SOME FEATURES OF THE GYROMAGNETIC EFFECT IN FERRODIELECTRICS AT LOW TEMPERATURES

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The temperature correction to the mechanical moment of the spin system in a ferrodielectric is calculated. It is shown that at a sufficiently low temperature, when the spin-spin interaction is appreciable, the temperature dependence of the magnetic and mechanical moments are different.

In gyromagnetic experiments on the determination of the electronic g factor, use is generally made of the fact that the ratio of the magnetic moment of the sample to the mechanical moment is equal to the corresponding ratio for the free electron. In ferrodielectrics, at sufficiently low temperatures, all the microscopic characteristics are determined by the spin-wave branch of the spectrum, which is due both to exchange and to relativistic magnetic interaction of the spins.

In the present paper, a formula is obtained in the spin-wave approximation for the temperature part of the mechanical moment of the spin system, and it is shown that in the range of temperatures where the dielectric interaction is appreciable, its ratio to the corresponding contribution to the magnetic moment differs from the single-electron g factor and depends on the temperature.

We note that because of the dipole interaction of the spins, the total magnetic moment of the system is not conserved, whereas under suitable symmetry of the lattice, the total mechanical moment should be conserved. Therefore, the mechanical moment of the system must be an integral of the motion, connected with the invariance of the spin system relative to the corresponding rotation.

1. We consider a uniaxial (or cubic) ferrodielectric in which a constant and homogeneous external field is applied along the chosen axis. The magnetic state of such a system can, as is well known,[1] be described classically by introducing the field of the magnetization vector $\mathbf{M}(\mathbf{r}, t)$ which satisfies the equation

$$\partial \mathbf{M} / \partial t = -g[\mathbf{M}\mathbf{H}_{\text{eff}}], \tag{1}^*$$

where g is the single particle gyromagnetic ratio and $\mathbf{H}_{\text{eff}}$ is the effective magnetic field, equal to the variational derivative of the energy with respect to the magnetization:

$$\mathbf{H}_{\text{eff}} = -\delta\mathscr{E} / \delta\mathbf{M}. \tag{2}$$

The energy of the system has the form

$$\mathscr{E} = \int \left(\frac{\alpha_{ik}}{2} \frac{\partial \mathbf{M}}{\partial x_i} \frac{\partial \mathbf{M}}{\partial x_k} + \frac{\beta}{2} \boldsymbol{\mu}^2 - \mathbf{M}\mathbf{H} - \frac{\mathbf{H}^2}{8\pi} \right) dV. \tag{3}$$

The first term of the integrand describes the exchange interaction, which is characterized by the tensor α_{ik}; the second term is the anisotropy energy density ($\beta > 0$ is the anisotropy constant, and μ is the component of the magnetization vector perpendicular to the easy axis); the last two terms correspond to the magnetic interaction of the spins with the external field $\mathbf{H}_0$ and with one another.

The magnetic dipole-dipole interaction is conveniently described by introducing the field **h**. At low temperatures, when the principal role is played by the long wave perturbations, one can neglect the retardation and assume that the field **h** satisfies the equations 1)

$$\text{rot}\, \mathbf{h} = 0, \quad \text{div}\, \mathbf{h} = 4\pi\, \text{div}\, \mathbf{M}. \tag{4}^*$$

The field **H** entering into Eq. (3) is the sum: $\mathbf{H} = \mathbf{H}_0 + \mathbf{h}$.

2. The spin-wave approximation corresponds to an expansion with accuracy up to quadratic terms of the energy density (3) in powers of the quantity μ, which is small in comparison with $\mathbf{M}_0$ —the magnetization—near the ground state.2) As a result of such an expansion, Eq. (1) becomes linear in μ:

$$\frac{\partial \boldsymbol{\mu}}{\partial t} = -g\left[\mathbf{M}_0, \mathbf{h} - \left(\frac{H_0}{M_0} + \beta\right)\boldsymbol{\mu} + \alpha_{ik} \frac{\partial^2 \boldsymbol{\mu}}{\partial x_i \partial x_k}\right]. \tag{5}$$

*$[\mathbf{M}\mathbf{H}_{\text{eff}}] \equiv \mathbf{M} \times \mathbf{H}_{\text{eff}}$.

1)The description used for the energy of magnetic interaction corresponds to the choice of **M** and **H** as independent variables.

*rot ≡ curl.

2)Equation (1) describes the change of the magnetization only in direction.

The dynamic characteristics of the system can most simply be determined by means of the Lagrangian density. If we introduce the components of the vector μ and the magnetic potential φ (from Eq. (4), $\mathbf{h} = \nabla\varphi$) as generalized coordinates, then it is easy to establish the fact that Eqs. (4) and (5) can be obtained as the Lagrangian equations for the field (φ, μ) if we choose the following expression as the Lagrangian density:

$$L = \frac{1}{2gM_0^2}[\dot{\mu}\mathbf{M}_0]\,\mu - \frac{\alpha_{ik}}{2}\frac{\partial\mu}{\partial x_i}\frac{\partial\mu}{\partial x_k} + \mu\mathbf{h} - \frac{1}{2}\left(\frac{H_0}{M_0}+\beta\right)\mu^2 + \frac{\mathbf{h}^2}{8\pi}. \tag{6}$$

Then, by means of standard procedures, [2] we can find the components of the energy-momentum tensor. In particular, the momentum density is shown to be equal to

$$p_i = \frac{1}{2gM_0^2}[\mu\mathbf{M}_0]\frac{\partial\mu}{\partial x_i}. \tag{7}$$

This makes it possible to determine the "orbital" magnetic moment of the field

$$\mathbf{J}_{\text{orb}} = \int [\mathbf{rp}]\,dV. \tag{8}$$

Differentiating $(\mathbf{J}_{\text{orb}})_z$ with respect to the time (the z axis is the same as the axis of easy magnetization) and using Eq. (5), we find, in the case of a uniaxial or cubic crystal,

$$\frac{d}{dt}(\mathbf{J}_{\text{orb}})_z = \int [\mathbf{h}\mu]_z\,dV.$$

On the other hand, it follows from Eq. (5) that

$$\frac{d}{dt}\int\frac{\mu^2}{2gM_0}dV = -\int [\mathbf{h}\mu]_z\,dV.$$

Therefore

$$\frac{d}{dt}(\mathbf{J}_{\text{orb}} + \mathbf{J}_{\text{sp}})_z = 0, \tag{9}$$

where

$$(\mathbf{J}_{\text{sp}})_z = \int\frac{\mu^2}{2gM_0}dV. \tag{10}$$

The moment (10) can be regarded as the spin moment of the field. It is equal to the time dependent contribution to the z component of the total spin of the system.

The conserved quantity $J = (\mathbf{J}_{\text{orb}} + \mathbf{J}_{\text{sp}})_z$ is the total mechanical moment of the spin system in the ferrodielectric. It is easy to see that when the dipole-dipole interaction is neglected, the moments $(\mathbf{J}_{\text{orb}})_z$ and $(\mathbf{J}_{\text{sp}})_z$ are separately conserved.[3)]

3. Quantization in the spin-wave approximation is obtained by replacement of the projections μ_x and μ_y by operators with the following commutation rule:[3]

$$[\mu_x(\mathbf{r}),\ \mu_y(\mathbf{r}')] = ig\hbar M_0\delta(\mathbf{r}-\mathbf{r}'). \tag{11}$$

Here the diagonalization of the Hamiltonian (3) in the same approximation is carried out by means of the generalized Fourier transformation:

$$\mu^+ \equiv \mu_x + i\mu_y = \sqrt{\frac{2g\hbar M_0}{V}}\sum_{\mathbf{k}}\{u_{\mathbf{k}}a_{\mathbf{k}}e^{i(\mathbf{kr}-\omega_{\mathbf{k}}t)} + v_{\mathbf{k}}^* a_{\mathbf{k}}^+ e^{-i(\mathbf{kr}-\omega_{\mathbf{k}}t)}\}, \tag{12}$$

where $a_{\mathbf{k}}^+$ and $a_{\mathbf{k}}$ are the Bose creation and annihilation operators, and the coefficients $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ and the frequency $\omega_{\mathbf{k}}$ are connected by the equations

$$A_{\mathbf{k}}u_{\mathbf{k}} + B_{\mathbf{k}}^* v_{\mathbf{k}} = \omega_{\mathbf{k}}u_{\mathbf{k}},\quad B_{\mathbf{k}}u_{\mathbf{k}} + A_{\mathbf{k}}v_{\mathbf{k}} = -\omega_{\mathbf{k}}v_{\mathbf{k}},$$
$$|u_{\mathbf{k}}|^2 - |v_{\mathbf{k}}|^2 = 1,$$
$$A_{\mathbf{k}} = gM_0\alpha_{ik}k_ik_k + g(H_0+\beta M_0) + 2\pi gM_0\frac{k_x^2+k_y^2}{k^2},$$
$$B_{\mathbf{k}} = 2\pi gM_0\frac{(k_x+ik_y)^2}{k^2}. \tag{13}$$

Substituting the expansion (12) in Eq. (3) we get in the quadratic approximation in μ:

$$\mathcal{H} = \mathcal{E}_0 + \sum_{\mathbf{k}}\hbar\omega_{\mathbf{k}}a_{\mathbf{k}}^+a_{\mathbf{k}},\qquad \omega_{\mathbf{k}} = \sqrt{A_{\mathbf{k}}^2 - |B_{\mathbf{k}}|^2}. \tag{14}$$

The mechanical moment J_z is reduced to the following form by means of this expansion:

$$J_z = J_{0z} - i\hbar\sum_{\mathbf{k}}a_{\mathbf{k}}^+[\mathbf{k}\nabla_{\mathbf{k}}]_z a_{\mathbf{k}} + \hbar\sum_{\mathbf{k}}a_{\mathbf{k}}^+a_{\mathbf{k}} \tag{15}$$

($\nabla_{\mathbf{k}}$ is the gradient in k-space).

The first component in (15) can be interpreted as the operator of the orbital momentum of the system of spin waves, and the second as the operator of the spin moment of this system. It is then seen that the spin of the spin wave is equal to unity (in units of ħ), while only one (+1) of the three projections of the spin on the z axis is realized.

As might be expected, the operator of the moment J_z commutes with the energy operator (14) when $\omega_{\mathbf{k}}$ does not depend on the angle $\varphi_{\mathbf{k}}$, that is, in the case of a uniaxial or cubic crystal. For diagonalization of the moment, we transform from

3)The procedure for obtaining the total moment in our case is similar to the introduction of the spin of the electron in relativistic theory, wherein the role of the spin-orbit interaction is played by the dipole-dipole interaction.

the quantum numbers k_x, k_y, k_z to the quantum numbers $k_\perp$, k_z, m, where $k_\perp = \sqrt{k_x^2 + k_y^2}$ and m is the eigenvalue of the operator $i[\mathbf{k} \times \nabla_\mathbf{k}]_z$. The transformation to the new representation reduces to the canonical transformation

$$a_\mathbf{k} = \frac{1}{\sqrt{2\pi}} \sum_{m=-\infty}^{\infty} a_{k_\perp, k_z, m} e^{im\varphi_\mathbf{k}},$$

as the result of which we get

$$J_z = J_{0z} + \hbar \sum_{k_\perp, k_z, m} (m+1) a^+_{k_\perp, k_z, m} a_{k_\perp, k_z, m}. \quad (16)$$

4. Since the frequency $\omega_\mathbf{k}$ in the new representation does not depend on m, averaging of Eq. (16) with the equilibrium Gibbs distribution reduces to the following result:

$$\bar{J}_z = J_{0z} + \hbar \sum_\mathbf{k} n_\mathbf{k}, \quad n_\mathbf{k} = (e^{\hbar\omega_\mathbf{k}/T} - 1)^{-1}. \quad (17)$$

Thus the temperature contribution ΔJ_z to the mechanical moment is equal (in units of ħ) to the total number of spin waves at the given temperature T.

So far as the temperature contribution to the magnetic moment is concerned, it is equal to[3]

$$\Delta M_z = \hbar \sum_\mathbf{k} \frac{\partial \omega_\mathbf{k}}{\partial H} n_\mathbf{k}. \quad (18)$$

It is seen from a comparison of (17) with (18) that upon neglect of the magnetic dipole-dipole interaction, the ratio $\Delta M_z/\Delta J_z$ is equal to g. Such a neglect is valid when the temperature T is large in comparison with the energy of the dipole-dipole interaction, which is equal in order of magnitude to $2\pi g\hbar M_0$.

In the case of low temperatures ($T \lesssim 2\pi g\hbar M_0$) the contribution of the dipole interaction becomes important, as a result of which the ratio of the temperature contributions to the magnetic and mechanical moment of the spin system of the ferromagnet will depend on the temperature.

We shall write down the results of the calculations of the spontaneous magnetic and mechanical moments in the region of temperatures $T \ll 2\pi g\hbar M_0$. As was shown earlier,[4] the temperature contribution to the magnetic moment is equal here to

$$M_{0z} - M_z(T) = \frac{\pi g\hbar}{48\sqrt{2}} \frac{V}{a^3} \left(\frac{T}{2\pi g\hbar M_0}\right)^{1/2} \left(\frac{T}{\Theta_c}\right)^{3/2}. \quad (19)$$

It follows from Eq. (17) that for $T \ll 2\pi g\hbar M_0$, the temperature dependence of ΔJ_z has the following form:

$$J_{0z} - J_z(T) = \frac{KV\hbar}{8\pi^2 a^3} \frac{T}{2\pi g\hbar M_0} \left(\frac{T}{\Theta_c}\right)^{3/2}, \quad (20)$$

where K is a numerical factor equal to the integral

$$K = \int_0^\infty \int_0^\infty dx\, dz \frac{(\sqrt{x^2+z^2} - z)^{1/2}}{\sqrt{x^2+z^2}} \frac{x\,dx}{e^x - 1}.$$

Comparison of Eqs. (19) and (20) shows that ΔM_z and ΔJ_z actually have different temperature dependences in the limiting case under consideration.[4)]

The difference in the temperature dependence of the magnetic and mechanical moments of the spin system can in principle be found by means of a gyromagnetic experiment in which the change of the moments takes place as the result of the change in the temperature in the absence of an external field.

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[1] L. D. Landau and E. M. Lifshitz, Z. Phys. Sowjetunion **8**, 157 (1935).

[2] L. D. Landau and E. M. Lifshitz, Teoriya polya (Theory of Fields) (Fizmatgiz, 1960).

[3] A. I. Akhiezer, V. G. Bar'yakhtar, and M. I. Kaganov, Usp. Fiz. Nauk **71**, 533 (1960), Soviet Phys. Uspekhi **3**, 567 (1961).

[4] M. I. Kaganov and V. M. Tsukernik, FMM **5**, 561 (1957).

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[4)] It must be noted that we have neglected the anisotropy energy, which is proper only under conditions of smallness of this energy in comparison with the temperature.