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ROTATION OF NORMAL AND SUPERFLUID FERMI SYSTEMS

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A selfconsistent scheme of calculation is constructed for the study of the rotation of charged Fermi systems, taking into account the intrinsic magnetic field of the currents that arise when the system rotates. A generalized Larmor theorem is formulated. A detailed study is made of the rotation of normal and superfluid systems in this scheme. It is shown that the moment of inertia of a London superfluid system is zero. Methods of Fermi-liquid theory are used to study the effect of the normal and superfluid interactions on the moment of inertia of a Fermi liquid. It is shown that the normal interaction in a normal Fermi system does not affect the moment of inertia. A consistent calculation is made of the effect of the normal interaction in the rotation of superfluid Fermi systems. Corrections associated with the intrinsic magnetic field are estimated for the case of rotation of atomic nuclei.

THERE have been a great many studies of the properties of Fermi systems in external magnetic fields. The analogy of the behavior of systems in a magnetic field and that of rotating systems must also be noted. By relying on this analogy and using methods of the theory of Fermi liquids, we can reach extremely general conclusions about the rotation of normal and superfluid Fermi systems.

1. Rotation of a system of charged particles gives rise to currents in it, which produce an intrinsic magnetic field acting on the system along with the external field. Therefore the Lagrangian of the system in a stationary reference system can be put in the form

$$\mathscr{L} = \sum_i \left\{ \frac{m\mathbf{v}_i^2}{2} + \frac{1}{2}\frac{q}{c}\mathbf{v}_i\mathbf{A}_i' \right\} + \mathscr{L}_{\text{int}}, \tag{1}$$

where the vector potential $\mathbf{A}'$ is determined by the currents produced by the motions of all particles except the i-th:

$$\nabla^2\mathbf{A}' = -4\pi c^{-1}\mathbf{j} = -4\pi c^{-1}q\mathbf{v}\rho,$$

where ρ is the density of particles in the system.

In order to get the Lagrangian $\mathscr{L}'$ in a reference system rotating around the z axis with angular velocity Ω_0, it is necessary to replace the velocity $\mathbf{v}_i$ in the Lagrangian (1) by $\mathbf{v}_i' + [\boldsymbol{\Omega}_0\mathbf{r}_i]$, where $\mathbf{v}'$ is the velocity of the particle relative to the rotating reference system. We get

$$\mathscr{L}' = \sum_i \left\{ \frac{m}{2}(\mathbf{v}_i' + [\boldsymbol{\Omega}_0\mathbf{r}_i])^2 + \frac{q}{2c}(\mathbf{v}_i' + [\boldsymbol{\Omega}_0\mathbf{r}_i])\mathbf{A}_i' \right\} + \mathscr{L}_{\text{int}}. \tag{3}^*$$

Here the vector potential $\mathbf{A}'$ obeys the equation

$$\nabla^2\mathbf{A}' = -4\pi c^{-1}\rho q(\mathbf{v}' + [\boldsymbol{\Omega}_0\mathbf{r}]) = -4\pi c^{-1}(\mathbf{j}' + \mathbf{j}_{\text{rig}}), \tag{4}$$

where $\mathbf{j}_{\text{rig}}$ is the current density of the rigid rotation. In this case the generalized momentum $\mathbf{p}_i'$ of the particle will be given by

$$\mathbf{p}_i' = \partial\mathscr{L}'/\partial\mathbf{v}_i' = m\mathbf{v}_i' + qc^{-1}\mathbf{A}_i \equiv \mathbf{p}_i \tag{5}$$

(we neglect the effect of retardation in the Lagrangian $\mathscr{L}_{\text{int}}$). In (5) $\mathbf{p}_i$ is the generalized momentum in the stationary reference system, and the potential $\mathbf{A} = \mathbf{A}_0 + \mathbf{A}'$ obeys the equation

$$\nabla^2\mathbf{A} = -\frac{4\pi}{c}\left(\mathbf{j}' + \frac{\rho q^2}{mc^2}\mathbf{A}_0\right), \tag{6}$$

where $\mathbf{A}_0$ is the effective vector potential of the rotation, given by

$$\mathbf{A}_0 = mcq^{-1}[\boldsymbol{\Omega}_0\mathbf{r}]. \tag{7}$$

The orbital angular momentum $\mathbf{L}'$ of the system in the rotating reference system will coincide with the orbital angular momentum $\mathbf{L}$ in the stationary reference system:

$$\mathbf{L}' = \sum_i [\mathbf{r}_i\mathbf{p}_i'] = \sum_i m[\mathbf{r}_i\mathbf{v}_i'] + \sum_i \frac{q}{c}[\mathbf{r}_i\mathbf{A}_i] \equiv \mathbf{L}. \tag{8}$$

On the other hand the Hamiltonian $\mathscr{H}'$ in the rotating reference system is connected with the Hamiltonian $\mathscr{H}$ in the stationary system by the well known relation

$$\mathscr{H}' = \sum_i \left\{ \frac{\mathbf{p}_i'^2}{2m} - \frac{q}{2mc}\mathbf{p}_i'\mathbf{A}_i' - \frac{q}{mc}\mathbf{p}_i'\mathbf{A}_{i0} \right\} + \mathscr{H}_{\text{int}} \equiv \mathscr{H} - \mathbf{L}\boldsymbol{\Omega}_0, \tag{9}$$

*$[\boldsymbol{\Omega}_i\mathbf{r}_0] \equiv \boldsymbol{\Omega}_i \times \mathbf{r}_0$.

so that $\mathbf{L} \equiv \mathbf{L}' = -\partial\mathcal{H}'/\partial\Omega_0$ and the moment of inertia J is given by

$$J = L_z' / \Omega_0 = -\partial^2\mathcal{H}' / \partial\Omega_0^2. \tag{10}$$

In an external uniform magnetic field $\mathbf{H}_0$ defined by the vector potential $\mathbf{A}_0 = {}^1/_2[\mathbf{H}_0\mathbf{r}]$ the Lagrangian of our system is of the form

$$\mathcal{L}_M = \sum_i \left\{ \frac{m\mathbf{v}_i^2}{2} + \frac{1}{2}\frac{q}{c}\mathbf{A}_i'\mathbf{v}_i + \frac{q}{c}\mathbf{A}_{i0}\mathbf{v}_i \right\} + \mathcal{L}_{\text{int}}, \tag{11}$$

where the potential $\mathbf{A}'$ obeys the equation

$$\nabla^2\mathbf{A}' = -\frac{4\pi}{c}\mathbf{j} = -\frac{4\pi}{c}\rho q\mathbf{v}. \tag{12}$$

A comparison of the Lagrangian (3) in the rotating reference system with the Lagrangian (11) shows that the usual Larmor theorem must be generalized: the behavior of a system of particles in a rotating reference system is equivalent in first order in the field to its behavior in a magnetic field which is the sum of the uniform magnetic field

$$\mathbf{H}_0 = 2mcq^{-1}\mathbf{\Omega}_0 \tag{13}$$

and the magnetic field produced by the rigid-body rotation of the system.

If the system as a whole is electrically neutral, such as a metal, for example, composed of a rigid positively charged crystal lattice and the conduction electrons, then the total rigid-body current is zero ($\mathbf{j}_{\text{rig}} = 0$), and we can use the ordinary Larmor theorem. For such systems the current moment of inertia J_{cur} defined by the first term of (8) can be expressed in terms of the orbital magnetic dipole moment of the system in an external magnetic field $\mathbf{H}_0$ given by (13),

$$\mathbf{M} = \frac{q}{2c}\sum_i [\mathbf{r}_i\mathbf{v}_i], \tag{14}$$

by a relation of the form

$$J_{\text{cur}} = \frac{4m}{r_{\text{Coul}}}\frac{M_z}{H_0} = \frac{4m}{r_{\text{Coul}}}\int \chi(r)\,dV, \tag{15}$$

where $\chi(r)$ is the local coefficient of orbital magnetization and r_{Coul} is the Coulomb radius of the particle

$$r_{\text{Coul}} = q^2 / mc^2. \tag{16}$$

If, on the other hand, the system is not electrically neutral, such as an atomic nucleus, for example, then its current density of rigid-body rotation is not zero and it is necessary to use the generalized Larmor theorem.

2. Let us consider the case of rotation of a normal Fermi system, in which the current density $\mathbf{j}'$ is zero in first order in the field. In such a system the moment of inertia is the sum of two terms:

$$J = L_z / \Omega_0 = J_{\text{rig}} + J_{\text{M}}, \tag{17}$$

where the definition of the rigid-body moment of inertia is

$$J_{\text{rig}} = \frac{m}{\Omega_0}\int \rho[\mathbf{r}[\mathbf{\Omega}_0\mathbf{r}]]_z\,dV = m\int \rho(x^2+y^2)\,dV, \tag{18}$$

and the moment of inertia J_M associated with the magnetic field of the rigid-body-rotation current is given by

$$J_{\text{M}} = \frac{q}{c\Omega_0}\int \rho\,[\mathbf{r}\mathbf{A}']_z dV. \tag{19}$$

For systems satisfying the ordinary Larmor theorem $J_M = 0$ and the moment of inertia is that of a rigid body. For those systems that do not obey the ordinary Larmor theorem we shall look for a solution of the equation (4) for the potential $\mathbf{A}'$, satisfying the usual boundary conditions, in the form

$$\mathbf{A}'(\mathbf{r}) = \frac{1}{c}\int \frac{\mathbf{j}_{\text{rig}}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}dV' = \frac{q}{c}\int \frac{\rho[\mathbf{\Omega}_0\mathbf{r}']}{|\mathbf{r}-\mathbf{r}'|}dV'. \tag{20}$$

For simplicity we consider a spherical system of radius R_0. Choosing the z axis in the direction of the radius vector $\mathbf{r}$ and taking the vector $\mathbf{\Omega}_0$ in the zy plane (we denote the angle between the vectors $\mathbf{r}$ and $\mathbf{\Omega}_0$ by θ_0), we get

$$\mathbf{A}'(\mathbf{r}) = \begin{cases} \dfrac{R_0^5}{15r^3\delta_L^2}\mathbf{A}_0, & r \geqslant R_0 \\ \dfrac{(5R_0^2-3r^2)}{30\delta_L^2}\mathbf{A}_0, & r \leqslant R_0 \end{cases}, \tag{21}$$

where δ_L is the London penetration depth,

$$\delta_L = (4\pi\rho r_{\text{Coul}})^{-1/2}. \tag{22}$$

Then the magnetic moment of inertia can be represented in terms of the rigid-body moment of inertia:

$$J_{\text{M}} = {}^2/_{21}(R_0 / \delta_L)^2 J_{\text{rig}}. \tag{23}$$

It can be seen from this that if the size of the system is sufficiently large in comparison with the London penetration depth the moment of inertia of a normal system can be considerably larger than the rigid-body moment of inertia. There is, however, only one type of stable systems existing in nature that do not obey the ordinary Larmor theorem—atomic nuclei. As will be shown below, for the proton components of an atomic nucleus the London penetration depth is approximately $4R_0$, so that

$$J_M \approx 0.006 J_{\text{rig}} \ll J_{\text{rig}}. \tag{24}$$

3. Let us now consider the rotation of superfluid Fermi systems which are large enough so that the effect of the boundary of the system on the moment of inertia can be neglected. For such systems the current density $\mathbf{j}'$ is not zero, but is a linear functional of the vector potential $\mathbf{A}$:

$$\mathbf{j}'(\mathbf{r}) = \int K(\mathbf{r}-\mathbf{r}')\mathbf{A}(\mathbf{r}')\,dV'. \tag{25}$$

In this case the equation (6) for the potential $\mathbf{A}$ can be rewritten in the form

$$\left\{\nabla^2\mathbf{A} + \frac{4\pi}{c}\int K(\mathbf{r}-\mathbf{r}')\mathbf{A}(\mathbf{r}')\,dV'\right\} = -\frac{4\pi}{c}\mathbf{j}_{\text{rig}}. \tag{26}$$

The solution of this equation is the sum of a solution $\mathbf{A}_1$ of the homogeneous equation and a solution $\mathbf{A}_2$ of the inhomogeneous equation.

The solution $\mathbf{A}_1$ of the homogeneous equation coincides with the vector potential of the system in question when it is in the uniform magnetic field (13). In this case there are two limiting cases for superfluid Fermi systems.

a) the London case, in which the penetration depth δ_L of the field into the system, Eq. (22), is much larger than the dimensions of a Cooper pair

$$r_C = v_0/\Delta \tag{27}$$

(where v_0 is the Fermi velocity and Δ is the energy gap). In this case the current density is very simply related to the field:

$$\mathbf{j}'(\mathbf{r}) = -\frac{\rho q^2}{mc}\mathbf{A}(\mathbf{r}); \tag{28}$$

b) The Pippard case, in which the penetration depth

$$\delta_P = \frac{4}{3\sqrt{3}}\left(\frac{mc^2 v_0}{3\pi^2\rho q^2\Delta}\right)^{1/3} = 0.6\delta_L\left(\frac{r_C}{\delta_L}\right)^{1/3} \tag{29}$$

is much smaller than the dimensions of a Cooper pair. In this case the dependence of the current $\mathbf{j}'$ on the field is of the nonlocal character (25). E Equations (25) and (28) join approximately at $\delta_L \approx \delta_P \approx r_C$.

The solution $\mathbf{A}_2$ of the inhomogeneous equation can be looked for in the form

$$\mathbf{A}_2(\mathbf{r}) = -\frac{4\pi}{c}\int G(\mathbf{r}-\mathbf{r}')\mathbf{j}_{\text{rig}}(\mathbf{r}')\,dV', \tag{30}$$

where $G(\mathbf{r}-\mathbf{r}')$ is the Green's function of Eq. (26). In the London case the Green's function $G(\mathbf{r}-\mathbf{r}')$, which is finite and continuous in all space, can be calculated by means of an integration in the complex plane:

$$G(\mathbf{r}-\mathbf{r}') = -\frac{1}{4\pi}\frac{\exp\{-|\mathbf{r}-\mathbf{r}'|/\delta_L\}}{|\mathbf{r}-\mathbf{r}'|}. \tag{31}$$

The solution $\mathbf{A}_2$, Eq. (30), is then of the form

$$\mathbf{A}_2(\mathbf{r}) = \frac{1}{c}\int\frac{\mathbf{j}_{\text{rig}}(\mathbf{r}')\exp\{-|\mathbf{r}-\mathbf{r}'|/\delta_L\}}{|\mathbf{r}-\mathbf{r}'|}\,dV'. \tag{32}$$

This solution goes over into the solution (20) for the normal Fermi system in the limit $\delta_L \to \infty$. In the Pippard case the calculation of the Green's function is much more complicated, but it can be expected that $G(\mathbf{r}-\mathbf{r}')$ will be qualitatively of the form (31) with δ_L replaced by δ_P. The magnitude of the potential $|\mathbf{A}_2(\mathbf{r})|$ in (32) is smaller than the magnitude of the potential $|\mathbf{A}'(\mathbf{r})|$ given by (20). Therefore by (24) we can neglect the effect of the magnetic field of the currents of rigid-body rotation on the properties of the system and use the ordinary Larmor theorem, even in the case of superfluid atomic nuclei.

We point out an extremely important peculiarity of the London case. If we substitute the current density $\mathbf{j}'$ from (28) in the formula (8) for the orbital angular momentum, the current moment of inertia is exactly cancelled by the term associated with the potential $\mathbf{A}$, so that the orbital angular momentum, and along with it the moment of inertia of the system, are equal to zero. Accordingly the presence of a nonzero moment of inertia of the system must be due either to the system's belonging to the Pippard case, or else to the effect of the finite size of the system on the superfluid properties.

4. The inclusion of an interaction between the particles can have an important effect on the moment of inertia of a Fermi system. Two types of interaction are distinguished in the theory of Fermi liquids: the superfluid interaction, determined by a diagram with four external lines, irreducible with respect to the particle-particle channel, and the normal interaction, determined by a four-line diagram which is irreducible with respect to the particle-hole channel. Among the papers devoted to the effect of the normal interaction on the moment of inertia of a Fermi liquid, we must point out a paper by Amado and Brueckner,[1] in which it is shown in first order of perturbation theory that the normal interaction does not change the moment of inertia of a system (this result can be extended easily to the case of a Fermi system of gaseous density), and a paper by Rockmore,[2] in which this same result is derived in the high-density approximation. The solution of this problem has not been obtained, however, in the most interesting case of intermediate density. The theory of the Fermi liquid developed in papers by Migdal[3] allows us to solve this problem in all orders of perturbation theory in a very simple way.

Let us consider a normal Fermi system containing a sufficiently large number of particles so that the quasiclassical approximation can be applied ($R_0 \gg 1/p_0$, where p_0 is the Fermi-system momentum). The moment of inertia of the system is given by the formula

$$J = \frac{L_z}{\Omega_0} = \frac{1}{\Omega_0}\mathrm{Sp}(\hat{L}_z\rho'), \tag{33}$$

where ρ' is the change of the one-particle density matrix under the influence of the perturbing Hamiltonian

$$\mathcal{H}' = \sum_i \frac{q}{mc}\mathbf{A}_i\mathbf{p}_i. \tag{34}$$

Expressing ρ' in terms of the complete vertex part $\tau(-q\mathbf{A}\cdot\mathbf{p}/\Omega_0 mc)$, we get in the momentum representation

$$J = 2\sum_{\mathbf{p},\mathbf{p}'} \frac{a(L_z)_{\mathbf{p}\mathbf{p}'}\tau_{\mathbf{p}'\mathbf{p}}(n_{\mathbf{p}'} - n_{\mathbf{p}})}{(\mathbf{p}'^2 - \mathbf{p}^2)/2m^*}, \tag{35}$$

where m^* is the effective mass and a is a renormalization constant.

Since only states in the neighborhood of the Fermi surface are important in the sum (35) and the bare field does not depend on the time, the vertex τ in (35) can be replaced by the static vertex τ^k. If we use the facts that in a sufficiently large system

$$\tau^k\left(-\frac{q}{\Omega_0 mc}\mathbf{A}\mathbf{p}\right) = -\frac{q}{\Omega_0 mc}\mathbf{A}(\mathbf{r})\tau^k(\mathbf{p}) \tag{36}$$

and that owing to gauge invariance[3]

$$\tau^k(\mathbf{p}) = -a\frac{\partial G^{-1}(p)}{\partial\mathbf{p}} = \frac{m\mathbf{p}}{m^*}, \tag{37}$$

we easily get the following expression for the vertex τ^k:

$$\tau^k\left(-\frac{q}{\Omega_0 mc}\mathbf{A}\mathbf{p}\right) = -\frac{q}{\Omega_0 m^* c}\mathbf{A}\mathbf{p}. \tag{38}$$

Substituting (38) in (35), we get a formula for the moment of inertia J of the system which is the same as the formula for the moment of inertia of a system of noninteracting particles. Since in a system of noninteracting fermions the current density $\mathbf{j}'$ is zero,[4] the normal interaction does not change the value of $\mathbf{j}'$ and does not affect the moment of inertia of the system.

5. The effect of the superfluid interaction on the moment of inertia of a Fermi system has been investigated in a number of papers, in particular in one by Migdal.[5] In these papers, however, the perturbation Hamiltonian chosen as a starting point was not the self-consistent Hamiltonian (34), which takes into account not only the bare rotation but also the intrinsic field of the currents, but the Hamiltonian

$$-\sum_i \frac{q}{mc}\mathbf{A}_{0i}\mathbf{p}_i \equiv -\sum_i \mathbf{L}_i\Omega_0; \tag{39}$$

moreover, the normal interaction was not consistently taken into account.

To include the effects of the normal interaction in the equations of Migdal,[5] it is necessary to insert instead of the bare vertex L_z the exact vertex $\tau(q\mathbf{A}\cdot\mathbf{p}/mc\Omega_0)$, and to solve, instead of the equation for $\Delta'_{\lambda_1\lambda_2}$, a system of equations for $\Delta'_{\lambda_1\lambda_2}$ and $\tau_{\lambda_1\lambda_2}$:

$$\tau_{\lambda_1\lambda_2} = \overset{\omega}{\tau}_{\lambda_1\lambda_2} + a^2\frac{dn}{d\mu}\sum_{\lambda'\lambda''}\langle\lambda_1\lambda_2|\Gamma^\omega|\lambda'\lambda''\rangle\{K_{\lambda'\lambda''}\tau_{\lambda'\lambda''} + M_{\lambda'\lambda''}\Delta'_{\lambda'\lambda''}\}, \tag{40}$$

$$\sum_{\lambda'\lambda''}(\Delta'_{\lambda'\lambda''}N_{\lambda'\lambda''} + O_{\lambda'\lambda''}\tau_{\lambda'\lambda''})\varphi_{\lambda'}^*(\mathbf{r})\varphi_{\lambda''}(\mathbf{r}) = 0, \tag{41}$$

where

$$K_{\lambda'\lambda''} = -[1 - g(x)] - g(x)(1 + \hat{P})/2,$$

$$N_{\lambda'\lambda''} = -\frac{1}{4\Delta^2}(\varepsilon_{\lambda'} - \varepsilon_{\lambda''})^2 g(x), \qquad M_{\lambda'\lambda''} = \frac{1}{2\Delta}(\varepsilon_{\lambda'} - \varepsilon_{\lambda''})g(x),$$

$$O_{\lambda'\lambda''} = \frac{1}{4\Delta}[(\varepsilon_{\lambda'} - \varepsilon_{\lambda''}) - (\varepsilon_{\lambda'} - \varepsilon_{\lambda''})\hat{P}]g(x),$$

$$g(x) = \frac{\sinh^{-1}x}{x(1+x^2)^{1/2}}, \qquad x = \frac{1}{2\Delta}(\varepsilon_{\lambda'} - \varepsilon_{\lambda''}) \tag{42}$$

($\hat{P}$ is the time-reversal operator).

In Eqs. (40)–(42) all of the calculations are made in an arbitrary λ representation, the choice of which is determined by the properties of the concrete system. We can look for the solution of Eq. (41) for $\Delta'_{\lambda'\lambda''}$ in the form

$$\Delta'_{\lambda'\lambda''} = -O_{\lambda'\lambda''}\tau_{\lambda'\lambda''}/N_{\lambda'\lambda''}. \tag{43}$$

When we substitute (43) in (40) and use the fact that $\hat{P}\tau = -\tau$, the terms in $g(x)$ cancel, and the equation for τ, Eq. (40), goes over into the analogous equation for the normal liquid.

By means of the requirement of gauge invariance the vertex τ^k can again be put in the form (38). We note that in the paper of Migdal[5] the perturbation Hamiltonian was chosen not in the form (39) but directly in the form $-(m/m^*)\mathbf{L}\cdot\mathbf{\Omega}_0$, which allowed inclusion of the effects of the normal interaction.

6. We shall estimate the correction to Migdal's calculations[5] of moments of inertia of atomic nuclei which are due to the difference between the potential $\mathbf{A}$ and the bare potential $\mathbf{A}_0$.

As was shown in Sec. 3, for nuclei we can neglect the effect of the magnetic field of rigid-body rotation and take as the potential $\mathbf{A}$ the quantity $\mathbf{A}_1$, which is the solution of Eq. (26) without the right member. To estimate the behavior of the potential $\mathbf{A}_1$ it is necessary to know to what type of superfluidity the atomic nucleus belongs.

For the proton component of an atomic nucleus the London pentration depth, calculated for an average distance between particles $r_0 = 1.2 \times 10^{-13}$ cm and for $Z/A \approx 0.4$ is $\delta_L^p \approx 26\ r_0$, which is four or five times the dimensions of heavy nuclei. If in the spirit of the theory of the Fermi liquid we introduce an effective electric charge of the neutrons, which arises owing to the interaction of neutrons with protons and is approximately 0.1 e, then the London penetration depth for the neutrons is of the order $\delta_L^n \approx 260\ r_0$, forty to fifty times the dimensions of heavy nuclei. In the Pippard case, on the other hand, the penetration depths for protons and neutrons are $\delta_P^p \approx 15\ r_0$ and $\delta_P^n \approx 150\ r_0$; here we have used the fact that the size r_C of a Cooper pair is

$$r_C = v_0 / \Delta \approx r_0 A^{2/3} \approx 25\, r_0 > R_0.$$

For the proton component of atomic nuclei $\delta_L^p \approx \delta_P^p \approx r_C$, so that we have the intermediate case. For the neutron component of atomic nuclei $\delta_L^n \gg r_C$, so that we have the London case. Since for the neutron component $\delta_L^n \gg R_0$, we can neglect the difference between the field $\mathbf{A}$ and the bare field $\mathbf{A}_0$, and in this case there is no correction to Migdal's calculations.[5] For the proton component of atomic nuclei, on the other hand, $(R_0/\delta) \sim 0.3$, so that there can be a finite correction to Migdal's calculations in [5].

To get a qualitative estimate of the size of this correction, we solve a model problem of the behavior of a spherical London superfluid system in the external field $\mathbf{H}_0$ of Eq. (13), for an arbitrary ratio of R_0 to δ_L (this sort of problem has previously been solved in the limiting cases $R_0 \gg \delta_L$ and $R_0 \ll \delta_L$[6,7]). The vector potential $\mathbf{A}(\mathbf{r})$, which is the solution of Eq. (26) without the right member and with the usual electrodynamic boundary conditions, is of the form

$$\mathbf{A}(\mathbf{r}) = \mathbf{A}_0 \left(1 + \frac{2\alpha}{R_0{}^3}\right) y^3 x^{-3} (\operatorname{sh} y - y \operatorname{ch} y)^{-1} (\operatorname{sh} x - x \operatorname{ch} x), \tag{44*}$$

*sh ≡ sinh, ch ≡ cosh.

in the exterior region of the sphere; here $x = r/\delta_L$, $y = R_0/\delta_L$, and α is the volume magnetization coefficient of the sphere, Eq. (15),

$$\alpha = \int \chi(r)\, dV = -\frac{3V}{8\pi}\left[1 + \frac{3}{y^2} - \frac{3}{y}\coth y\right]. \tag{45}$$

In the limiting cases $R_0 \gg \delta_L$ and $R_0 \ll \delta_L$ the coefficient α goes over into the well known magnetization coefficients for a massive superconducting sphere, $\alpha = -3V/8\pi$,[6] and for a superconducting sphere of small radius, $\alpha = -(1/40\,\pi)(R_0/\delta_L)^2 V$.[7]

If we recall that the exact perturbation Hamiltonian (34) differs from Migdal's Hamiltonian by the replacement of $\mathbf{A}$ by $\mathbf{A}_0$ (sic), we can obtain the correction to Migdal's calculations by replacing the bare angular velocity $\mathbf{\Omega}_0$ by the effective angular velocity $\mathbf{\Omega}(r) = (q/mc)$ curl $\mathbf{A}$. Calculation of $\mathbf{\Omega}(r)$ with the potential (44) shows that $\Omega(r)$ is equal to Ω_0 for $r = R_0$ and decreases to $(1-k)\Omega_0$ for $r = 0$, where the coefficient k is positive and proportional to $(R_0/\delta_L)^2$. Owing to this we can expect that the proton moment of inertia of an atomic nucleus is diminished as compared with Migdal's results by a quantity of the order of $(R_0/\delta_L)^2 \approx 0.09$. Since the accuracy of the theory of Fermi liquids for atomic nuclei is to the order $A^{-1/3} = 0.16$, this correction is scarcely to be detected experimentally.

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