SOME FEATURES OF THE PHOTOCONDUCTIVITY SPECTRUM OF SEMICONDUCTORS

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The dependence of semiconductor photoconductivity arising because of strongly nonequilibrium current carriers on the frequency of external radiation is investigated. Expressions are obtained for photoconductivity as a function of this frequency. Photoconductivity oscillations are observed when the interaction time between an electron and optical phonons is sufficiently small; the form of the oscillations is then found to depend on the specific energy dependences of the carrier lifetime and the relaxation time for impurity scattering. It is shown that when the energy of the photo-produced electrons is a multiple of the optical phonon energy negative photoconductivity arises, the photocurrent direction being opposed to that of the external field. In this case the current is a nonlinear function of the field strength.

1. INTRODUCTION. FORMULATION OF THE PROBLEM

MUCH interest is being shown at the present time in various semiconductor phenomena involving nonequilibrium current carriers; only electrons are specifically referred to in the present study. The electron energy distribution ordinarily differs little from the quasi-equilibrium distribution that is characterized by the effective temperature. Also, as a rule, the stabilization time for equilibrium between electrons and lattice vibrations is considerably shorter than the electron lifetime in the conduction band. Therefore electrons reach equilibrium rapidly, so that semiconductor conduction is determined mainly by thermal electrons.[1]

The situation changes considerably, however, when the lifetime τ_e of a nonequilibrium electron becomes shorter than both the interelectron interaction time τ_{ee} and the relaxation time τ_{ac} for acoustic phonon scattering. In this case electrons do not achieve equilibrium with the lattice during their lifetime in the conduction band and they possess energy distribution that differs greatly from quasi-equilibrium. The strongly nonequilibrium states can lead to several qualitatively new effects. We have previously[2] suggested that negative conductivity may exist under such conditions. In several recent investigations[3-5] photoconductivity was measured at very low temperatures that would appear to provide the foregoing situation. It was observed that photoconductivity oscillates as the external radiation source frequency increases and that the frequency of these oscillations coincides with that of longitudinal optical phonons.

The foregoing result makes it interesting to construct a theory of photoconductivity for such strongly nonequilibrium states and to analyze the mentioned effect; the present study is devoted to this end. Our theory of photoconductivity will be based on a solution of the kinetic equation for photoelectrons that interact only with optical phonons and impurities, neglecting interelectronic interactions and interactions with acoustic phonons. We also assume that the temperature is sufficiently low ($kT \ll \hbar\omega_0$, where $\hbar\omega_0$ is the frequency of optical phonons), and that photoelectrons are injected into the conduction band by a monochromatic source of intensity J and distribution $g(\Omega)$, where Ω is the frequency of the radiation source. We assume that the electric field $\mathbf{E}$ applied to the semiconductor is not too high, i.e., $\beta = eE\tau/\sqrt{\hbar\omega m} \ll 1$, where e, m, and τ are, respectively, the charge, effective mass, and total relaxation time of an electron, and $\hbar\omega$ is the energy with which electrons are introduced into the conduction band. This means that the energy acquired by an electron through acceleration in the field $\mathbf{E}$ is considerably smaller than the energy with which it originally appears in the conduction band.

We note that the familiar method of solving the kinetic equation using a series expansion of the electron distribution function in powers of the electric field can be applied in our problem only subject to the condition

$$\beta = \frac{eE\tau(\omega)}{(\hbar\omega m)^{1/2}} \ll \frac{\omega - \omega_0}{\omega}.$$

When ω is close to ω_0 this condition is violated and the indicated method is inapplicable. Therefore in our investigation of electron photoconductivity at energies near that of optical phonons we shall employ the method used by Keldysh[6] in his study of impact ionization within semiconductors. The kinetic equation will be transformed into an integral equation, which will then be solved exactly for a definite case.

In Sec. 2 we present the general formulas for calculating the photocurrent and the symmetric part of the electron distribution, and we also describe the method of solving the kinetic equation. In Sec. 3 we consider the conditions that produce photocurrent oscillations as the source frequency is varied. We consider negative conductivity far from "resonance" in Sec. 4 and at "resonance" ($\omega = \omega_0$) in Sec. 5. The principal results are summarized and analyzed in Sec. 6. We shall henceforth assume $\hbar = 1$.

2. DETERMINATION OF THE PHOTOCURRENT. GENERAL

The photocurrent is defined by

$$\mathbf{j} = -e\int \frac{2d\mathbf{p}}{(2\pi)^3}\mathbf{v}f(\mathbf{p}), \tag{1}$$

where $\mathbf{p}$ is the electron quasimomentum in the conduction band, $\mathbf{v} = \nabla_{\mathbf{p}}\epsilon_{\mathbf{p}}$, and $\epsilon_{\mathbf{p}}$ is the electron energy. The electron distribution function $f(\mathbf{p})$ satisfies the equation

$$-e\mathbf{E}\nabla_{\mathbf{p}}f(\mathbf{p}) + S_{\mathbf{p}}^{(-)}f(\mathbf{p}) = S_{\mathbf{p}}^{(+)}\{f\}, \tag{2}$$

where $\mathbf{E}$ is the external electric field of the semiconductor, and the "removal" and "admission" operators on electrons with respect to a state of given $\mathbf{p}$ are under our conditions

$$S_{\mathbf{p}}^{(-)} = 2\pi\int\frac{2d\mathbf{k}}{(2\pi)^3}\left\{|\hat{M}^{(op)}_{\mathbf{p},\mathbf{p}-\mathbf{k}}|^2\delta(\varepsilon_{\mathbf{p}}-\omega_{\mathbf{k}}-\varepsilon_{\mathbf{p}-\mathbf{k}}) + |\hat{M}^{(im)}_{\mathbf{p},\mathbf{p}-\mathbf{k}}|^2\delta(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}-\mathbf{k}})\right\} + \frac{1}{\tau_e(\varepsilon_{\mathbf{p}})},$$

$$S_{\mathbf{p}}^{(+)}\{f\} = 2\pi\int\frac{2d\mathbf{k}}{(2\pi)^3}f(\mathbf{p}+\mathbf{k})\left\{|\hat{M}^{(op)}_{\mathbf{p},\mathbf{p}+\mathbf{k}}|^2\delta(\varepsilon_{\mathbf{p}}+\omega_{\mathbf{k}}-\varepsilon_{\mathbf{p}+\mathbf{k}}) + |\hat{M}^{(im)}_{\mathbf{p},\mathbf{p}+\mathbf{k}}|^2\delta(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}+\mathbf{k}})\right\} + Ig(\varepsilon_{\mathbf{p}}-\omega). \tag{3}$$

Here $M^{(op)}$ and $M^{(im)}$ are the matrix elements of interactions between electrons and optical phonons or impurities, respectively; $\tau_e(\epsilon_{\mathbf{p}})$ is the electron lifetime in the conduction band. The term $Ig(\epsilon_{\mathbf{p}} - \omega)$ describes photoelectron production near $\epsilon_{\mathbf{p}} = \omega$ through the absorption of external monochromatic radiation; I is associated with the source strength J and the semiconductor absorption coefficient $k(\omega)$ by the relation

$$I = Jk(\omega)\pi^2p^{-2}|\partial\varepsilon_{\mathbf{p}}/\partial\mathbf{p}|.$$

The source width is $\Gamma \ll \omega$, so that

$$g(x) \neq 0 \quad \text{for} \quad |x| \sim \Gamma, \quad g(x) \approx 0 \quad \text{for} \quad |x| \gg \Gamma, \qquad \int_{-\infty}^{\infty} g(x)\,dx = 1. \tag{4}$$

We shall henceforth assume a quadratic dispersion law for electrons: $\epsilon_{\mathbf{p}} = p^2/2m$ and $M^{(op)}_{\mathbf{p},\mathbf{p}-\mathbf{k}} = M_0$ (independent of the quasimomentum). We note that the following calculation could be performed without making these assumptions; the resulting expressions would be essentially equivalent but more complicated, thus hindering their analysis. Finally, we assume $\omega_{\mathbf{k}} = \omega_0$ for the optical phonon spectrum; this is well satisfied if the electron quasimomentum is much smaller than the quasimomentum at the Brillouin zone boundary. We then obtain

$$S_{\mathbf{p}}^{(-)} = \frac{1}{\tau_e(\varepsilon_{\mathbf{p}})} + \frac{1}{\tau_{im}(\varepsilon_{\mathbf{p}})} + \frac{1}{\tau_{op}(\varepsilon_{\mathbf{p}})} = \frac{1}{\tau(\varepsilon_{\mathbf{p}})},$$

$$S_{\mathbf{p}}^{(+)}\{f\} = \frac{f_0(\varepsilon_{\mathbf{p}})}{\tau_{im}(\varepsilon_{\mathbf{p}})} + \frac{f_0(\varepsilon_{\mathbf{p}}+\omega_0)(\varepsilon_{\mathbf{p}}+\omega_0)^{1/2}}{\tau_{op}(\varepsilon_{\mathbf{p}})(\varepsilon_{\mathbf{p}}-\omega_0)^{1/2}} = S_{\mathbf{p}}^{(+)}\{f_0(\varepsilon_{\mathbf{p}})\}, \tag{5}$$

where the integral for elastic collisions with impurities is written in the τ approximation, so that $\tau_{im}(\epsilon_{\mathbf{p}})$ is the relaxation time for impurity scattering (see [7], for example).

The probability of spontaneous phonon emission is

$$\frac{1}{\tau_{op}(\varepsilon_{\mathbf{p}})} = \begin{cases} A\sqrt{\varepsilon_{\mathbf{p}}-\omega_0}, & \varepsilon_{\mathbf{p}} > \omega_0 \\ 0, & \varepsilon_{\mathbf{p}} \leqslant \omega_0 \end{cases}, \tag{6}$$

where $A = (2m)^{3/2}\pi^{-1}|M_0|^2$. The symmetric part $f_0(\epsilon_{\mathbf{p}})$ of the stationary nonequilibrium distribution function is

$$f_0(\varepsilon_{\mathbf{p}}) = \frac{\pi^2}{m\sqrt{2m\varepsilon_{\mathbf{p}}}}\int\frac{2d\mathbf{p}'}{(2\pi)^3}f(\mathbf{p}')\delta(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}'}). \tag{7}$$

It can now be seen from (2) and (5) that the right-hand side of the kinetic equation depends only on $f_0(\epsilon_{\mathbf{p}})$. If the right-hand side of (2) is considered to be inhomogeneous, the solution of the kinetic equation can be represented by

$$f(\mathbf{p}) = \int_0^{\infty} dt\, S^{(+)}_{\mathbf{p}+e\mathbf{E}t}\{f_0\}\exp\left(-\int_0^t S^{(-)}_{\mathbf{p}+e\mathbf{E}t'}\,dt'\right), \tag{8}$$

which may be verified indirectly. Substituting (8) into (7), we obtain the integral equation

$$f_0(\varepsilon_{\mathbf{p}}) = \frac{\pi^2}{m\sqrt{2m\varepsilon_{\mathbf{p}}}}\int\frac{2d\mathbf{p}'}{(2\pi)^3}\delta(\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p}'})$$

$$\times \int_0^\infty dt\, S^{(+)}_{\mathbf{p}'+e\mathbf{E}t}\{f_0\} \exp\left(-\int_0^t S^{(-)}_{\mathbf{p}'+e\mathbf{E}t'}\, dt'\right), \quad (9)$$

which determines the photocurrent when used in conjunction with Eqs. (1) and (8).

We now indicate the method of solving (9). When an electron is scattered by impurities its energy is not affected, whereas the electron loses the energy ω_0 when an optical phonon is emitted. Since the external source is highly monochromatic, the electrons will be grouped mainly near $\epsilon_{\mathbf{p}} = \omega$, where they are produced by the action of the source, and at $\epsilon_{\mathbf{p}} = \omega - k\omega_0$, $k = 1, 2, \ldots$, which they reach following the emission of one, two, etc. phonons. If ω lies within the interval $n\omega_0 \geq \omega \geq (n-1)\omega_0$, where n is an integer, the symmetric part of the distribution function can be represented by a sum:

$$f_0(\varepsilon_{\mathbf{p}}) = \sum_{l=1}^{n} f_{n,l}(\varepsilon_{\mathbf{p}}), \quad (10)$$

where $f_{n,l}(\epsilon_{\mathbf{p}})$ will differ greatly from zero near $\epsilon_{\mathbf{p}} = \omega - (n-l)\omega_0$. This enables us to transform (9) into a system of coupled integral equations. After making the substitution $\mathbf{p}' \to \mathbf{p}' - e\mathbf{E}t$ in (9) and utilizing (10) we obtain

$$f_{n,n}(\varepsilon_{\mathbf{p}}) = \frac{\pi^2}{m\sqrt{2m\varepsilon_{\mathbf{p}}}} \int \frac{2d\mathbf{p}'}{(2\pi)^3} \int_0^\infty dt\, \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'-e\mathbf{E}t}) \times \left\{\frac{f_{n,n}(\varepsilon_{\mathbf{p}'})}{\tau_{im}(\varepsilon_{\mathbf{p}'})} + Ig(\varepsilon_{\mathbf{p}'} - \omega)\right\} \exp\left[-\int_0^t \frac{dt'}{\tau(\varepsilon_{\mathbf{p}'-e\mathbf{E}t'})}\right], \quad (11)$$

$$f_{n,l}(\varepsilon_{\mathbf{p}}) = \frac{\pi^2}{m\sqrt{2m\varepsilon_{\mathbf{p}}}} \int \frac{2d\mathbf{p}'}{(2\pi)^3} \int_0^\infty dt\, \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'-e\mathbf{E}t}) \times \left\{\frac{f_{n,l}(\varepsilon_{\mathbf{p}'})}{\tau_{im}(\varepsilon_{\mathbf{p}'})} + A\sqrt{\varepsilon_{\mathbf{p}'} + \omega_0}\, f_{n,l+1}(\varepsilon_{\mathbf{p}'} + \omega_0)\right\} \times \exp\left[-\int_0^t \frac{dt'}{\tau(\varepsilon_{\mathbf{p}'-e\mathbf{E}t'})}\right]. \quad (12)$$

Equation (11) does not include the term associated with electron transitions to a state with $\epsilon_{\mathbf{p}} = \omega$ from $\epsilon_{\mathbf{p}} = \omega + \omega_0$ due to the emission of optical phonons, because there are no photoelectrons in $\epsilon_{\mathbf{p}} = \omega + \omega_0$, whereas for $\epsilon_{\mathbf{p}} = \omega - (n-l)\omega_0$ with $l < n$ electron transitions can occur only through optical phonon emission by electrons of higher-lying energies. The system represented by (11) and (12) will be solved here for small electric fields (Secs. 3, 4) and when $\omega = \omega_0$ (Sec. 5).

3. OSCILLATIONS OF PHOTOCONDUCTIVITY

We now consider the case of small electric fields permitting a series expansion in powers of **E**; the appropriate criterion will be given below. Confining ourselves to the first terms, we obtain from (8) and (9):

$$f_0(\varepsilon_{\mathbf{p}}) = \frac{S_{\mathbf{p}}^{(+)}\{f_0(\varepsilon_{\mathbf{p}})\}}{S_{\mathbf{p}}^{(-)}}, \quad (13)$$

$$f(\mathbf{p}) = \frac{S_{\mathbf{p}}^{(+)}\{f_0(\varepsilon_{\mathbf{p}})\}}{S_{\mathbf{p}}^{(-)}} + \frac{e\mathbf{E}\mathbf{p}}{m}\frac{1}{S_{\mathbf{p}}^{(-)}}\frac{d}{d\varepsilon_{\mathbf{p}}}\frac{S_{\mathbf{p}}^{(+)}\{f_0(\varepsilon_{\mathbf{p}})\}}{S_{\mathbf{p}}^{(-)}} = f_0(\varepsilon_{\mathbf{p}}) + \frac{e\mathbf{E}\mathbf{p}}{m}\tau(\varepsilon_{\mathbf{p}})\frac{df_0(\varepsilon_{\mathbf{p}})}{d\varepsilon_{\mathbf{p}}}, \quad (14)$$

corresponding to the customary method of solving the kinetic equation. The equation (1) for the current then becomes simply

$$\mathbf{j} = \mathbf{E}\frac{e^2 2\sqrt{2m}}{3\pi^2}\int_0^\infty d\varepsilon\, f_0(\varepsilon)\frac{d}{d\varepsilon}[\tau(\varepsilon)\varepsilon^{3/2}], \quad (15)$$

and the system of integral equations (11) and (12) is transformed into a system of coupled algebraic equations:

$$f_{n,n}(\varepsilon_{\mathbf{p}})\left[\frac{1}{\tau_e(\varepsilon_{\mathbf{p}})} + \frac{1}{\tau_{op}(\varepsilon_{\mathbf{p}})}\right] = Ig(\varepsilon_{\mathbf{p}} - \omega),$$

$$f_{n,l}(\varepsilon_{\mathbf{p}})\left[\frac{1}{\tau_e(\varepsilon_{\mathbf{p}})} + \frac{1}{\tau_{op}(\varepsilon_{\mathbf{p}})}\right] = f_{n,l+1}(\varepsilon_{\mathbf{p}} + \omega_0)\cdot\frac{(\varepsilon_{\mathbf{p}} + \omega_0)^{1/2}}{\tau_{op}(\varepsilon_{\mathbf{p}})(\varepsilon_{\mathbf{p}} - \omega_0)^{1/2}},$$

which are solved easily, yielding

$$f_{n,l}(\varepsilon_{\mathbf{p}}) = IC_{n,l}(\varepsilon_{\mathbf{p}})g[\varepsilon_{\mathbf{p}} - \omega + (n-l)\omega_0],$$

$$C_{n,l}(\varepsilon_{\mathbf{p}}) = A^{n-l}\varepsilon_{\mathbf{p}}^{-1/2}\prod_{k=0}^{n-l}\sqrt{\varepsilon_{\mathbf{p}} + k\omega_0} \times \left[\frac{1}{\tau_e(\varepsilon_{\mathbf{p}} + k\omega_0)} + \frac{1}{\tau_{op}(\varepsilon_{\mathbf{p}} + k\omega_0)}\right]^{-1},$$

$$n = \mathrm{E}\{\omega/\omega_0 + 1\}. \quad (16)$$

We note that $f_{n,l}(\epsilon_{\mathbf{p}})$ repeats the line shape of the external radiation source. This can occur only when the broadening of the electron distribution in the electric field is considerably smaller than the source width ($\beta \ll \Gamma/\omega$), which is a sufficient condition for the series expansions of (8) and (9) in powers of the field **E**.

We now calculate the current from (15) by using (16). Assuming $\omega - (n-1)\omega_0 \gg \Gamma$ and taking outside of the integrand the functions that vary slowly compared with $g[\epsilon_{\mathbf{p}} - \omega + (n-l)\omega_0]$, we obtain

$$\mathbf{j} = \mathbf{E}\left(\frac{e^2 2\sqrt{2m}}{3\pi^2}\right) I\sum_{l=1}^{n}\left\{C_{n,l}(\varepsilon)\frac{d}{d\varepsilon}[\tau(\varepsilon)\varepsilon^{3/2}]\right\}_{\varepsilon=\omega-(n-l)\omega_0}, \quad (17)$$

which represents the photoconductivity as a function of the frequency ω.

When $\tau_e(\epsilon) \gg \tau_{op}(\epsilon)$, which occurs in a broad region of ϵ except near ω_0, the principal contri-

bution to (17) comes from $C_{n,1}(\epsilon)$, so that the photocurrent is transformed into

$$\mathbf{j} = \mathbf{E}\left(\frac{e^2 2\sqrt{2m}}{3\pi^2}\right)\frac{I\tau_e[\omega-(n-1)\omega_0]\sqrt{\omega}}{[\omega-(n-1)\omega_0]^{1/2}} \times \left\{\frac{d}{d\varepsilon}\tau(\varepsilon)\varepsilon^{3/2}\right\}_{\varepsilon=\omega-(n-1)\omega_0}. \quad (18)$$

Thus the photocurrent oscillates regularly with the frequency ω_0, since $\omega - (n-1)\omega_0$ varies in the range $[0, \omega_0]$. Physically this means that the photocurrent depends on the behavior of electrons in the energy interval $[0, \omega_0]$, where they do not interact with optical phonons and remain a large part of their lifetime. When $\omega > \omega_0$ the electrons lose energy rapidly, emit optical phonons, and drop to a lower energy range.

We observe that the electric field for which (17) is valid must fulfill the condition

$$\beta = \frac{eE\tau(\omega)}{\sqrt{m\omega}} \ll \frac{\omega-(n-1)\omega_0}{\omega}. \quad (19)$$

Indeed, in the derivation of (17) no detailed knowledge of the distribution function $f_0(\epsilon_{\mathbf{p}})$ was required. The only essential requirement was that this function should vary much more rapidly with $\epsilon_{\mathbf{p}}$ than the remainder of the integrand in (15). Equation (19) is a sufficient condition for this purpose.

The case $\omega \to (n-1)\omega_0$ requires the special treatment that will be given in the following section.

4. NEGATIVE CONDUCTIVITY IN WEAK ELECTRIC FIELDS

For simplicity we confine ourselves to the case n = 2, i.e., $\omega_0 < \omega < 2\omega_0$. The electric current (15) is divided into two parts:

$$\mathbf{j} = \mathbf{j}_+ + \mathbf{j}_-,$$

$$\mathbf{j}_+ = \mathbf{E}\left(\frac{e^2 2\sqrt{2m}}{\pi^2}\right) \times \int_0^\infty d\varepsilon \sqrt{\varepsilon} f_0(\varepsilon)\left[\tau(\varepsilon) - \frac{2}{3}\varepsilon\tau^2(\varepsilon)\frac{d}{d\varepsilon}(\tau_e^{-1}+\tau_{im}^{-1})\right],$$

$$\mathbf{j}_- = -\mathbf{E}\left(\frac{4e^2\sqrt{2m}}{3\pi^2}\right)\int_0^\infty d\varepsilon\, \varepsilon^{3/2} f_0(\varepsilon)\tau^2(\varepsilon)\frac{d}{d\varepsilon}\tau_{op}^{-1}(\varepsilon). \quad (20)$$

The current $\mathbf{j}_+$ is parallel to $\mathbf{E}$, while $\mathbf{j}_-$ is antiparallel since the derivative of $\tau_{op}^{-1}(\epsilon)$ is positive (Eq. (6)). The procedure used to calculate (17) is also used to calculate $\mathbf{j}_+$ and $\mathbf{j}_-$:

$$j_+ = \frac{eE\tau^*(\omega)}{m} en_1 + \frac{eE\tau^*(\omega-\omega_0)}{m} en_2, \quad (21)$$

$$j_- = -\sqrt{\frac{2\omega}{m}} e\Delta n = -\frac{eE\tau(\omega)}{3m} en_1(\tau(\omega)A\sqrt{\omega})\left(\frac{\omega}{\omega-\omega_0}\right)^{1/2}, \quad (22)$$

where we have introduced the notation

$$\tau^*(\omega) = \tau(\omega) - \frac{2}{3}\omega\tau^2(\omega)\frac{d}{d\omega}(\tau_e^{-1}+\tau_{im}^{-1}),$$

$$n_1 = \frac{Jk(\omega)\tau_e(\omega)}{1+\tau_e(\omega)\tau_{op}^{-1}(\omega)},$$

$$n_2 = \frac{Jk(\omega)\tau_e(\omega-\omega_0)\tau_e(\omega)\tau_{op}^{-1}(\omega)}{1+\tau_e(\omega)\tau_{op}^{-1}(\omega)}$$

$$\Delta n = \frac{\sqrt{2}}{3} n_1\tau^2(\omega) eE\sqrt{\frac{\omega}{m}}\frac{d}{d\omega}\tau_{op}^{-1}(\omega). \quad (23)$$

Here $\tau^*(\omega)$ characterizes the electron accelerating time in the field $\mathbf{E}$, n_1 and n_2 are the electron densities at energies near ω and $\omega - \omega_0$, and Δn is the density of electrons participating in the negative current.

When we assume $\tau^*(\omega) \approx \tau(\omega)$ the current ratio becomes

$$\frac{j_+}{j_-} = \frac{3\sqrt{\omega-\omega_0}}{\tau(\omega)A\omega} + \frac{3(\omega-\omega_0)\tau_e(\omega-\omega_0)\tau(\omega-\omega_0)}{\omega\tau^2(\omega)}. \quad (24)$$

As ω approaches ω_0 this ratio diminishes and becomes smaller than unity for

$$\left(\frac{\omega-\omega_0}{\omega}\right)^{1/2} < \frac{2\tau(\omega)A\sqrt{\omega}(\sqrt{1+\xi}-1)}{3\xi}, \quad (25)$$

where $\xi = {}^4/_3\,\tau(\omega-\omega_0)\tau_e(\omega-\omega_0)A^2\omega$. For the most realistic case, when $\xi \gg 1$, Eq. (25) is replaced by

$$\frac{\omega-\omega_0}{\omega} < \frac{\tau^2(\omega)}{3\tau_e(\omega-\omega_0)\tau(\omega-\omega_0)}. \quad (26)$$

In the case of InSb, for example, we have

$$\tau(\omega) \approx \tau_{im}(\omega) \sim \omega^{3/2} \sim 10^{-12}\ \text{sec}$$

for $\omega \approx 0.013$ eV and $\tau_e \sim 10^{-10}$ sec.[3] Therefore negative conduction should occur when $\omega - \omega_0 \lesssim (0.01-0.1)\omega$.

5. NEGATIVE CONDUCTIVITY AT RESONANCE

It is interesting to analyze the behavior of the negative conductivity as ω approaches ω_0. The condition (19) is then violated and the customary method of solving the kinetic equation through a series expansion in powers of $\mathbf{E}$ cannot be applied. A complete solution of the problem is obtained by solving the integral equations (11) and (12) for $f_0(\epsilon_{\mathbf{p}})$. The inapplicability of the usual method results from the fact that although the energy ac-

quired by an electron from the electric field is much smaller than its mean energy, this additional energy is sufficiently large to produce a sharp change in the probability of optical phonon emission [Eq. (6)] and therefore in the electron energy distribution near ω_0.

Before proceeding to solve the equation for $f_0(\epsilon_{\mathbf{p}})$ it will be useful to analyze the expression obtained for the current when the solution of (9) is substituted into (1). Then, making the substitution $\mathbf{p} \to \mathbf{p} - e\mathbf{E}t$, we obtain

$$\mathbf{j} = \mathbf{j}_1 + \mathbf{j}_2,$$

$$\mathbf{j}_1 = \mathbf{E}\frac{e^2}{m}\int \frac{2d\mathbf{p}}{(2\pi)^3} S_{\mathbf{p}}^{(+)}\{f_0\}\int_0^\infty dt \cdot t \exp\left(-\int_0^t \frac{dt'}{\tau(\varepsilon_{\mathbf{p}-e\mathbf{E}t'})}\right),$$

$$\mathbf{j}_2 = -\frac{e}{m}\int \frac{2d\mathbf{p}}{(2\pi)^3}\mathbf{p} S_{\mathbf{p}}^{(+)}\{f_0\}\int_0^\infty dt \exp\left(-\int_0^t \frac{dt'}{\tau(\varepsilon_{\mathbf{p}-e\mathbf{E}t'})}\right). \quad (27)$$

If we assume for the combined relaxation time $\tau = \text{const}$, then $\mathbf{j}_2 = 0$ and $\mathbf{j}_1 = e^2\mathbf{E}\tau n m^{-1}$. It is thus seen that the current $\mathbf{j}_1$ results from the fact that electrons derive momentum from the electric field directly; $\mathbf{j}_1$ therefore always has the direction of the field.

The current $\mathbf{j}_2$ is of entirely different origin. The asymmetry inducing $\mathbf{j}_2$ is associated with the dependence of the relaxation time $\tau(\epsilon_{\mathbf{p}-e\mathbf{E}t})$ on the electron energy in the electric field. The direction of $\mathbf{j}_2$ will be determined by the specific form of $\tau(\epsilon_{\mathbf{p}})$. It is easily seen from (6) that $\tau_{op}(\epsilon_{\mathbf{p}})$ results in a negative current, which becomes appreciable if $f_0(\epsilon_{\mathbf{p}})$ has a maximum near $\epsilon_{\mathbf{p}} \approx \omega_0$. In this case the electric field participates in creating $\mathbf{j}_2$ as a factor governing the distribution of the external source energy among the electrons and optical phonons, so that the electron and phonon subsystems acquire momenta that are equal in magnitude but opposite in direction.

We shall now calculate the negative current for a specific case which is the reverse of that given by (19):

$$\omega - \omega_0 \ll eE\tau\sqrt{\omega/m}.$$

We can assume $\omega = \omega_0$ and a rigorously monochromatic source: $g(x) = \delta(x)$. The calculation becomes especially simple for fields satisfying

$$(\tau_0(\omega)A\sqrt{\omega})\left(\frac{eE\tau_0(\omega)}{\sqrt{m\omega}}\right)^{1/2} \ll 1, \quad \tau_0^{-1} = \tau_{im}^{-1} + \tau_e^{-1}. \quad (28)$$

The exponential in (27) can then be written as

$$\exp\left[-\int_0^t \frac{dt'}{\tau(\varepsilon_{\mathbf{p}-e\mathbf{E}t'})}\right] = \left\{1 + \frac{e\mathbf{E}\mathbf{p}}{m}\frac{d}{d\varepsilon_{\mathbf{p}}}\frac{1}{\tau_0(\varepsilon_{\mathbf{p}})} - A\int_0^t \left(\varepsilon_{\mathbf{p}} - \omega_0 - \frac{e\mathbf{E}\mathbf{p}}{m}t'\right)^{1/2} dt'\right\}\exp\left[-\frac{t}{\tau_0(\varepsilon_{\mathbf{p}})}\right]. \quad (29)$$

By substituting (29) into (27) we shall distinguish a current determined by interactions between electrons and optical phonons. For this purpose we retain the last term in (29), integrate twice by parts, and obtain

$$\mathbf{j}_- = -\frac{e^2}{2m^2}\int \frac{2d\mathbf{p}}{(2\pi)^3}\mathbf{p} S_{\mathbf{p}}^{(+)}\{f_0(\varepsilon_{\mathbf{p}})\} \times \tau_0^2(\varepsilon_{\mathbf{p}})A\int_0^\infty dt \frac{\mathbf{E}\mathbf{p}\exp[-t/\tau_0(\varepsilon_{\mathbf{p}})]}{[\varepsilon_{\mathbf{p}} - \omega_0 - e\mathbf{E}\mathbf{p}t/m]^{1/2}}. \quad (30)$$

Electrons contribute to the negative conductivity only if $\epsilon_{\mathbf{p}} \approx \omega_0$, so that

$$S_{\mathbf{p}}^{(+)}\{f_0(\varepsilon_{\mathbf{p}})\} = \frac{f_0(\varepsilon_{\mathbf{p}})}{\tau_{im}(\varepsilon_{\mathbf{p}})} + I\delta(\varepsilon_{\mathbf{p}} - \omega) \quad (31)$$

and we shall now regard $f_0(\epsilon_{\mathbf{p}})$ as the symmetric part of the distribution function satisfying (11) in the vicinity of $\epsilon_{\mathbf{p}} = \omega_0$. It is then sufficient to replace the exponential in (11) by $\exp[-t/\tau_0(\epsilon_{\mathbf{p}})]$. Integration over the angle variables converts (11) into

$$f_0(\varepsilon) = \int_0^\infty d\varepsilon' K(\varepsilon, \varepsilon')\{a(\varepsilon')f_0(\varepsilon') + b(\omega)\delta(\varepsilon' - \omega)\},$$

$$K(\varepsilon, \varepsilon') = \frac{1}{2\sqrt{\varepsilon}}\int_1^\Lambda \frac{dz}{z}\exp\left(-\frac{2m|\sqrt{\varepsilon} - \sqrt{\varepsilon'}|}{eE\tau_0(\varepsilon')}z\right),$$

$$\Lambda = \frac{\sqrt{\varepsilon} + \sqrt{\varepsilon'}}{|\sqrt{\varepsilon} - \sqrt{\varepsilon'}|},$$

$$a(\varepsilon) = \frac{\sqrt{m}}{\sqrt{2}\,eE\tau_{im}(\varepsilon)}, \quad b(\omega) = \frac{I\sqrt{m\omega}}{\sqrt{2}\,eE}. \quad (32)$$

When $e\mathbf{E}\tau_0(\omega) \ll \sqrt{m\omega}$ the upper limit of integration in the kernel $K(\epsilon, \epsilon')$ can be replaced by infinity, thus enabling us to neglect small terms $\sim \exp(-\sqrt{2m\omega}/e\mathbf{E}\tau_0)$. By making the change of variables

$$x = \frac{\varepsilon - \omega}{\sqrt{2}\beta\omega}, \quad \beta = \frac{eE\tau_0(\omega)}{\sqrt{m\omega}}$$

and by extending (in the same approximation) the lower limit of integration over x′ to $-\infty$, we obtain for the distribution function in the vicinity of $\epsilon = \omega$ the Milne integral equation[8]

$$f_0(x) = {}^1/_2\int_{-\infty}^\infty dx'[hf_0(x') + Q\delta(x')]E_1(|x - x'|),$$

$$E_1(|x - x'|) = \int_1^\infty \frac{dz}{z}e^{-|x-x'|z}, \quad h = \frac{\tau_0(\omega)}{\tau_{im}(\omega)}, \quad Q = I\tau_0(\omega),$$

which is solved by a Fourier transform. Standard operations yield the exact solution

$$f_0(x)=\frac{Q}{2}E_1(|x|)+\frac{Q\xi_0(1-\xi_0)}{h(h+\xi_0^2-1)}e^{-\xi_0|x|}$$
$$+\frac{Q}{2}\int_1^\infty\frac{\eta(h,\xi)e^{-|x|\xi}d\xi}{1-\eta(h,\xi)}, \qquad (33)$$

where ξ_0 is the root of the transcendental equation

$$1=\frac{h}{2\xi_0}\ln\frac{1+\xi_0}{1-\xi_0},$$

and

$$\eta(h,\xi)=1-\frac{\pi^2h^2}{4\xi^2}-\left(1-\frac{h}{2\xi}\ln\frac{\xi+1}{\xi-1}\right)^2.$$

The first term in (33) describes the distribution for unscattered electrons and becomes dominant if $\tau_e(\omega)\ll\tau_{im}(\omega)$, when the "absorption" of electrons prevails over scattering by impurities. The second term describes the distribution of scattered electrons and becomes dominant in the opposite case, when $\tau_{im}(\omega)\ll\tau_e(\omega)$, where we have

$$f_0(x)=\frac{3Q}{2\xi_0}e^{-\xi_0|x|};\quad \xi_0^2=\frac{3\tau_0(\omega)}{\tau_e(\omega)}. \qquad (34)$$

The latter distribution corresponds to the diffusion approximation for the kinetic equation, so that the analogue of the diffusion length in energy space is

$$L=eE[\omega\tau_e(\omega)\tau_0(\omega)/6m]^{1/2},$$

which is the maximum broadening of the electron distribution near $\epsilon_p=\omega$. The third term is unimportant because for $x\ll1$ it is considerably smaller than the first term, while for $x\gg1$ it is considerably smaller than the second term.

We note that for arbitrary values of ω satisfying the inequality $\beta\omega\ll\omega-\omega_0$ the symmetric part of the distribution function will have a similar form after replacing $\tau_0(\omega)$ by $\tau(\omega)$.

When $\tau_e(\omega)\ll\tau_{im}(\omega)$ we can neglect the first term in (31) as compared with the second term. A complete integration for $\omega=\omega_0$ then yields

$$j_-=-\frac{\sqrt{\pi}}{10}\frac{eE\tau_e(\omega_0)}{m}eJk(\omega_0)\tau_e(\omega_0)[\tau_e(\omega_0)$$
$$\times A\sqrt{\omega_0}]\left[\frac{\sqrt{2m\omega_0}}{eE\tau_e(\omega_0)}\right]^{1/2}. \qquad (35)$$

We see that except for a numerical factor this result coincides in the same limiting case with (22) when we replace $\omega-\omega_0$ with $eE\tau_e(\omega_0)(\omega_0/m)^{1/2}$ in the latter equation.

In the opposite case $\tau_e(\omega)\gg\tau_{im}(\omega)$, however, by neglecting the second term in (31) compared with the first term and substituting the distribution (34) into (30) we obtain after a similar integration

$$j_-=-\frac{\sqrt{\pi}\,3^{-3/4}}{5\sqrt{2}}\frac{eE\tau_{im}(\omega_0)}{m}eJk(\omega_0)\tau_e(\omega_0)$$
$$\times[\tau_{im}(\omega_0)A\sqrt{\omega}]\left[\frac{\tau_{im}(\omega_0)}{\tau_e(\omega_0)}\right]^{1/4}\left[\frac{\sqrt{2m\omega_0}}{eE\tau_{im}(\omega_0)}\right]^{1/2}. \qquad (36)$$

with $eE\tau_{im}(\omega_0/m)^{1/2}$ replaced by $\omega-\omega_0$, this result will differ from (22) only by a numerical coefficient and by the factor $(\tau_{im}/\tau_e)^{1/4}$ that results from additional broadening of the symmetric part of the distribution function in the electric field. By virtue of (25) it follows that negative conduction also occurs at "resonance." We note that the dependence of (35) or (36) on the electric field is nonlinear, having the form $j\sim\sqrt{E}$.

6. ANALYSIS OF RESULTS. CONCLUSION

1. Negative conductivity permits the following simple physical interpretation. In phase space electrons are produced in a narrow spherical layer close to the sphere $p^2/2m=\omega\approx\omega_0$. When an electric field is switched on the electron quasimomentum becomes $\mathbf{p}'=\mathbf{p}-e\mathbf{E}\tau$ and their energy becomes $\epsilon_{\mathbf{p}}'\approx\epsilon_{\mathbf{p}}-(e\mathbf{E}\cdot\mathbf{p}\tau/m)$. Electrons moving against the field acquire more energy and begin to interact more efficiently with optical phonons [Eq. (6)] than do the electrons moving in the direction of the field E. Since in this case electrons interacting with optical phonons lose almost all their energy and quasimomentum, an excess of electrons with the quasimomentum $(2m\omega)^{1/2}$ will appear, moving mainly in the field direction. This results in a negative current $j_-\sim-e\Delta n(2\omega/m)^{1/2}$.

Under stationary conditions the electron excess Δn can be determined from the balance equation $\Delta n/\tau=n_1W$, where W is the probability that electrons will be produced with an uncompensated momentum:

$$W\sim\tau_{op}^{-1}\left(\varepsilon_p+\frac{eEp\tau}{m}\right)-\tau_{op}^{-1}\left(\varepsilon_p-\frac{eEp\tau}{m}\right),$$

which for $eEp\tau m^{-1}\ll\epsilon_{\mathbf{p}}-\omega_0$ gives $W\sim2eEp\tau m^{-1}\times(d\tau_{op}^{-1}/d\epsilon)$, so that we have

$$j_-\sim-e(2\omega/m)^{1/2}n_1eEp\tau^2m^{-1}(d\tau_{op}^{-1}/d\varepsilon),$$

which coincides with (22) except for a numerical coefficient.

For exact resonance, $\epsilon_{\mathbf{p}}=\omega_0$, we have $W\sim A(eEp\tau/m)^{1/2}$, so that

$$j_-\sim e(2\omega/m)^{1/2}n_1\tau A(eEp\tau/m)^{1/2},$$

which coincides with (35) when $\tau\approx\tau_e$. It is thus found that negative conductivity results only from

a nonequilibrium energy distribution of electrons and from the threshold character of the interaction between electrons and optical phonons.

2. It was shown in Sec. 3 that the main contribution to photoconductivity comes from electrons having energies in the interval $[0, \omega_0]$ into which electrons move at a rapid rate from higher energies as optical phonons are emitted. Therefore the negative conductivity considered in Secs. 4 and 5 will appear whenever the frequency ω is nearly a multiple of an optical phonon frequency. Under these conditions the photoconductivity spectrum can contain dips at the points $\omega = k\omega_0$, $k = 1, 2, \ldots$, with their widths determined by (25). The experimentally observed dips at the maximum of oscillations in the photoconductivity spectrum[3] appear to be of just this nature.

3. The foregoing analysis of the negative current permits an important generalization. Smallness of the lifetime τ_e compared with τ_{ac} and τ_{ee} is not a necessary condition if the inequalities $\tau_{ac}(\omega) \gg \tau_{op}(\omega)$ and $\tau_{ee}(\omega) \gg \tau_{op}(\omega)$ hold true. This means that electrons can contribute to the negative current before their energy distribution is smeared out by acoustic phonons and by interelectronic interactions.

4. In electric fields that are high enough to violate condition (28) the solution of our problem requires special treatment. However, a qualitative analysis shows that in this case the negative current ceases to depend on the electric field. In such high fields almost all electrons moving against the field direction lose their momentum as they emit optical phonons so that any further increase of the field strength does not essentially affect the electron momentum distribution. For this reason the field dependence of the positive current should be weakened when ω is close to ω_0.

5. The expressions (17) and (18) for the photoconductivity that were derived in Sec. 3 contain semiconductor properties which were not averaged over energy, such as the electron lifetime in the conduction band, and the relaxation times for scattering from impurities and optical phonons. Therefore a measurement of the photoconductivity spectrum can furnish sufficiently full information regarding these properties if we also take into account that the entire foregoing analysis of the photocurrent can be performed with an arbitrary electron dispersion and an arbitrary law for the interaction between electrons and optical phonons.

6. An analysis of (24) for the ratio between the positive and negative currents shows that when ω approaches ω_0 this ratio becomes arbitrarily small. This result is associated with the fact that in calculating the photocurrent we neglected the broadening of electron levels resulting from the finite lifetimes of electrons and optical phonons, so that the minimum value of this ratio will depend on the specific parameters for a given semiconductor.

7. It has been suggested that negative conductivity can result from the existence of "negative masses."[9] We emphasize that the negative conductivity discussed in the present article and in [2] is of a very different character. Specifically, the interaction between electrons and optical phonons results in negative conductivity, whereas this interaction is undesirable for conductivity based on "negative masses."

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[1] S. M. Ryvkin, Fotoelektricheskie yavleniya v poluprovodnikakh (Photoelectric Phenomena in Semiconductors), Fizmatgiz, 1963.

[2] V. F. Elesin and É. A. Manykin, JETP Letters **3**, 26 (1966), transl. p. 15. Soviet Phys.—JETP Letters **3**, 15.

[3] M. A. Habegger and H. Y. Fan, Phys. Rev. Letters **12**, 99 (1964).

[4] H. J. Stocker, C. R. Stannard, H. Kaplan, and H. Levinstein, Phys. Rev. Letters **12**, 163 (1964); D. N. Nasledov, Yu. G. Popov, and Yu. S. Smetannikova, FTT **6**, 3728 (1964), Soviet Phys. Solid State **6**, 2989 (1965).

[5] C. Benoita a la Guillaume and J. Cernogor, J. Phys. Chem. Solids **24**, 293 (1963); I. A. Kurova and N. N. Ormont, FTT **6**, 3708 (1964), Soviet Phys. Solid State **6**, 2970 (1965); V. A. Besfamil'naya, I. A. Kurova, N. N. Ormont, and V. V. Ostroborodova, JETP **48**, 1588 (1965), Soviet Phys. JETP **21**, 1065 (1965).

[6] L. V. Keldysh, Doctoral dissertation, Phys. Inst. Acad. Sci. USSR, 1965.

[7] A. I. Ansel'm, Vvedenie v teoriyu poluprovodnikov (Introduction to the Theory of Semiconductors), Fizmatgiz, 1962.

[8] P. M. Morse and H. Feshbach, Methods of Theoretical Physics, vol. I, McGraw-Hill Book Co., New York, 1953 (Russ. transl., IIL, 1958).

[9] H. Krömer, Phys. Rev. **109**, 1856 (1958); D. C. Mattis and M. J. Stevenson, Phys. Rev. Letters **3**, 18 (1959), P. Kaus, Phys. Rev. Letters **3**, 20 (1959).

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166