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STABILITY OF A FOCUSED CHARGED BEAM IN AN ABSORBING MEDIUM AGAINST TRANSVERSE OSCILLATIONS

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The transverse stability of a focused beam of charged particles moving through an absorbing medium is analyzed. Various kinds of coherent oscillations (of the center of gravity, dimensions, and shape) are found together with their characteristic frequencies and growth rates. The possibility of beam stabilization by energy spread or nonlinearity in the betatron oscillations is examined. The effect due to the plasma produced as a result of ionization of the residual gas is determined. These questions are of interest in the design and use of accelerators and storage rings for high-intensity beams.

THE experimental data pertaining to the existence of transverse instabilities in a beam of particles formed in a cyclical accelerator or storage ring[1,2] has been examined theoretically in papers by the present authors[3] and by Laslett et al.[4] under certain restrictions. It has been found that an instability associated with oscillations of the center of gravity of the beam can arise by virtue of the finite conductivity of the walls of the vacuum chamber.

It is desirable, however, to consider the problem from a more general point of view. First of all, one should find all the possible kinds of oscillations that can arise by virtue of the space charge; then, it would be desirable to establish the relative danger of these oscillations and to determine the appropriate frequencies and growth rates. It should be noted that in [3,4] consideration has been given only to the motion of the center of gravity; oscillations of the higher moments have been neglected. It is also important to note that in practice an instability can arise by virtue of an interaction between the beam and any absorbing medium, not necessarily the walls. For instance, the role of such a medium can be played by the ionized residual gas. The present paper is devoted to a consideration of these problems.

We neglect the curvature of the chamber and assume that the motion takes place along one of the the transverse axes, specifically the vertical axis (z). In order to introduce the oscillation along the x axis we carry out an averaging over this coordinate and assume that the dependence of the charge density on this coordinate remains unchanged. The y axis is taken parallel to the walls of the chamber and it is assumed that the motion in this direction is characterized by a constant velocity.

EQUATION FOR THE CHARGE DENSITY

Under the assumptions given above there exists an equilibrium particle distribution which is independent of z and the time t. The equilibrium distribution function F_0 satisfies the relations

$$\int_{-\infty}^{\infty} F_0\,dp_x = Ng(p_y)W(x)\Phi(z,\,p_z),\qquad \int_{-\infty}^{\infty} W^2(x)\,dx = \frac{1}{w},\tag{1}$$

where the functions $g(p_y)$ and $\Phi(z, p_z)$ characterize respectively the dependence on the longitudinal momentum and on the variables that describe the vertical oscillations; N is the number of particles per unit length of the chamber and w is the width of the beam. We introduce the following notation: f is the small equilibrium increment to the distribution function; **E** and **H** are the electric and magnetic fields produced by this deviation; $\xi(\xi_0, t)$ represents the coordinates and the momentum components of a particle which has an initially given ξ_0 and moves in the field produced by external sources (focusing field) and the equilibrium beam, $\xi_0(\xi, t)$ represents the corresponding inverse functions ($\xi = x, y, z, p_x, p_y, p_z$). Starting from the kinetic equation we obtain the following expression for the Fourier component of the functions f, **E**, and **H**:[3]

$$f_{\omega k} = -e\int_0^{\infty} e^{i(\omega-kv_y)\tau}\left\{\frac{\partial F_0}{\partial \mathbf{p}}\left(\mathbf{E}_{\omega k}+\frac{1}{c}[\mathbf{v}\mathbf{H}_{\omega k}]\right)\right\}\xi_0(\xi,\,\tau)\,d\tau.\tag{2*}$$

The dependence of all quantities on y and t is written in the form $\exp[i(ky - \omega t)]$. The subscripts k and ω will be omitted below.

*$[\mathbf{v}\mathbf{H}_{\omega k}] \equiv \mathbf{v}\times\mathbf{H}_{\omega k}$.

We assume that the beam thickness $h \ll w$ and $|kh| \ll 1$. Then, in the region occupied by the beam, the field components E_z and H_x are large compared with the others and can be determined from the equations

$$\frac{dE_z}{dz} = 4\pi\varepsilon^{-1}\rho(x, z), \quad \frac{dH_x}{dz} = \frac{4\pi\mu}{c} j_y(x, z), \qquad (3)$$

where ϵ and μ are the dielectric constant and magnetic permeability of the medium through which the beam moves, while ρ and j are the nonequilibrium charge density and current density. We have $j_y = V\rho$, where $V = \beta c$ is the mean longitudinal velocity of the particles so that $H_x = \epsilon\mu\beta E_z$. We substitute (2) in (3) and integrate the resulting expression over all momenta and over the x coordinate. Then, making use of (1) and assuming that the function Φ is stationary and that the particle motion along the z axis is periodic and nonrelativistic, we obtain an equation for E_z:

$$E_z(z) = \frac{4e^2N}{w}(\varepsilon^{-1} - \mu\beta^2) \sum_{n=1}^{\infty} \int_{-\infty}^{\infty} g(p_y)\,dp_y \int_{-\infty}^{z} dz'$$

$$\times \int_{-\infty}^{\infty} \frac{n\omega_z\,dp_z}{(\omega - kv_y)^2 - n^2\omega_z^2} \cdot \int_{-\pi}^{\pi} \left\{\frac{\partial\Phi}{\partial p_z} E_z\right\} \qquad (4)$$

$$\times \left(z_0\left(z', p_z, \frac{\vartheta}{\omega_z}\right),\ p_{z0}\left(z', p_z, \frac{\vartheta}{\omega_z}\right)\right)\sin n\vartheta\,d\vartheta,$$

where ω_z is the frequency of the betatron oscillations with the effect of the field of the equilibrium beam taken into account.

We assume that the particle density in the beam is small (this criterion will be refined below). In this case (4) will have a solution only when the denominator of one of the terms in the series tends to zero while the other terms can be neglected. We shall assume, for the time being, that the beam is monoenergetic and that the betatron oscillations are linear. The function Φ depends on the square of the amplitude of the oscillations U and (4) becomes

$$E(z) = \frac{4e^2Nn(\varepsilon^{-1} - \mu\beta^2)}{\pi m\gamma hw[(\omega - kV)^2 - n^2\omega_z^2]} \int_{-\infty}^{\infty} K(z, z')E(z')\,dz';$$

$$K(z, z') = h\int_{-\pi}^{\pi} \Phi\left(\frac{z^2 - 2zz'\cos\vartheta + z'^2}{\sin^2\vartheta}\right)|\sin n\vartheta|\,d\vartheta, \qquad (5)$$

where m is the mass of the particle, $\gamma = (1-\beta^2)^{-1/2}$, and in contrast with (1) and (4), the function $\Phi(U)$ is now normalized so that

$$\int_0^{\infty} \Phi(U)\,dU = 1.$$

The quantity n in (5) is understood to be an integer which corresponds to the region in which the denominator in (4) approaches zero.

DISPERSION EQUATION

The equation in (5) is a Fredholm equation with a real symmetric kernel. Hence, we can write

$$(\omega - kV)^2 - n^2\omega_z^2 = \frac{4e^2Nn}{\pi m\gamma hw\lambda}(\varepsilon^{-1} - \mu\beta^2), \qquad (6)$$

where the dimensionless number λ is one of the characteristic values of (5) and depends on the form of the distribution function. It is assumed that the right side of (6) is small compared with $c\omega_z^2$. Making use of the conditions

$$\omega\,\mathrm{Im}\,\varepsilon(\omega) \geqslant 0, \quad \omega\,\mathrm{Im}\,\mu(\omega) \geqslant 0,$$

we find that when $\lambda > 0$ an instability is possible in the region $|kV| > n\omega_z$, and that the solution which corresponds to growing waves can be written in the form

$$\mathrm{Re}\,\omega = kV - \frac{kV}{|kV|}\left[n\omega_z + \frac{2e^2N}{\pi m\gamma\omega_z hw\lambda}\mathrm{Re}(\varepsilon^{-1} - \mu\beta^2)\right], \qquad (7a)$$

$$\mathrm{Im}\,\omega = \frac{2e^2N}{\pi m\gamma\omega_z hw\lambda}\left(\frac{|\mathrm{Im}\,\varepsilon|}{|\varepsilon|^2} + \beta^2|\mathrm{Im}\,\mu|\right), \qquad (7b)$$

where $\epsilon(\omega)$ and $\mu(\omega)$ are to be taken at $\omega = kV - n\omega_z kV/|kV|$. In the coordinate system fixed in the beam the frequency of the perturbation is approximately a multiple of the betatron frequency. If $\lambda < 0$, the growth rate is given as before by (7b) while the oscillation frequency becomes

$$\mathrm{Re}\,\omega = kV + n\omega_z - \frac{2e^2N\,\mathrm{Re}(\varepsilon^{-1} - \mu\beta^2)}{\pi m\gamma\omega_z hw|\lambda|} \quad \text{for } kV > -n\omega_z, \qquad (8a)$$

$$\mathrm{Re}\,\omega = kV - n\omega_z + \frac{2e^2N\,\mathrm{Re}(\varepsilon^{-1} - \mu\beta^2)}{\pi m\gamma\omega_z hw|\lambda|} \quad \text{for } kV < n\omega_z. \qquad (8b)$$

Thus, the existence of dissipation in the medium surrounding the beam leads to an instability, the boundary region of the instability being determined by the sign of the quantity λ. It is shown in the Appendix that when $\Phi'(U) < 0$ all the λ are positive. The focus beam then represents an ensemble of moving oscillators. The condition $\Phi'(u) < 0$ means that it is the lower energy levels that are primarily filled in the system. The instability of this beam is the result of the anomalous Doppler effect which arises when $0 < V_{ph} < V$ where V_{ph} is the phase velocity of the wave, as indicated by

(7a). The energy which drives the transverse oscillations and is responsible for the growing field comes from the longitudinal motion, and the absorbing medium plays the role of an intermediate agency. If the beam is at rest with respect to the medium this instability cannot arise.

If the condition $\Phi'(U) < 0$ does not hold, there will be both positive and negative characteristic values and, in accordance with (8), both waves of the above type and waves characterized by a phase velocity greater than V (or directed in the opposite direction) can be unstable. The mechanism responsible for this instability is the normal Doppler effect. In this case the growing electromagnetic field is produced as a result of the excess of induced emission over absorption. Hence, the instability can also develop in a beam at rest, and the effect of the instability is to smooth the distribution function.

CHARACTERISTIC FUNCTIONS OF THE BEAM OSCILLATIONS OF THE HIGHER MOMENTS

The asymptotic behavior of the perturbation is determined by root of the dispersion equation with the largest imaginary part. According to (7) and (8) the effect of space charge is reduced as $|\lambda|$ increases. Estimates of the minimum characteristic value and corresponding characteristic distribution function for the equation in (5) for the important case $\Phi'(U) < 0$ are given in the Appendix. Making use of the results obtained there, we write the frequency and growth rate of the oscillations:

$$|\mathrm{Re}\,\omega| = |kV| - n\omega_z - \frac{3n}{4n^2-1}\frac{\omega_L{}^2}{2\gamma\omega_z}\mathrm{Re}(\varepsilon^{-1} - \mu\beta^2), \quad (9a)$$

$$\mathrm{Im}\,\omega = \frac{3n}{4n^2-1}\frac{\omega_L{}^2}{2\gamma\omega_z}\left(\frac{|\mathrm{Im}\,\varepsilon|}{|\varepsilon|^2} + \beta^2|\mathrm{Im}\,\mu|\right) \quad (9b)$$

(ω_L is the mean plasma frequency of the beam). In this case the particle distribution in the nonequilibrium deviation is given by

$$\rho(z) \sim \frac{d}{dz}\sin\left(n \arccos\frac{2z}{h}\right) \quad \text{for} \quad |2z| < h,$$
$$\rho(z) = 0 \quad \text{for} \quad |2z| > h.$$

If $\Phi(U)$ is a power function which vanishes when $U > h^2/4$, then the expression for ρ is exact and the equilibrium charge density $\rho_0(z)$ is proportional to $[1 - (2z/h)^2]^{1/2}$. When n = 1 the nonequilibrium density is proportional to $\rho_0'(z)$ so that the perturbation leads to the displacement of the beam without a change in cross section. When n = 2 the beam perturbation consists of a uniform compression or extension along the z axis with $\rho(z) \sim [z\rho_0(z)]'$. If n > 2, oscillations of the higher moments appear but these do not cause a displacement or change in dimensions of the beam and are less dangerous.

The results given here apply so long as the terms proportional to the particle density can be regarded as corrections, that is to say, so long as the inequality $\omega_L^2\gamma^{-1}|\epsilon^{-1} - \mu\beta^2| \ll \omega_z^2$ is satisfied. In vacuum this relation yields the simple criterion $\omega_L^2 \ll \omega_z^2\gamma^3$.

As an example we consider the passage of a beam through an isotropic plasma; the dielectric constant and the permeability are given by

$$\varepsilon = 1 - \frac{\omega_p{}^2}{\omega^2}\left(1 - i\frac{\nu_{\mathrm{eff}}}{\omega}\right), \quad \mu = 1$$

(ω_p is the plasma frequency and ν_{eff} is the effective collision frequency in the plasma). In a toroidal geometry, waves can propagate for which the chamber length is an integer multiple of the wavelength. Assuming this to be the case and introducing the symbol Ω for the mean angular velocity of the particles and $Q = \omega_z/\Omega$ for the number of betatron oscillations per turn, we get from (9b) as an approximation

$$(\mathrm{Im}\,\omega)_p = \frac{3n}{4n^2-1}\frac{\omega_L{}^2}{2\gamma\Omega Q}\frac{\Omega\nu_{\mathrm{eff}}}{\omega_p{}^2}(l - nQ) \times\left\{\frac{\nu_{\mathrm{eff}}}{\Omega^2(l-nQ)^2} + \left[1 - \frac{\Omega^2}{\omega_p{}^2}(l-nQ)^2\right]^2\right\}^{-1}, \quad (10)$$

where $l > nQ$ is an integer. The expression in (10) is a sensitive function of the frequency of the perturbation $\mathrm{Re}\,\omega \approx \Omega(l - nQ)$ and is a maximum when $\mathrm{Re}\,\omega \approx \omega_p$. If $\Omega/\omega_p \gg 1$ this condition can only be satisfied for a minimum $l > nQ$, whence it follows that perturbations with the longest possible wavelength are favored for the oscillations of any moment. A strong functional dependence $\epsilon(\omega)$ can cause a significant increase in the growth rate for any one of the oscillation modes.

A plasma can be formed in an accelerator or in a storage ring by ionization of the residual gas. In the case of an electron accelerator, which we shall use as an example, it is only necessary to take account of the positive ions, which are bound by the attraction of the beam. Hence ν_{eff} denotes the frequency of collisions between ions and neutral atoms: $\nu_{\mathrm{eff}} \approx V_T S_{\mathrm{eff}} N_n$ where V_T is the thermal velocity of the ions ($\sim 10^5$ cm/sec), S_{eff} is the effective geometric cross-section of the atoms ($\sim 10^{-15}$ cm^2) and N_n is the concentration of neutral atoms. We make use of the results of earlier work[3] and substitute the numerical values for a number of quantities which are usually approximately the same for different accelerators; in this way we find that the ratio of the growth rates for the "plasma" instability and the "wall" instability is of order

$$\frac{(\mathrm{Im}\,\omega)_p}{(\mathrm{Im}\,\omega)_w} \approx \frac{10^5 a^3 P}{hw\Omega^{1/2}} \frac{(\mathrm{Re}\,\omega/\omega_p)^2}{[1-(\mathrm{Re}\,\omega/\omega_p)^2]^2}, \qquad (11)$$

where P is the chamber pressure (in mm Hg); the chamber height a as well as h and w are measured in centimeters and Ω is measured in $\sec^{-1}$. It is assumed that $\mathrm{Re}\,\omega = \Omega(l - nQ) \neq \omega_p$.

Thus, the relative influence of the conductivity of the medium increases with increasing chamber dimensions and pressure of the residual gas and with reduction in the cross-section area of the beam. For the values of these parameters usually encountered, the first factor of the right hand side of (11) is small compared with unity and it follows that the effect of the medium is important when $\mathrm{Re}\,\omega \approx \omega_p$. The ion density required for this condition to obtain lies in the range 10^7–10^{10} cm^{-3} if the rotational frequency is 10^7–10^9 $\sec^{-1}$.

EFFECT OF ENERGY SPREAD AND NONLINEARITY OF THE BETATRON OSCILLATIONS

In order to take these effects into account we make use of (4), retaining only one term in the series as before, and making use of perturbation theory. In this way we find[3, 4]

$$1 = -\Lambda \int_0^\infty \int_{-\infty}^\infty \frac{\Phi'(U)\,U\,dU\,g(p_y)\,dp_y}{(\omega - kv_y)^2 - n^2\omega_z^2}, \qquad (12)$$

where Λ is the right side of the dispersion equation (6) with the nonlinearity and energy spread neglected.

We shall investigate the effect of the nonlinearity separately, limiting our analysis to the important case $\Phi'(U) < 0$. We seek a solution close to $kV + n\omega_z$, $n = \pm1, \pm2, \ldots$. We first introduce the following notation: u_0 is the square of the amplitude at which the function $U\Phi'(U)$ reaches a maximum; $\delta + i\zeta = \omega - kV - n\omega_z(u_0)$ is the deviation in the oscillation frequency from the value obtained without the beam field taken into account; $\delta_0 + i\zeta_0 = \Lambda[2n\omega_z(u_0)]^{-1}$ is the same deviation for the linear oscillations; $u = U - u_0 - (u + u_0)\Phi'(u + u_0)$. Using (12) we have

$$\frac{1}{\delta_0 + i\zeta_0} = \int_{-u_0}^{\infty} \frac{f(u)\,du}{\delta + i\zeta - nu\,\partial\omega_z/\partial u}. \qquad (13)$$

Let us consider the right side as a function of the complex variable $\delta + i\zeta$. The lines ζ = const > 0 is mapped by closed curves C_ζ located in the lower half plane, and the line $C_0 = C_{\zeta \to +0}$ encloses all the $C_{\zeta > 0}$ lines. When $\zeta \to \infty$ the curves contract to the origin. Hence the system is unstable if the point $(\delta_0 + i\zeta_0)^{-1}$ lies inside C_0. If this is not the case, the oscillations are characterized by constant amplitude (if $(\delta_0 + i\zeta_0)^{-1}$ lies on C_0) or are not excited at all. Hence, the system is stable if $\zeta_0 < 0$, that is to say, the existence of a nonlinearity does not lead to the appearance of new regions of instability.

Computing the right side of (13) for $\epsilon \to +0$ and using the criterion given above, we obtain the stability condition:

$$n\frac{\partial\omega_z}{\partial u}\frac{\delta_0 u_{1,2}}{\delta_0^2 + \zeta_0^2} \geqslant \mathrm{P}\int \frac{f(u)\,du}{1 - u/u_{1,2}}, \qquad (14a)$$

where P denotes the principal value of the integral while the quantities $u_2 < 0$ and $u_2 > 0$ are solutions of the equation

$$f(u_{1,2}) = \frac{\zeta_0\,|n\partial\omega_z/\partial u|}{\pi(\delta_0^2 + \zeta_0^2)}. \qquad (14b)$$

If the function f(u) approaches zero rapidly as $|u|$ increases and $\delta_0 \ll |\delta_0|$, the parameters $u_{1,2}$ represent the maximum deviations of the amplitude from the mean value while $nu_{1,2}\,\partial\omega_z/\partial u$ represents the corresponding deviations of the characteristic frequency from $kV + n\omega_z(u_0)$. It follows from (14a) that stabilization of this beam requires the presence of a rather intense component with the characteristic frequency equal to the frequency of the wave $kV + n\omega_z(u_0) + \delta_0$.

As an example, we consider the distribution

$$\Phi(U) = \frac{1}{\Delta u} e^{-U/\Delta u}.$$

When $\epsilon_0 \ll |\delta_0|$ one of the stability conditions is

$$-n\frac{\partial\omega_z}{\partial u}\Delta u/\delta_0 > 1,$$

this condition being possible when $\partial\omega_z/\partial u < 0$. The function f(u) does not have a sharp boundary for positive values of the argument and the parameter u_2 must be determined from the equation

$$\frac{1}{\Delta u}\left(1 + \frac{u_2}{\Delta u}\right)\exp\left\{-\left(1 + \frac{u_2}{\Delta u}\right)\right\} = \frac{\zeta_0}{\pi\delta_0^2}\left|n\frac{\partial\omega_z}{\partial u}\right|. \qquad (15a)$$

The corresponding stability condition is given by

$$\frac{1}{\delta_0} n\frac{\partial\omega_z}{\partial u} u_2 \geqslant \mathrm{P}\int_0^\infty \frac{xe^{-x}\,dx}{1 + (1 - x)\Delta u/u_2}, \qquad (15b)$$

this being possible when $\partial\omega_z/\partial u > 0$. If the right side of (15a) is small, then $\Delta u/u_2 \ll 1$ and the approximate criterion for stability is

$$n\frac{\partial\omega_z}{\partial u}u_2/\delta_0 > 1.$$

Thus, for this distribution, in order to obtain stabilization it is feasible to take $\partial\omega_z/\partial u > 0$, this result being connected with the presence of particles exhibiting large oscillation amplitudes: $U > \Delta u$.

In a cyclic accelerator the characteristic frequency of the beam also depends on the energy: $\omega = \Omega(l - nQ)$. Deviations of the characteristic frequency from the mean value are given by

$$\left[\frac{\partial\Omega}{\partial\mathscr{E}}(l-nQ)-n\Omega\frac{\partial Q}{\partial\mathscr{E}}\right]\Delta\mathscr{E}_{1,2},$$

where $\Delta\mathscr{E}_{1,2}$ are the deviations from the mean energy. In order to obtain a stability criterion this quantity must be substituted in (14b) in place of $nu_{1,2}\partial\omega_z/\partial u$ and the energy distribution function must be substituted in place of f(u).

APPENDIX

ANALYSIS OF THE INTEGRAL EQUATION

We write the equation for the electric field (5) in the form $E = \lambda\hat{G}E$ and set up the quadratic form

$$J(q)=\int q(z)\hat{G}q(z)\,dz,$$

where q(z) is an arbitrary continuous function. After a number of transformations we find

$$J(q)=-\frac{h}{n}\int_0^\infty\Phi'(U)\,U\,dU\left[\int_{-\pi}^{\pi}q(\sqrt{U}\cos\vartheta)\sin n\vartheta\sin\vartheta\,d\vartheta\right]^2. \quad (A.1)$$

it is then evident that when $\Phi'(U) < 0$ the quantity J(q) is positive for any function q(z), this being the necessary and sufficient condition for making all the characteristic values of the equation positive (for example, [5]). In this case (A.1) can be used to determine the minimum characteristic value. For this purpose we treat J(q) as a functional which is determined from the class of normalized functions. The quantity λ_{min} can then be determined from the equation

$$1=\lambda_{min}\max J(q),\qquad \|q\|=1, \quad (A.2)$$

in which that function for which J(q) reaches a maximum value is the characteristic function corresponding to the characteristic value λ_{min}. Using the theorem of the mean in (A.1) we have

$$J(q)=\frac{4h}{n}\left[\int_0^\pi q\left(\frac{h}{2}\cos\vartheta\right)\sin n\vartheta\sin\vartheta\,d\vartheta\right]^2. \quad (A.3)$$

Using the Bunyakovskiĭ (Schwartz) inequality, we have from (A.3)

$$J(q)\leqslant\frac{4h}{n}\int_0^\pi q^2\left(\frac{h}{2}\cos\vartheta\right)\sin\vartheta\,d\vartheta\int_0^\pi\sin^2 n\vartheta\sin\vartheta\,d\vartheta. \quad (A.4)$$

The integral of the function q^2 is computed from the normalization condition. In this way we obtain the upper bound on the functional. Substituting this in (A.3) we have

$$\frac{1}{\lambda_{min}}=\frac{32n}{4n^2-1} \quad (A.5)$$

The functional reaches the upper bound if

$$q(x)\sim\sin\left(n\arccos\frac{2z}{h}\right),$$

as can be shown quite simply by direct calculation. If we write $\omega_L^2 = 128e^2N/3\pi mhw$, then (9) follows directly from (A.5) and (7).

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