \mathbf{M}_2 - \frac{1}{2} \beta [(\mathbf{n}\mathbf{M}_1)^2 + (\mathbf{n}\mathbf{M}_2)^2] - (\mathbf{M}_1 + \mathbf{M}_2, \mathbf{H}) - \frac{\mathbf{H}^2}{8\pi} \right\}.$$

The first four terms describe here the exchange interaction of the sublattices. The remaining ones correspond to the energy of magnetic anisotropy of the ionic sublattices of the uniaxial antiferromagnet.

Using the equations of motion $i\partial M^\pm/\partial t = [M^\pm, \mathcal{H}]$ and also the commutation rules for M_1 and M_2, neglecting the magnetic field of the spin wave, we can easily obtain the Green's function of the spin wave in the momentum representation:

$$D(\omega, \mathbf{k}) = \frac{\omega_s^2(\mathbf{k}) / 2\delta\mu_0 M_0}{\omega^2 - \omega_s^2(\mathbf{k})}.$$

The quantity

$$\omega_s(\mathbf{k}) = 2\mu_0 M_0 [2\delta\beta + 2\delta(\alpha - \alpha_{12}) \mathbf{k}^2]^{1/2}$$

($\delta\mu_0 M_0 \sim \Theta$, $\mu_0 M_0 (\alpha - \alpha_{12}) \sim \Theta a^2$, a is the crystal lattice constant) corresponds to the frequency of the spin wave in an antiferromagnetic dielectric.

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