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THE JOSEPHSON TUNNEL EFFECT IN SUPERCONDUCTORS WITH PARAMAGNETIC IMPURITIES

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A theory of the Josephson effect in superconductors with paramagnetic impurities is developed for $T = 0$ and the absence of a potential difference between the superconductors. It is shown that the superconducting tunnel current differs from zero in the region of ordinary ($\omega_g \neq 0$) as well as gapless ($\omega_g = 0$) superconductivity, and the dependence of the peak value of the current on the concentration of paramagnetic impurities is found. Non-magnetic impurities which do not affect the electron spin direction during scattering also do not affect the magnitude of the dc Josephson current.

THE interest in the investigation of superconductors with paramagnetic impurities is associated with the circumstance that, as was shown by Abrikosov and Gor'kov,[1] the energy gap in the spectrum of single-particle states ω_g vanishes in such superconductors in a certain region of concentrations ($0.91 n_c < n < n_c$ at $T = 0$)[1)], whereas the ordering parameter Δ is non-zero, i.e., the metal remains a superconductor (ideal conductor and ideal diamagnet). The first confirmation of these theoretical conclusions was obtained by an investigation of the tunnel effect.[2] The tunnel effect is still one of the most effective ways of studying gapless superconductivity.

In this paper we investigate the features of the Josephson[3] superconducting tunnel current in superconductors containing paramagnetic impurities. For simplicity we restrict our attention to the case $T = 0$, since all fundamental properties of the Josephson current are already manifested in this case. In addition, we shall consider the potential difference V between the superconductors also to be zero, i.e., we are concerned only with the the dc Josephson current.[2)]

[1)]The symbol n_c represents the critical paramagnetic-impurity concentration at which T_c goes to zero.

[2)]The qualitative conclusions for $V \neq 0$ are the same as for the ordinary Josephson effect. Namely, when $V \neq 0$ (but $eV \ll \Delta$), the Josephson current oscillates in time according to the law

$$J = J_s \sin(2eVt/\hbar + \varphi_0). \qquad \text{(A)}$$

However, in the gapless region ($\omega_g = 0$), a large quasi-particle current arises, which leads to a strong damping of these oscillations.

In the case of ordinary superconductors (we shall call them BCS superconductors), the energy gap ω_g and the ordering parameter Δ coincide: $\Delta = \omega_g$. Then we have the following formula, obtained by a number of authors,[3-5] for the magnitude of the maximum Josephson current (at $T = 0$):

$$J_s = R_{NN}^{-1}\Delta_1 K(\sqrt{1 - \Delta_1^2/\Delta_2^2}). \qquad (1)$$

Here R_{NN} is the resistance of the tunnel junction in the normal state, Δ_1 and Δ_2 are the energy gaps of the superconductors constituting the tunnel junction ($\Delta_1 \leq \Delta_2$), and $K(x)$ is a complete elliptical integral of the first kind. In particular, for a tunnel junction of two identical superconductors ($\Delta_1 = \Delta_2 = \Delta$), we have

$$J_s = \tfrac{1}{2}\pi\Delta R_{NN}^{-1}. \qquad (2)$$

In the case of superconductors with paramagnetic impurities considered by Abrikosov and Gor'kov[1] (we may call these AG superconductors), the magnitudes of ω_g and Δ do not coincide, and ω_g can vanish while $\Delta \neq 0$. The theory to be presented below shows that the Josephson current does not vanish in the gapless region ($\omega_g = 0$); however, it is diminished in comparison with its value in the absence of impurities. This is quite obvious, since the superconducting tunnel current is analogous to the ordinary superconducting current which exists so long as $\Delta \neq 0$. The formulas obtained below show that the investigation of the dependence of the superconducting tunnel current J_s on the concentration of paramagnetic impurities n gives the possibility of studying the dependence of the ordering parameter Δ on n.

In calculating the superconducting tunnel current we shall follow the method of Ambegaokar and Baratoff.[5] According to[5] the magnitude of the Josephson current is given by the expression[3)]

$$J = -4\mathrm{Re}\sum_{kq} |T_{kq}|^2 \int_{-\infty}^{t} dt' \{\langle a_{k\uparrow}^{+}(t)\, a_{-k\downarrow}^{+}(t')\rangle \langle b_{q\uparrow}(t)\, b_{-q\downarrow}(t')\rangle - \langle a_{-k\downarrow}^{+}(t')\, a_{k\uparrow}^{+}(t)\rangle \langle b_{-q\downarrow}(t')\, b_{q\uparrow}(t)\rangle\}, \quad (3)$$

where T_{kq} are the matrix elements of the transition of an electron from the left metal to the right one with a change in momentum from **k** to **q**; $a_k^+(t)$, $b_k^+(t)$ are the creation operators for electrons in the left and right metals in the Heisenberg representation.

The averages in (3), as noted in[5], are not ordered in time. However, when $t' < t$ we have, for example,

$$\begin{aligned} \langle a_{k\uparrow}{}^{+}(t)\, a_{-k\downarrow}{}^{+}(t')\rangle &= e^{2i\mu t} F^{+}(k, t-t'), \\ \langle b_{q\uparrow}(t)\, b_{-q\downarrow}(t')\rangle &= -e^{-2i\mu t} F(q,\, t-t'), \end{aligned} \quad (4)$$

where F^+ and F are the Gor'kov functions,[6] which are chronologically ordered, and μ is the chemical potential, the same for both metals. Since in (3) the integral over t' is taken from $-\infty$ to t, i.e., $t > t'$, the averages in (3) can be replaced by the Gor'kov F functions in accordance with Eqs. (4).

It should be realized further that in the determination of the functions F^+ there is an arbitrariness, because the equations that these functions satisfy (the Gor'kov equations) are invariant relative to the multiplication of F^+ by $e^{i\varphi}$ and F by $e^{-i\varphi}$, where $e^{i\varphi}$ is an arbitrary phase factor. This invariance holds for both the BCS[6] and the AG[1] superconductors (see Eq. (2) in[1], rewritten in the ordinary time representation, $t = -i\tau$). Taking this into account, we write Eq. (3) in the form

$$J = 4\,\mathrm{Re}\sum_{kq} |T_{kq}|^2 \int_{-\infty}^{t} dt' [F^{+}(k, t-t')\, F(q, t-t') - \bar{F}^{+}(k, t-t')\, \bar{F}(q, t-t')]\, e^{i(\varphi_1-\varphi_2)}, \quad (5)$$

where $\varphi_1 - \varphi_2$ is the difference in the phases of the F functions (the wave functions of the "superconducting pairs") in the left and right metals.

In (5), in which one can set $t = 0$ since it is clear that in actuality J is independent of t, it is convenient to go over from the ordinary Green functions F^+, F to the thermodynamic functions $\mathcal{F}^+$, $\mathcal{F}$, which have simpler analytical properties.

3)Throughout we use the system of units in which $\hbar = e = 1$, where e is the electron charge.

The possibility of this replacement can be proved in the following way.

We transform to the energy representation for the operators $a_{k\uparrow}^+$, $a_{-k\downarrow}^+$, $b_{q\uparrow}$, and $b_{-q\downarrow}$ in (4), as a result of which we obtain (for $t < 0$)

$$\begin{aligned} F^{+}(k, -t) &= \sum_{s} (a_{k\uparrow}^{+})_{0s}\, (a_{-k\downarrow}^{+})_{s0}\, e^{i(\varepsilon_s+\mu)t}, \\ F(q, -t) &= -\sum_{\sigma} (b_{q\uparrow})_{0\sigma}\, (b_{-q\downarrow})_{\sigma 0}\, e^{i(\varepsilon_\sigma-\mu)t}, \end{aligned} \quad (6)$$

where $\epsilon_s = E_{s1} - E_{01}$, $\epsilon_\sigma = E_{\sigma 2} - E_{02}$ are positive energies of excitation in the left and right metals: $\epsilon_s > 0$, $\epsilon_\sigma > 0$ (E_0 is the energy of the ground state of the superconductor, E_s and E_σ the energies of the excited states). On the basis of (6) we obtain

$$\int_{-\infty}^{0} F^{+}(k, -t)\, F(q, -t)\, dt = -\sum_{s\sigma} (a_{k\uparrow}^{+})_{0s}\, (a_{-k\downarrow}^{+})_{s0}\, (b_{q\uparrow})_{0\sigma}\, (b_{-q\downarrow})_{\sigma 0} \frac{1}{i(\varepsilon_s + \varepsilon_\sigma)}. \quad (7)$$

We now introduce the thermodynamic Green functions[6] (for $T = 0$)

$$\begin{aligned} \mathcal{F}^{+}(k, \tau - \tau') &= \langle T \bar{a}_{k\uparrow}(\tau)\, \bar{a}_{-k\downarrow}(\tau')\rangle, \\ \mathcal{F}(q, \tau - \tau') &= -\langle T b_{q\uparrow}(\tau)\, B_{-q\downarrow}(\tau')\rangle, \end{aligned} \quad (8)$$

where

$$A(\tau) = e^{(H-\mu N)\tau} A e^{-(H-\mu N)\tau}, \quad \bar{A}(\tau) = e^{(H-\mu N)\tau} A^{+} e^{-(H-\mu N)\tau}.$$

Going over to the energy representation in these expressions, we obtain in analogous fashion for $\tau < 0$

$$\begin{aligned} \mathcal{F}^{+}(k, -\tau) &= \sum_{s} (a_{k\uparrow}^{+})_{0s}\, (a_{-k\downarrow}^{+})_{s0}\, e^{\varepsilon_s \tau}, \\ \mathcal{F}(q, -\tau) &= -\sum_{\sigma} (b_{q\uparrow})_{0\sigma}\, (b_{-q\downarrow})_{\sigma 0}\, e^{\varepsilon_\sigma \tau}. \end{aligned} \quad (9)$$

Consequently, $\mathcal{F}^+(k, -\tau)$ and $\mathcal{F}(q, -\tau)$ are regular functions for $\tau < 0$ (and also, as can be shown, for $\tau > 0$). On the basis of (9) we obtain

$$\int_{-\infty}^{0} \mathcal{F}^{+}(k, -\tau)\, \mathcal{F}(q, -\tau)\, d\tau = -\sum_{s\sigma} (a_{k\uparrow}^{+})_{0s}\, (a_{-k\downarrow}^{+})_{s0}\, (b_{q\uparrow})_{0\sigma}\, (b_{-q\downarrow})_{\sigma 0} \frac{1}{\varepsilon_s + \varepsilon_\sigma}. \quad (10)$$

Comparing this expression with (7), we see that

$$\int_{-\infty}^{0} F^{+}(k, -t)\, F(q, -t)\, dt = \frac{1}{i} \int_{-\infty}^{0} \mathcal{F}^{+}(k, -\tau)\, \mathcal{F}(q, -\tau)\, d\tau. \quad (11)$$

Consequently, in the expression for the Josephson current (5) it is possible to go over from the ordinary Green functions to the thermodynamic ones, which are more convenient for what is to follow. Performing this transformation and transforming to the Fourier representation, we write (5) in the following form (here and later $F(k, \omega)$ is the Fourier component of the thermodynamic Green function $\mathscr{F}(k, \tau)$)[4)]:

$$J = -\frac{2}{\pi^2}\operatorname{Re}\sum_{kq}|T_{kq}|^2\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}d\omega'\frac{F(k,\omega)F(q,\omega')}{\omega+\omega'-i\delta}e^{i(\varphi_1-\varphi_2)},$$

$$\delta = +0. \tag{12}$$

Using the properties of the functions $F(k, \omega)$ (they are even and real), we obtain

$$J = J_s \sin(\varphi_1 - \varphi_2), \tag{13}$$

where

$$J_s = \frac{2}{\pi^2}\operatorname{Im}\sum_{kq}|T_{kq}|^2\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}d\omega'\frac{F(k,\omega)F(q,\omega')}{\omega+\omega'-i\delta} \tag{14}$$

or

$$J_s = \frac{2}{\pi}\sum_{kq}|T_{kq}|^2\int_{-\infty}^{\infty}F(k,\omega)F(q,\omega)\,d\omega. \tag{15}$$

We remark that the transition from the case $T = 0$ considered in this paper to $T > 0$ is by replacing in (15) of the integration over ω by a summation over discrete frequencies $\omega_n = (2n + 1)\pi T$.

The quantities $|T_{kq}|^2$ appearing in (14) and (15) can be expressed in terms of the parameters of the tunnel junction in the normal state (when both metals are non-superconducting). According to [4,5,7], the resistance of the tunnel junction in the normal state R_{NN} is determined from the formula

$$R_{NN} = \{4\pi N_1(0)N_2(0)\langle|T|^2\rangle\}^{-1}, \tag{16}$$

where $N_1(0)$ and $N_2(0)$ are the state densities when $\epsilon = \mu$, and $\langle|T|^2\rangle$ is the average value of $|T_{kq}|^2$ over the angles of the vectors **k** and **q** on the Fermi surface. Since the integrals over energy in (14) and (15) converge in an interval $\Delta\epsilon \sim \Delta$ much less than μ, $\langle|T|^2\rangle$ can also be factored out of Eqs. (14) and (15). Eliminating this factor by means of (16), we convert Eq. (14) into the form

$$J_s = R_{NN}^{-1}\frac{1}{2\pi^3}\operatorname{Im}\int_{-\infty}^{\infty}d\xi_k\int_{-\infty}^{\infty}d\xi_q\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}d\omega'\frac{F(k,\omega)F(q,\omega')}{\omega+\omega'-i\delta}, \tag{17}$$

4)The functions $\tilde{F}^+$ and $\tilde{F}$ appearing in (5) are averages of Fermi operators, taken in anti-chronological order. By writing out a spectral representation similar to (6) for these functions, and using the property of the quantity T, viz, $T_{kq} = T^*_{-k,-q}$, which comes from the invariance of the tunnel Hamiltonian relative to time inversion,[4,7] it is easy to show that the second term in (5) gives a contribution that coincides with that of the first term (taken with opposite sign).

and Eq. (15) correspondingly to the form

$$J_s = R_{NN}^{-1}\frac{1}{2\pi^2}\int_{-\infty}^{\infty}d\xi_k\int_{-\infty}^{\infty}d\xi_q\int_{-\infty}^{\infty}d\omega\,F(k,\omega)F(q,\omega) \tag{18}$$

($\xi_k = k^2/2m - \mu$ is the energy of a normal electron, reckoned from the Fermi energy).

The obtained expressions (17) and (18) reduce the calculation of the amplitude of the Josephson current J_s at $T = 0$ to finding the thermodynamic Green functions $F(k, \omega)$ and $F(q, \omega)$ of the isolated superconductors. Before applying these formulas to the treatment of superconductors with paramagnetic impurities, we show that the usual results for the Josephson current[3-5] in the absence of impurities follow from them.

For BCS superconductors F has the form[6] (Δ is the gap)

$$F(k,\omega) = \Delta/(\omega^2 + \xi_k^2 + \Delta^2), \tag{19}$$

whence

$$\int_{-\infty}^{\infty}F(k,\omega)\,d\xi_k = \pi\Delta/\sqrt{\omega^2+\Delta^2}. \tag{20}$$

The rest of the calculation may be carried out in two ways, based, respectively on the use of Eqs. (17) and (18). We present both methods in order to make clear the transition to the case of AG superconductors, where it is possible to use only one of them.

The first method consists of the following. Substituting (20) into (17), we obtain

$$J_s = R_{NN}^{-1}\frac{1}{2\pi}\operatorname{Im}\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}d\omega'$$
$$\times\frac{1}{\omega+\omega'-i\delta}\frac{\Delta_1}{\sqrt{\omega^2+\Delta_1^2}}\frac{\Delta_2}{\sqrt{\omega'^2+\Delta_2^2}}. \tag{21}$$

Deforming the contour of integration in the plane of the complex variable ω as shown in Fig. 1, we transform the integral over the real axis

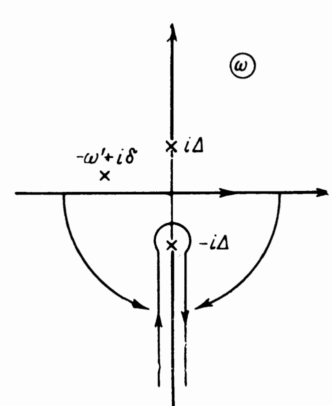

FIG. 1

from $-\infty$ to $+\infty$ to an integral over the boundaries of the cut going from $-i\Delta$ to $-i\infty$. Replacing ω by $-iu$ and then u by $(\Delta^2 + \Omega^2)^{1/2}$, we reduce (21) to

$$J_s = R_{NN}^{-1}\frac{1}{2\pi}\int_{-\infty}^{\infty} d\Omega_1$$

$$\times\int_{-\infty}^{\infty} d\Omega_2\frac{\Delta_1\Delta_2}{\sqrt{\Omega_1^2+\Delta_1^2}\sqrt{\Omega_2^2+\Delta_2^2}(\sqrt{\Omega_1^2+\Delta_1^2}+\sqrt{\Omega_2^2+\Delta_2^2})}. \tag{22}$$

This expression coincides with the formula obtained in [3,4]. Repeating Anderson's calculation,[4] we bring (22) to the form of the elliptic integral

$$J_s = R_{NN}^{-1}\frac{2\Delta_1\Delta_2}{\Delta_1+\Delta_2}K\left(\frac{|\Delta_1-\Delta_2|}{\Delta_1+\Delta_2}\right) \tag{23}$$

The second method consists of substituting (20) in (18), which immediately yields

$$J_s = R_{NN}^{-1}\int_0^{\infty}\frac{\Delta_1}{\sqrt{\Delta_1^2+\omega^2}}\frac{\Delta_2}{\sqrt{\Delta_2^2+\omega^2}}d\omega. \tag{24}$$

This integral is equal to (cf. Eq. (1))

$$J_s = R_{NN}^{-1}\Delta_1 K\left(\sqrt{1-\frac{\Delta_1^2}{\Delta_2^2}}\right), \quad \Delta_1<\Delta_2. \tag{25}$$

Using the properties of elliptic integrals (see [8]), it is easy to show that (23) and (25) are identical.

In calculating the tunnel current in superconductors with paramagnetic impurities, we shall make use of the second method, i.e., we start from Eq. (18).

The expression for the functions $F(k, \omega)$ of a superconductor with paramagnetic impurities was obtained by Abrikosov and Gor'kov.[1] On the basis of [1] we have

$$F(k,\omega) = \tilde{\Delta}/(\tilde{\omega}^2+\xi_k^2+\tilde{\Delta}^2), \tag{26}$$

where $\tilde{\omega}$ and $\tilde{\Delta}$ are defined by the equations

$$\tilde{\omega} = \omega + \frac{1}{2\tau_1}\frac{u}{\sqrt{1+u^2}}, \qquad \tilde{\Delta} = \Delta + \frac{1}{2\tau_2}\frac{1}{\sqrt{1+u^2}},$$

$$u = \frac{\tilde{\omega}}{\tilde{\Delta}}. \tag{27}$$

The presence of two relaxation times τ_1 and τ_2 is due to the existence of two types of scattering respectively without spin flip (n-scattering) and with spin flip (s-scattering). The probability of the latter is, according to [1],

$$1/\tau_s = 1/2\tau_1 - 1/2\tau_2. \tag{28}$$

Equation (27) leads to the relation

$$\frac{\omega}{\Delta} = u\left(1-\frac{1}{\Delta\tau_s}\frac{1}{\sqrt{1+u^2}}\right). \tag{29}$$

Integrating Eq. (26) for $F(k, \omega)$ over ξ_k and substituting it into (18), we obtain the following formula for the Josephson current:

$$J_s = R_{NN}^{-1}\int_0^{\infty}\frac{\tilde{\Delta}_1}{\sqrt{\tilde{\Delta}_1^2+\tilde{\omega}^2}}\frac{\tilde{\Delta}_2}{\sqrt{\tilde{\Delta}_2^2+\tilde{\omega}^2}}d\omega. \tag{30}$$

Using (27) this expression can also be rewritten in the more compact form

$$J_s = R_{NN}^{-1}\int_0^{\infty}\frac{1}{\sqrt{u_1^2(\omega)+1}}\frac{1}{\sqrt{u_2^2(\omega)+1}}d\omega, \tag{31}$$

where the dependence of u on ω is given implicitly by relation (29).

Before proceeding further with the calculation, we shall draw a few conclusions. Firstly, in the absence of scattering with spin flip: $\tau_S = \infty$ (i.e., for purely diamagnetic impurities having no localized magnetic moments), we obtain on the basis of (29) $u = \omega/\Delta$, from which it is seen that the integral (31) coincides with Eq. (24), which holds in the absence of impurities. Consequently, diamagnetic impurities have no effect on the magnitude of the critical Josephson current, just as they have no effect on the thermodynamics of superconductors.[6] (Of course, in our treatment, effects associated with the influence of impurities on the energy spectrum of the metal [9] or with a decrease of the anisotropy of the gap [10] drop out. We consider an isotropic metal with a quadratic dispersion law that is unchanged by the presence of impurities.)

Secondly, from (31) (and also from the more general formulas (17) and (18)) it is clear that the Josephson current is non-zero as long as the F function (i.e., the quantity Δ) differs from zero and does not vanish at the point at which the energy gap ω_g becomes zero. Thus, for weak superconductivity, just as for ordinary superconductivity, what is more important is not the presence of a gap in the spectrum of single-particle excitations, but the phenomenon of condensation associated with the emergence of pairs of electrons (described by the F function) in states with exactly identical momenta.

Calculation of the integral (31) for $\Delta_1 \neq \Delta_2$ and $\tau_{S1} \neq \tau_{S2}$ is extremely complicated and requires, evidently, numerical methods. However, simple analytical expressions can be obtained in the case of identical superconductors ($\Delta_1 = \Delta_2 = \Delta$, $\tau_{S1} = \tau_{S2} = \tau_S$). We have from (31) in this case

$$J_s = R_{NN}^{-1}\int_0^{\infty}\frac{d\omega}{u^2(\omega)+1}. \tag{32}$$

Further, it is possible to go over from integration over ω to integration over u, in accordance with (29). A particularly simple expression is obtained in the region $\Delta\tau_s > 1$, in which $\omega_g \neq 0$:

$$J_s = \frac{\pi}{2}\Delta R_{NN}^{-1}\left(1 - \frac{4}{3\pi}\frac{1}{\Delta\tau_s}\right), \quad \Delta\tau_s \geqslant 1 \tag{33}$$

(we recall that Δ also depends on the concentration of impurities [1]).

In particular, at the point $\Delta\tau_s = 1$, for which the energy gap ω_g goes to zero, we have

$$J_s' = \frac{\pi}{2}\Delta\left(1 - \frac{4}{3\pi}\right)R_{NN}^{-1}. \tag{34}$$

At this point $\Delta = e^{-\pi/4}\Delta_0$ (see [1]), where Δ_0 is the value of the ordering parameter in the absence of impurities. Consequently,

$$J_s' = e^{-\pi/4}\left(1 - \frac{4}{3\pi}\right)J_s^0 = 0.26J_s^0,$$

where $J_s^0 = \frac{1}{2}\pi\Delta_0 R_{NN}^{-1}$ is the maximum value of the Josephson current in the absence of impurities.

In the gapless region ($\Delta\tau_s < 1$), the integral (32) equals

$$J_s = \frac{\pi}{2}\Delta R_{NN}^{-1}\left\{1 - \frac{2}{\pi}\tan^{-1}\sqrt{(\Delta\tau_s)^{-2} - 1} - \frac{4}{3\pi}\frac{1}{\Delta\tau_s}\right.$$
$$\left.\times\left[1 - \frac{3}{2}(1 - \Delta^2\tau_s^2)^{1/2} + \frac{1}{2}(1 - \Delta^2\tau_s^2)^{3/2}\right]\right\},$$
$$\Delta\tau_s < 1. \tag{35}$$

The dependence of J_s on the concentration of paramagnetic impurities (more precisely, on n/n_c), plotted on the basis of (33) and (35) with account taken of the dependence of Δ on n tabulated in [11,12], is shown in Fig. 2. On the same graph is shown for comparison the dependence of Δ and ω_g on n as taken from [11,12]. It is seen that J_s decreases with increasing impurity concentration more rapidly than Δ, but more slowly than ω_g.

In conclusion we remark that, as shown earlier,[7] calculation of the scattering of electrons within the dielectric layer of the tunnel junction, which leads to a reversal of the electron spin as it crosses through the barrier, also leads to a decrease in the Josephson tunnel current compared to the theoretical values of J_s^0 given by Eqs. (1) and (2).

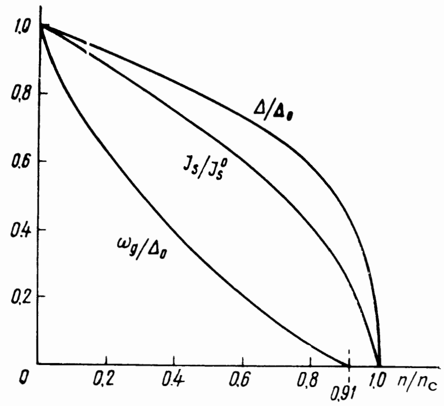

FIG. 2

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