ELECTROMAGNETIC PROPERTIES OF BARYONS AND MESONS IN A NONRELATIVISTIC QUARK MODEL

S. B. GERASIMOV

Joint Institute for Nuclear Research

Submitted to JETP editor December 30, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) **50**, 1559–1564 (June, 1966)

The electromagnetic properties (magnetic moments, electromagnetic mass splitting, radiative decays, electromagnetic radii) of mesons and baryons are treated in the framework of a nonrelativistic quark model, on the assumption that the medium-strong interaction that breaks SU(3) symmetry leads to a change of the mass and magnetic moment of the "strange" quark. An expression is derived for the electric quadrupole moments of vector mesons which are caused by relativistic effects.

IT is well known that most of the predictions of the unitary symmetries can be obtained in the framework of a nonrelativistic quark model[1,2] for the baryons and mesons. In papers by Struminskiĭ,[3] Dolgov et al.,[4] and Azimov et al.[5] the quark model has been applied to the calculation of magnetic moments, of the electromagnetic mass difference of baryons, and of the radiative decays of vector mesons. Here the general expression for the electromagnetic current of a system of quarks,

$$\mathbf{j} = \sum_i \mathbf{j}_i + \sum_{i<k} \mathbf{j}_{ik} + \ldots \qquad (1)$$

was approximated by a sum of one-particle operators [i.e., only the first term of the sum (1) was retained], and in addition it was assumed that the magnetic moments of the various quarks are proportional to their charges:

$$\mu_u : \mu_d : \mu_s = e_u : e_d : e_s = 2 : -1 : -1. \qquad (2)$$

In the present paper the electromagnetic properties of baryons and mesons are discussed on the following assumptions:

1) It is assumed that the turning on of the medium-strong interaction that breaks SU(3) symmetry leads to a change of the mass of the "strange" quark (in accordance with Zweig[2]) and the violation of the relations (2) for the magnetic moments of the quarks.

2) The contribution of interaction currents, i.e., of two-particle and three-particle (in the case of baryons) operators in (1) can be neglected.

3) The total orbital angular momentum is a good quantum number and is equal to zero for the multiplets of baryons ($^1/_2{}^+$ and $^3/_2{}^+$) and of mesons (0^- and 1^-).

4) The wave functions of particles in the rest system transform according to the laws of SU(6) symmetry—that is, it is assumed that in first approximation the effect of the symmetry-breaking interaction on the wave functions can be neglected.

THE ELECTROMAGNETIC PROPERTIES OF BARYONS

1. Following Struminskiĭ,[3] we can express the magnetic moments of baryons in terms of those of u, d, and s quarks by the usual formulas of nonrelativistic quantum mechanics. For the proton, neutron, and Λ particle we have

$$\mu_p = {}^1/_3(4\mu_u - \mu_d), \qquad (3a)$$

$$\mu_n = {}^1/_3(4\mu_d - \mu_u), \qquad (3b)$$

$$\mu_\Lambda = \mu_s. \qquad (3c)$$

From (3a) and (3b) and the experimental values $\mu_p = 2.79$, $\mu_n = -1.91$ (all values of magnetic moments will be given in nuclear magnetons) we find that the relation $\mu_u : \mu_d = 2 : -1$ is satisfied with good accuracy. The experimental value of the magnetic moment of the Λ particle, as the average of the results of several experiments, is[6] $\mu\Lambda = -0.73 \pm 0.17$. The experimental errors are rather large, but nevertheless some decrease of μ_s as compared with $\mu_d = -0.93$ seems compatible with the idea of Zweig[2] that the mass of the s quark is increased as a result of the breaking of SU(3) symmetry. In what follows we shall assume $\mu_s = -0.73$, and consequently

$$\mu_u : \mu_d : \mu_s = 2 : -1 : -0.8. \qquad (4)$$

The other values of magnetic moments of stable

Table I

Theory	Magnetic moment					
	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-	Ω^-
SU (6)	2.79	0.93	−0.93	−1.86	−0.93	−2.79
According to [7]	2.20	0.73	−0.73	−1.32	−0.66	−1.56
Quark model and Eq. (4)	2.72	0.86	−1	−1.59	−0.66	−2.19

(against strong interactions) mesons are given in Table I.

For comparison there are also given the values of μ_B which correspond to exact SU(6) symmetry, and the values of the magnetic moments which follow from the suggestion of Bég and Pais[7] that there is a correction for the mass splitting

$$\mu_B{}' = \mu_B{}^{SU(6)} m_p/m_B. \tag{5}$$

As can be seen from Table I, there is a rather strong discrepancy between our approach and Eq. (5). In the case of the Σ^- particle there is even a different direction of the change of the result of SU(6) symmetry: the magnetic moment μ_{Σ^-} is increased, not decreased, in absolute value. We note that if we interpret the decrease of μ_s directly in terms of an increase of the mass of the s quark—that is, suppose that $\mu_d : \mu_s = m_s : m_d$—then, taking $\Delta m = m_s - m_d \approx 150-190$ MeV, we can use (4) to get values of the masses of the quarks as constituents of baryons:

$$m_u = m_d \approx 550-700 \text{ MeV}, \quad m_s \approx 700-890 \text{ MeV}.$$

2. In the quark model the magnetic moments of the transitions which determine the probabilities of the radiative decays $\Sigma^0 \to \Lambda + \gamma$ and $B^{10} \to B^8 + \gamma$ are

$$\begin{aligned}
&\mu(\Sigma^0\Lambda) = -\sqrt{3}\,\mu_d = -{}^1\!/_2\sqrt{3}\,\mu_n,\\
&\mu(\Delta_\delta{}^+p) = \mu(\Delta_\delta{}^0n) = -{}^2\!/_3\sqrt{3}\,\mu(\Sigma_\delta{}^0\Lambda) = {}^2\!/_3\sqrt{2}\,(\mu_u - \mu_d)\\
&\quad = {}^2\!/_3\sqrt{2}\,\mu_p,\\
&\mu(\Sigma_\delta{}^+\Sigma^+) = \mu(\Xi_\delta{}^0\Xi^0) = {}^2\!/_3\sqrt{2}(\mu_s - \mu_u),\\
&\mu(\Sigma_\delta{}^-\Sigma^-) = \mu(\Xi_\delta{}^-\Xi^-) = {}^2\!/_3\sqrt{2}\,(\mu_d - \mu_s),\\
&\mu(\Sigma_\delta{}^0\Sigma^0) = {}^1\!/_3\sqrt{2}\,(\mu_u + \mu_d - 2\mu_s).
\end{aligned} \tag{6}$$

It follows from (4) and (6) that the decays $\Sigma_\delta^- \to \Sigma^- + \gamma$ and $\Xi_\delta^- \to \Xi^- + \gamma$, which are forbidden in exact SU(6) symmetry, now have finite widths.

3. According to a paper by Dolgov and others,[4] the electromagnetic splitting of the baryon masses is due to the difference Δ of the electromagnetic masses of the u and d quarks, the Coulomb interaction $\epsilon\sum e_i e_k$, and the interaction of the magnetic moments of the quarks, $M\sum \mu_i \mu_k$.

Following this approach, we make a new calculation of the parameters Δ, ϵ, and M, using the relation (4) and more accurate values of the experimentally observed mass differences of the hyperons:[8] $(\Sigma^- - \Sigma^0)_{exp} = 4.99 \pm 0.12$ MeV $(\Sigma^- - \Sigma^+)_{exp} = 7.89 \pm 0.12$ MeV. We have

$$\Delta = 2 \text{ MeV}, \quad \varepsilon = 1.3 \text{ MeV}, \quad M = 0.6 \text{ MeV}. \tag{7}$$

The greatest difference from the results of Dolgov et al.[4] occurs for the mass differences $\Xi^- - \Xi^0$ and $\Xi_\delta^- - \Xi_\delta^0$:

$$\begin{aligned}
\Xi^- - \Xi^0 &= \Delta + 2\varepsilon + 3.2M \approx 6.5 \text{ MeV},\\
\Xi_\delta{}^- - \Xi_\delta{}^0 &= \Delta + 2\varepsilon - 1.6M \approx 3.6 \text{ MeV}.
\end{aligned} \tag{8}$$

The corresponding values in [4] are $\Xi^- - \Xi^0 = 6.4$ MeV and $\Xi_\delta^- - \Xi_\delta^0 = 3.4$ MeV. The experimental values are $(\Xi^- - \Xi^0)_{exp} = 6.5 \pm 1$ MeV and $(\Xi_\delta^- - \Xi_\delta^0)_{exp} = 5.7 \pm 3$ MeV.[9] We see that the electromagnetic mass splitting is not much affected by the replacement of the relation (2) by Eq. (4).

4. If we define the electromagnetic radii of the particles by means of the formulas

$$\begin{aligned}
\langle r^2\rangle^B_{\text{elec}} &= \frac{1}{Q_B}\left\langle B \left| \int \rho(\mathbf{r}) r^2 d^3r \right| B \right\rangle\\
&= \frac{1}{Q_B}\sum_i \langle B | e_i(\mathbf{r}_i - \mathbf{R})^2 | B\rangle,\\
\langle r^2\rangle^B_{\text{mag}} &= \frac{1}{\mu_B}\left\langle B \left| \int \mu_z(\mathbf{r}) r^2 d^3r \right| B \right\rangle\\
&= \frac{1}{\mu_B}\sum_i \langle B | \mu_{iz}(\mathbf{r}_i - \mathbf{R})^2 | B\rangle
\end{aligned} \tag{9}$$

(where $Q_B \neq 0$ and $\mu_B \neq 0$ are the charge and the magnetic moment of baryon B, $\mathbf{r}_i$ is the coordinates of the i-th quark, $\mathbf{R} = \sum_i m_i \mathbf{r}_i / \sum_i m_i$, and the averaging is taken over the ground-state wave function), then in the limit of exact SU(6) symmetry the charge radii of all neutral particles are zero, and the magnetic and charge radii of the charged baryons are equal to these same quantities for the proton.

The breaking of SU(3) symmetry, expressed in an increase of the mass of the "strange" quark, has the consequences that

a) the charge radii of neutral particles which include strange quarks in their composition become different from zero;

b) the electromagnetic radii of charged strange particles are different from the proton radius.

From the formulas (9) and the condition of complete antisymmetry of the radial wave functions of baryons we can derive

$$\langle r^2\rangle^{\Lambda}_{\text{elec}} = \langle r^2\rangle^{\Sigma^0}_{\text{elec}} = \frac{y^2+y-2}{(y+2)^2}\langle r^2\rangle^{p}_{\text{elec}} \approx \frac{1}{3}\delta\langle r^2\rangle^{p}_{\text{elec}},$$

$$\langle r^2\rangle^{\Sigma^0\to\Lambda}_{\text{mag}} = 3\frac{y^2+y+1}{(y+2)^2}\langle r^2\rangle^{p}_{\text{mag}} \approx \left(1+\frac{1}{3}\delta\right)\langle r^2\rangle^{p}_{\text{mag}},$$

$$\langle r^2\rangle^{\Sigma^0\to\Lambda}_{\text{mag}} \approx \langle r^2\rangle^{p}_{\text{mag}} + \langle r^2\rangle^{\Lambda}_{\text{elec}}, \tag{10}$$

where $y = m_s/m_d = m_s/m_u = 1+\delta$, $\delta > 0$, $m_{s,u,d}$ being the masses of the s, u, and d quarks. At present the most accessible way to check (10) is to study the decay $\Sigma^0 \to \Lambda + e^+ + e^-$. The study of this reaction is also interesting for another reason. The splitting of the masses of Σ^0 and Λ is due to dynamical factors beside the renormalization of the parameters of the s quark, and it can be expected that they will appear most noticeably in the reaction of conversion decay of Σ^0. We note that owing to the orthogonality of the spin and unitary-spin wave functions of Σ^0 and Λ the equation

$$\langle r^2\rangle^{\Sigma^0\to\Lambda}_{\text{elec}} = 0 \tag{11}$$

is also valid for $y \neq 1$.

THE ELECTROMAGNETIC PROPERTIES OF MESONS

1. The magnetic moments of vector mesons and the magnetic moments of the transitions that describe radiative decays $V \to P + \gamma$ are given in the quark model by

$$\mu(\rho^+) = -\mu(\rho^-) = \mu(\omega\pi^0) = \sqrt{3}\,\mu(\rho\eta) = \mu_u - \mu_d,$$
$$\mu(K^{*+}) = -\mu(K^{*-}) = \mu_u - \mu_s, \qquad \mu(K^{*0}) = -\mu(\overline{K}^{*0}) = \mu_d - \mu_s,$$
$$\mu(\rho\pi) = \sqrt{3}\,\mu(\omega\eta) = \mu_u + \mu_d,$$
$$\mu(K^*_{\text{ch}}K_{\text{ch}}) = \mu_u + \mu_s, \qquad \mu(K^*_{\text{neut}}K_{\text{neut}}) = \mu_d + \mu_s,$$
$$\mu(\varphi\eta) = -{}^2/_3\sqrt{6}\,\mu_s, \qquad \mu(\varphi\pi^0) = 0, \tag{12}$$

where

$$\mu(K^*_{\text{ch}}K_{\text{ch}}) = \mu(K^{*\pm}K^{\pm}),$$
$$\mu(K^*_{\text{neut}}K_{\text{neut}}) = \mu(K^{*0}K^0) = \mu(\overline{K}^{*0}\overline{K}^0),$$
$$\mu(\rho\pi) = \mu(\rho^{\pm 0}\pi^{\pm 0}).$$

In the derivation of (12) it has been assumed that the physical particles φ and ω arise as the result of mixing of singlet and octet, so that φ contains only strange quarks and ω only nonstrange quarks. Moreover, we have not taken into account the possible mixing[10] of η (548 MeV) and X^0 (959 MeV). By means of (12) one can derive a number of sum rules for the magnetic moments with arbitrary ratios of μ_u, μ_d, and μ_s. If we suppose that the relation (4) is valid also for the case of mesons, then we can express all of the magnetic moments in terms of $\mu(\omega\pi^0)$, while the width of the decay $\omega \to \pi^0 + \gamma$ is known from experiment.[11]

Table II

$V \to P + \gamma$	$\frac{\mu(VP)}{\mu(\omega\pi^0)}$	Γ, MeV $x=1$	Γ, MeV $x=0.8$
$\omega \to \pi^0 + \gamma$	1	1 ± 0.2	1 ± 0.2
$K^*_{\text{ch}} \to K_{\text{ch}} + \gamma$	$^1/_3(2-x)$	0.058	0.083
$K^*_{\text{neut}} \to K_{\text{neut}} + \gamma$	$-^1/_3(1+x)$	0.23	0.19
$\varphi \to \eta + \gamma$	$^2/_9\sqrt{6}x$	0.25	0.16

In Table II we give those values of magnetic moments and widths which differ from those obtained in the paper by Azimov et al.,[5] where it was assumed that Eq. (2) holds (x denotes the ratio μ_s/μ_d). It can be seen from the table that a change of μ_s has a rather marked effect on the widths of radiative decays.

We also point out that in contrast with those corresponding to exact SU(6) symmetry, the magnetic moments $\mu(K^{*0})$ and $\mu(\overline{K}^{*0})$ are now different from zero and will make a definite contribution to the cross section for photoproduction of K^{*0} and $\overline{K}^{*0}$ mesons.

2. The electromagnetic splitting of the meson masses has been treated in the quark model in the paper of Azimov et al.[12] for $\mu_u:\mu_d:\mu_s = 2:-1:-1$. We have already seen that a change of the magnetic moment of the s quark has only a weak effect on the size of the electromagnetic mass splitting of the baryons. The same will evidently be true also for mesons. At present there are no experimental data for vector mesons, and nothing with which the theoretical predictions of [12] can be compared.

3. By means of formulas of the type of (9) it is not hard to get ratios between the charge radii of pseudoscalar mesons:

$$\langle r^2\rangle_{K^+} = \frac{4}{3}\frac{1+2y^2}{(1+y)^2}\langle r^2\rangle_\pi \approx \left(1+\frac{1}{3}\delta\right)\langle r^2\rangle_\pi,$$

$$\langle r^2\rangle_{K^0} = \langle r^2\rangle_{K_1^0\to K_2^0} = \frac{4}{3}\frac{1-y^2}{(1+y)^2}\langle r^2\rangle_\pi \approx -\frac{2}{3}\delta\langle r^2\rangle_\pi,$$

$$\langle r^2\rangle_{K^0} = \langle r^2\rangle_{K_1^0\to K_2^0} \approx 2(\langle r^2\rangle_\pi - \langle r^2\rangle_{K^+}), \tag{13}$$

where

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle), \qquad |K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle).$$

The meanings of the notations y and δ are the same in (13) as in (10), but their numerical values here can differ from those for baryons. We note that in the framework of exact SU(3) symmetry, with $m_u = m_d = m_s$, it follows from (13) that $\langle r^2\rangle_{K^0} = \langle r^2\rangle_{K_1^0 \to K_2^0} = 0$. This fact can be tested experimentally by means of the process of regeneration of K_1^0 mesons in a beam of K_2^0 mesons.[13]

The formulas (10) and (13) for the electromagnetic radii of baryons and mesons have been derived on the assumption that the quarks are point particles. It is easy, however, to extend them to the case in which the quarks have a structure of their own. Here we only point out that the last equations in (10) and (13) remain valid if the electromagnetic radii of u, d, and s quarks are equal.

4. In addition to charge and magnetic moment, vector mesons can also have an electric quadrupole moment. In the present model the charge distribution is spherically symmetric, and therefore the "intrinsic" quadrupole moments of the mesons are zero. There can, however, be an observable quadrupole moment of a meson, caused by the relativistic "trembling" of the particle as a whole. We estimate its size in the following way.

In a slowly varying electromagnetic field we can neglect effects of the structure of the particle and describe its behavior by means of a phenomenological relativistically invariant Lagrangian of local field theory. Moreover, we shall take the total magnetic moment μV of the vector meson to be equal to the value obtained according to the quark model. It can be shown (cf., e.g., [14]) that if a vector particle has an "anomalous" (in the usual field-theoretical sense) magnetic moment κ,

$$\mu_V = (e/2m_V)(1+\varkappa), \tag{14}$$

then its electric quadrupole moment is given by

$$Q_V = -\frac{e}{m_V^2}\varkappa = -2\frac{\mu_V}{m_V} + \frac{e}{m_V^2}, \tag{15}$$

where e and m_V are the charge and mass of the vector meson.

If, for example, we take for the ρ meson $\mu(\rho^+) = \mu(p) = 2.79\ e/2m_p$, then from (15) we get

$$Q_\rho \approx -1.26e/m_V^2. \tag{16}$$

In a similar way one can find the electric quadrupole moments of other vector mesons.

[1] M. Gell-Mann, Physics Letters **8**, 214 (1964).
[2] G. Zweig, Preprint CERN 8419 TH, 412, 1964.
[3] B. V. Struminskiĭ, Preprint OIYaI (Joint Inst. Nucl. Research) P-1939, 1965.
[4] A. D. Dolgov, L. B. Okun, I. Ya. Pomeranchuk, and V. V. Solovyev, Physics Letters **15**, 84 (1965).
[5] Ya. I. Azimov, V. V. Anisovich, A. A. Ansel'm, G. S. Danilov, and I. T. Dyatl ov, JETP Letters **1**, No. 2, 50 (1965), transl. **1**, 72 (1965).
[6] D. A. Hill, K. K. Li, E. W. Jenkins, T. F. Kycia, and H. Ruderman, Phys. Rev. Letters **15**, 85 (1965).
[7] M. A. B. Bég and A. Pais, Phys. Rev. **137**, B1514 (1965).
[8] P. Schmidt, Phys. Rev. **140**, B1328 (1965).
[9] G. M. Pjerrou, P. E. Schlein, W. E. Slater, L. T. Smith, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **14**, 275 (1965).
[10] R. H. Dalitz and D. G. Sutherland, Nuovo Cimento **37**, 1777 (1965).
[11] A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, Ja. Kirz, and M. Roos, Revs. Modern Phys. **36**, 977 (1964).
[12] Ya. I. Azimov, V. V. Anisovich, A. A. Ansel'm, G. S. Danilov, and I. T. Dyatlov, JNP **2**, 583 (1965), Soviet Phys. JNP **2**, 417 (1966).
[13] Ya. B. Zel'dovich, JETP **36**, 1381 (1959), Soviet Phys. **9**, 984 (1959).
[14] T. D. Lee, Phys. Rev. **128**, 899 (1962). J. A. Young and S. A. Bludman, Phys. Rev. **131**, 2326 (1963).

Translated by W. H. Furry
187