# *EMISSION OF TWO PHOTONS INTO A GIVEN ANGLE IN ELECTRON COLLISIONS*

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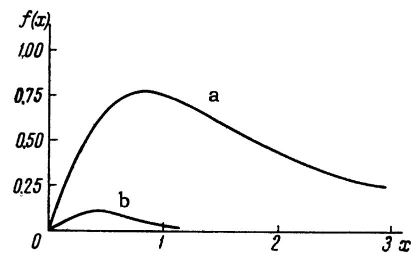
Submitted to JETP editor January 10, 1966

J. Exptl. Theoret. Phys. (U.S.S.R.) 50, 1611-1616 (June, 1966)

The process of the emission of two photons of arbitrary energy into a given angle in electron collisions is considered. The cross section $d\sigma_{\omega_1\omega_2}$ has the form of a power series in $\omega_1$ and $\omega_2$. In the case in which the angular dimension of the detectors $\vartheta_0$ is of the order of $1/\gamma$, the coefficients of the powers of $\omega_{1,2}$ were calculated numerically. In the case $\vartheta_0 \gg 1/\gamma$, an analytic expression was obtained for the cross section.

1. The process of emission of two photons of arbitrary energy in electron collisions has been studied by Galitskiĭ and one of the authors.[1-3] This process is of great interest for current colliding-beam experiments for two reasons: 1) it can be used as a monitor in the recording of collisions of beams; 2) in the case of electron-positron collisions, the process of double bremsstrahlung competes with the process of two-quantum annihilations; this is connected with the fact that the cross section of the double bremsstrahlung, in contrast with the cross section of the double bremsstrahlung, in contrast with the cross section of the two-quantum annihilation, does not fall off with increase in the energy of the colliding particles.

The cross section of double bremsstrahlung[1)] $d\sigma_{\omega_1\omega_2}$ had previously[1-3] been found with an accuracy up to terms of order $\gamma^{-2}$ ($\gamma = \epsilon/m$, $\epsilon$ is the energy of the initial electrons in the center-of-mass system). This cross section was integrated over the final states of the electrons (inasmuch as electrons are not recorded) and over all angles of flight of the photons (thus assuming that the angular dimensions of the detectors of the photons are much larger than the characteristic angle of emission $1/\gamma$). However, in real experiments on the study of double bremsstrahlung, which take place at relatively low energies, the angular dimensions of the detectors $2\vartheta_0$ are comparable with the quantity $1/\gamma$ (for example in the VÉP-1 installation at Novosibirsk,[4] $\epsilon = 43$ MeV, $\vartheta \sim 3/\gamma$). In this connection, the cross section of emission of two photons into a given angle is of interest. For simplicity we assume that the angles of the two detectors are identical: $\vartheta_{10} = \vartheta_{20} = \vartheta_0$, although, as is shown at the end of the article, it is easy to generalize the approach to include the case of detectors of different dimensions.




2. For a qualitative understanding of the situation arising in the given problem, let us consider the emission of classical photons.[1] In this case, it is easy to obtain the cross section of emission into a given angle, inasmuch as the contribution of each of the classical currents is integrated independently. We consider the integral over the square of the momentum transfer $\Delta^2/m^2 = 4x^2$ [Eq. (53) of the earlier paper[2]]. The integrand $f(x)$ is shown in the drawing for the cases 1) all angles of emission of the photons are permissible ($f(x) = \Phi^2(x^2)/x^3$) —curve a; 2) the angle of emission of the photons is not larger than $\vartheta_0 = 1/\gamma$ — curve b. It is seen that the principal contribution to the integral is made by the region $x \sim 1$. Inasmuch as the characteristic angle of emission of the photons is $\vartheta_k \sim 1/\gamma$, we cut off for $\vartheta = 1/\gamma$, the significant part of the region of integration over the angle of emission of the photon, which appreciably decreases the integrand in the integration over x, all the more since this applies to each of the two photons. Moreover, for $x > 1$, the probability of emission of the photon is small for small $\vartheta_k$ and reaches a maximum only for $\vartheta_k \sim 1/\gamma$, which leads to an additional suppression of the integrand when $\vartheta_0 = 1/\gamma$, which thus falls off more rapidly when $x > 1$.

[1)]The notation used in this paper will be that of[1-3].

As a result, the cross section of emission into the angle $\vartheta_0 = 1/\gamma$ amounts to only a small fraction of the emission cross section integrated over all photon-emission angles.

3. We now proceed to a consideration of the emission of photons of any energy into a given angle. With this aim, we transform Eq. (3) of [3] to the following form:[2)]

$$d\sigma = \frac{2\alpha^4}{(2\pi)^3}\frac{d\xi_1}{\xi_1}\frac{d\xi_2}{\xi_2}\int\frac{d\Delta^2}{\Delta^4}dR_1\,dR_2, \qquad (1)$$

where

$$dR_1 = \left[-\frac{1}{\varkappa_3^2} + \frac{\Delta^2[1+(1-\xi_1)^2]+4(1-\xi_1)}{2\varkappa_1\varkappa_3} - \frac{(1-\xi_1)^2}{\varkappa_1^2}\right]$$
$$\times\delta(p_1+\Delta-p_3-k_1)\,\xi_1\frac{d^3p_3}{\varepsilon_3}d\varkappa_1\,d\varkappa_3\,d\varphi_1,$$
$$dR_2 = dR_1(p_1\to p_2,\quad p_3\to p_4,$$
$$k_1\to k_2,\quad \Delta\to-\Delta,\quad \varphi_1\to\varphi_2); \qquad (2)$$

Here $p_1, p_2$ ($p_3, p_4$) are the momenta of the initial (final) electrons, $k_1, k_2$ the momenta of the photons,

$$\xi_i = \omega_i/\varepsilon,\quad \varkappa_1 = -(k_1p_1),\quad \varkappa_2 = -(k_2p_2),$$
$$\varkappa_3 = -(k_1p_3),\quad \varkappa_4 = -(k_2p_4),$$

and $\varphi_{1,2}$ are the azimuthal angles of flight of photons in the center-of-mass system under the condition that the respective directions of the vectors $p_1, p_2$ are taken for the polar axes. Inasmuch as integration over the angles of flight of the photons (variables $\kappa_1, \kappa_2$) is within specified limits, it is expedient to change the previously used order of integration[2,3] and carry out the integration over the variables $\kappa_3$ and $\kappa_4$ first.

Carrying out integration over $d^3p_3\,d\varphi_1$, we obtain

$$\int\delta(p_1+\Delta-k_1-p_3)\frac{d^3p_3}{\varepsilon_3}d\varphi_1 = \frac{2}{g\sin\varphi_1}, \qquad (3)$$

where $g\sin\varphi_1 = \sqrt{U_1}$; $U_1$ is a quadratic form of $\kappa_3$:

$$U_1 = c\varkappa_3^2 + b\varkappa_3 + a. \qquad (4)$$

The limits of integration with respect to $\kappa_3$ are determined by the zeroes of the function $U_1$. This fact is easily understood from simple kinematic considerations. The fact is that $\varphi_1$ is the angle between the planes $(\mathbf{p}_1, \Delta)$ and $(\mathbf{p}_1, \mathbf{k}_1)$, while the angle between the vectors $\mathbf{p}_1$ and $\mathbf{k}_1$ is a constant [inasmuch as the quantity $\kappa_1 = -(p_1k_1)$ is fixed]. When $\varphi_i$ changes from zero to $2\pi$, the function $\kappa_3 = \kappa_3(\cos\varphi_1)$ takes on all values inside the interval of integration; therefore the limits of integration over $\kappa_3$ are determined by the condition $d\kappa_3/d\varphi_1 = 0$, from which follows $\sin\varphi_1 = 0$. Here, all four vectors $\mathbf{p}_1$, $\mathbf{p}_3$, $\mathbf{k}_1$, and $\Delta$, lie in the same plane.

In accord with the statement of the problem, we shall compute the principal term in the expansion of the cross section in powers of $\epsilon^{-2}$ [2,3]. Inasmuch as we shall carry out the integration over $\kappa_1$ to the upper limit $\kappa_{1\,max} \ll \epsilon^2$, we discard terms of order $\kappa_1/\epsilon^2$ in the expression for $U_1$ (the coefficients a, b, c in Eq. (4)). Furthermore, in the case of arbitrary angles of photon emission, the upper limit of integration over $\kappa_1$ is proportional to $\epsilon^2$ and therefore, in view of the convergence of the integral we can set the upper limit equal to infinity within the assumed accuracy. In view of this fact, the main contribution to the integral over $\kappa_1$ is made by small $\kappa_1$ ($\kappa_1 \sim 1$), so that one can always discard terms of the order $\kappa_1/\epsilon^2$.[2] As a consequence of the fact that at the upper boundary of the region of integration over the variables $\kappa_3$ and $\kappa_4$, these quantities are of the order of $\kappa_1$, $\kappa_2$, $\Delta^2$ (but not $\epsilon^2$), and with account of the fact that the fundamental contribution to the integral cross section is made by $\kappa_1, \kappa_2 \sim 1$ (see above) and $\Delta^2 \sim 1$,[2,3] we also neglect terms $\sim \kappa_4/\epsilon^2$. Making the approximations shown above, we obtain

$$c = -(1-\xi_1)^2,\quad b = 2(1-\xi_1)\left[\varkappa_1 + \frac{\xi_1\Delta^2}{2}\right],$$
$$a = -\left(\varkappa_1 - \frac{\xi_1\Delta^2}{2}\right)^2 - \xi_1^2\Delta^2. \qquad (5)$$

It is seen that the function $U_1$ does not contain the quantity $\kappa_4$. Thus, with the specified accuracy, it has been shown to be possible to carry out the integration over variables that pertain to each of the photons independently.[2,3]

Integrating $R_1$ with respect to $\kappa_3$, we get

$$\int_{\varkappa_3} dR_1 = 2\pi\xi_1\left[-\frac{b}{2(-a)^{3/2}} + \frac{\Delta^2[1+(1-\xi_1)^2]+4(1-\xi_1)}{2\varkappa_1(-a)^{1/2}} - \left(\frac{1-\xi_1}{\varkappa_1^2}\right)\right]d\varkappa_1. \qquad (6)$$

We also have

$$\int_{\varkappa_4} dR_2 = \int_{\varkappa_3} dR_1\begin{pmatrix}\varkappa_1\to\varkappa_2\\ \xi_1\to\xi_2\end{pmatrix}. \qquad (7)$$

We now proceed to integration over $\kappa_1$. The lower limit of integration is determined by the condition $\vartheta_k = 0$, whence, with accuracy up to terms in $\epsilon^{-2}$, we get $\kappa_{1\,min} = \xi_1/2$. The upper limit of integration in $\kappa_1$ is determined by the limiting angle of emission of the photon $\vartheta_0$:

$$\varkappa_{1max} = \xi_1\varepsilon^2(1-\beta\cos\vartheta_0) = \xi_1\varkappa_0.$$

If we put $\vartheta_0 = n/\epsilon$, then $\kappa_0 = n^2/2$ for $n \ll \epsilon$.

[2)]Here and in what follows, m = 1.

Carrying out the elementary integration with respect to $\kappa_1$ (and correspondingly $\kappa_2$), we obtain the following expression for the cross section of the double bremsstrahlung:

$$d\sigma = \frac{8r_0^2\alpha^2}{\pi}\frac{d\xi_1}{\xi_1}\frac{d\xi_2}{\xi_2}\int\frac{dx}{x^3}$$
$$\times\Big\{(1-\xi_1)\Phi(x^2)+\xi_1^2\frac{x}{\sqrt{1+x^2}}\ln(x+\sqrt{1+x^2})$$
$$-F(x,\varkappa_0,\xi_1)\Big\}\Big\{(1-\xi_2)\Phi(x^2)$$
$$+\xi_2^2\frac{x}{\sqrt{1+x^2}}\ln(x+\sqrt{1+x^2})-F(x,\varkappa_0,\xi_2)\Big\}, \tag{8}$$

where

$$F(x,\varkappa_0,\xi_1)=\frac{1}{4}\Big[\frac{(1-\xi_1)+[2(1-\xi_1)+\xi_1^2]x^2}{x\sqrt{1+x^2}}$$
$$\times\ln\left(\frac{2x(1-x^2)-x\varkappa_0+\sqrt{(1+x^2)R_0}}{\varkappa_0[\sqrt{1+x^2}-x]}\right)$$
$$-(1-\xi_1)\left(1+\frac{1}{\varkappa_0}+\frac{1-\varkappa_0+2x^2}{\sqrt{R_0}}\right)\Big], \tag{9}$$

$$R_0=(2x^2-\varkappa_0)^2+4x^2. \tag{10}$$

It is clear that as $\kappa_0 \to \infty$, the function $F(x, \kappa_0, \xi_1) \to 0$, so that Eq. (8) transforms to Eq. (4) of [3]. It is easy to see that as $x \to 0$, the component $F(x, \kappa_0, \xi_1)$, similar to the remaining part of the expression lying in the curly brackets, is proportional to $x^2$, so that the lower limit of integration over x can be set equal to zero as before. Similarly, the upper limit of integration over x can be set equal to infinity.

4. The single integrals entering into Eq. (8) cannot be computed in analytic form; therefore they were calculated by means of an M-20 electronic computer.

The cross section of the double bremsstrahlung into a given angle is represented in the form

$$d\sigma_{\omega_1\omega_2}=\frac{8r_0^2\alpha^2}{\pi}\Big\{\Big(1-\frac{\omega_1}{\varepsilon}\Big)\Big(1-\frac{\omega_2}{\varepsilon}\Big)\eta_1(n)+\Big[\Big(1-\frac{\omega_1}{\varepsilon}\Big)\frac{\omega_2^2}{\varepsilon^2}$$
$$+\Big(1-\frac{\omega_2}{\varepsilon}\Big)\frac{\omega_1^2}{\varepsilon^2}\Big]\eta_2(n)+\frac{\omega_1^2\omega_2^2}{\varepsilon^4}\eta_3(n)\Big\}\frac{d\omega_1}{\omega_1}\frac{d\omega_2}{\omega_2}. \tag{11}$$

Numerical values of the function $\eta_m(n)$ are given in Table I.

5. In the range of values $1 \ll n \ll \epsilon$, it is possible to obtain an analytic expression for the desired cross section. In this case the asymptotic expressions for the coefficient $\eta_m(n)$ in Eq. (11)

**Table I.** Values of the coefficient $\eta_m(n)$ in Eq. (11)

| $n$ $(\vartheta_0=\frac{n}{\varepsilon})$ | $\eta_1(n)$ Elect. computer | $\eta_1(n)$ Eq. (12) | $\eta_2(n)$ Elect. computer | $\eta_2(n)$ Eq. (12) | $\eta_3(n)$ Elect. computer | $\eta_3(n)$ Eq. (12) |
|---|---|---|---|---|---|---|
| 1 | 0,081 | — | 0,065 | — | 0,053 | — |
| 2 | 0.406 | — | 0.311 | — | 0,237 | — |
| 3 | 0.743 | — | 0,555 | — | 0.412 | — |
| 4 | 1,02 | 1.07 | 0,744 | 0.748 | 0,543 | 0.534 |
| 5 | 1,23 | 1.24 | 0,885 | 0,881 | 0,638 | 0.631 |
| 10 | 1.77 | 1.77 | 1.23 | 1.23 | 0.863 | 0.862 |

have the form

$$\eta_1(n)=\frac{5}{4}+\frac{7}{8}\zeta(3)-\frac{1}{n^2}\Big[10\ln^2 n-\frac{\pi^2}{2}+\frac{11}{2}\Big],$$
$$\eta_2(n)=\frac{1}{2}+\frac{7}{8}\zeta(3)-\frac{1}{2n^2}\Big[10\ln^2 n+5\ln n-\frac{\pi^2}{2}+\frac{9}{2}\Big],$$
$$\eta_3(n)=\frac{7}{8}\zeta(3)-\frac{1}{2n^2}\Big[5\ln^2 n+5\ln n-\frac{\pi^2}{4}+\frac{5}{2}\Big]. \tag{12}$$

As is seen from Table I, beginning with n = 4 the results obtained with the help of Eq. (12) are in excellent agreement with the results of the numerical calculation.

6. We have considered the case of symmetric detectors. In the case of nonsymmetric detectors, one can use Eq. (8) directly, but each function $F(x, \kappa_{0i}, \xi_i)$ will depend on its own limiting angle. Chief interest centers on the case when the angular dimensions of one of the counters are very large ($\vartheta_{20} \gg 1/\epsilon$), and those of the second are small ($\vartheta_{10} \sim 1/\epsilon$); then for one of the photons, one can carry out integration over all the angles of flight of the photon (as in [1-3]), and for the emission of the second photon we can make use of the approach of the present paper. The cross section of the process is represented in the form

$$d\sigma_{\omega_1\omega_2}=\frac{8r_0^2\alpha^2}{\pi}\Big\{\Big(1-\frac{\omega_1}{\varepsilon}\Big)\Big(1-\frac{\omega_2}{\varepsilon}\Big)\mu_1(n)+\Big(1-\frac{\omega_2}{\varepsilon}\Big)\frac{\omega_1^2}{\varepsilon^2}\mu_2(n)$$
$$+\Big(1-\frac{\omega_1}{\varepsilon}\Big)\frac{\omega_2^2}{\varepsilon^2}\mu_3(n)+\frac{\omega_1^2\omega_2^2}{\varepsilon^4}\mu_4(n)\Big\}\frac{d\omega_1}{\omega_1}\frac{d\omega_2}{\omega_2}. \tag{13}$$

The values of the functions $\mu_m(n)$ are shown in Table II. For the case $1 \ll n \ll \epsilon$ it is easy to obtain the asymptotic expressions for these functions:

$$\mu_1(n)=\frac{5}{4}+\frac{7}{8}\zeta(3)-\frac{3}{n^2}[2\ln^2 n+1],$$
$$\mu_2(n)=\frac{1}{2}+\frac{7}{8}\zeta(3)-\frac{1}{n^2}\Big[3\ln^2 n+\ln n+\frac{3}{2}\Big],$$
$$\mu_3(n)=\frac{1}{2}+\frac{7}{8}\zeta(3)-\frac{1}{n^2}[3\ln^2 n+2\ln n+1],$$
$$\mu_4(n)=\frac{7}{8}\zeta(3)-\frac{3}{2n^2}\Big[\ln^2 n+\ln n+\frac{1}{2}\Big]. \tag{14}$$

**Table II.** Values of coefficients $\mu_m(n)$ in Eq. (13)

| $n$ $\left(\vartheta_0 = \frac{n}{\varepsilon}\right)$ | $\mu_1(n)$ | | $\mu_2(n)$ | | $\mu_3(n)$ | | $\mu_4(n)$ | |
|---|---|---|---|---|---|---|---|---|
| | Elect. computer | Eq. (14) | Elect. computer | Eq. (14) | Elect. computer | Eq. (14) | Elect. computer | Eq. (14) |
| 1 | 0.299 | — | 0.281 | — | 0.215 | — | 0.199 | — |
| 2 | 0.805 | — | 0.628 | — | 0.568 | — | 0.441 | — |
| 3 | 1.16 | — | 0.859 | — | 0.811 | — | 0.601 | — |
| 4 | 1.40 | 1.40 | 1.01 | 1.01 | 0.972 | 0.956 | 0.705 | 0.696 |
| 5 | 1.56 | 1.56 | 1.12 | 1.12 | 1.08 | 1.07 | 0.776 | 0.770 |
| 10 | 1.95 | 1.95 | 1.35 | 1.35 | 1.34 | 1.34 | 0.930 | 0.930 |

These expressions, as also Eq. (12), beginning with n = 4, are in excellent agreement with the results of numerical calculation (see Table II).

The authors express their gratitude to A. P. Onuchin for discussion of the problems associated with the experiment and G. I. Rusov and E. Z. Borovskiĭ for help in the numerical calculations.

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Translated by R. T. Beyer
193