OPTICAL PHONON PRODUCTION AND GALVANOMAGNETIC EFFECTS FOR A LARGE-ANISOTROPY ELECTRON DISTRIBUTION

I. I. VOSILIUS and I. B. LEVINSON

Kaunas Polytechnic Institute; Institute of Physics and Mathematics, Academy of Sciences, Lithuanian S.S.R.

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It is shown that for low-temperature scattering by optical phonons there exists an electric field strength range in which spontaneous emission of a phonon leads to complete stoppage of the electron. In such fields, all electrons possess velocities directed along the field. The optical phonons produced are monochromatic in the momenta: the momentum direction is parallel to the field and its magnitude is identical with the momentum of an electron with an energy equal to that of the optical phonon. A transverse magnetic field rotates the beam of generated monochromatic phonons in a plane perpendicular to the magnetic field. With increase of the magnetic field strength, the angle of rotation grows to 90°, after which this monochromatic generation ceases. A sharp decrease of the dissipative current and a maximal Hall current should appear at a certain value of the magnetic field.

1. An overwhelming majority of papers on the behavior of electrons of a semiconductor in a strong electric field start from the assumption that the momentum distribution function has little anisotropy, i.e., that

$$f(\mathbf{p}) = f_0(\varepsilon) + f_1(\varepsilon)\cos\vartheta, \quad f_1(\varepsilon) \ll f_0(\varepsilon), \quad (1.1)$$

where ϵ is the electron width and ϑ the angle between its momentum and the field **E**. Davydov has shown[1] that this approximation is valid if the scattering of the electrons is almost elastic.

An approximation which is the opposite of (1.1) is

$$f(\mathbf{p}) = 2f_0(\varepsilon)\delta(\cos\vartheta - 1). \quad (1.2)$$

Such a "needle-like" distribution was used by Baraff in a problem connected with impact ionization[2]. The factor 2 is necessary in order that $f_0(\epsilon)$ in (1.2) represent in the usual fashion, just as in (1.1), a normalized distribution over the energies. It is natural to expect the distribution (1.2) in the case of maximally inelastic scattering, that is, when the electron stops after scattering. The correctness of this statement is confirmed by the work of Baraff[3,4].

We shall show that in scattering by optical phonons (with frequency ω_0), at low temperatures, when $u = \omega_0/T \gg 1$, there exists an interval of fields E in which a needle-like distribution is realized. We introduce the characteristic fields $E_0^{\pm}$, defined by the relations

$$eE_0^{\pm}\tau^{\pm} = p_0. \quad (1.3)$$

Here e is the electron charge, p_0 the momentum of an electron with energy $\epsilon = \omega_0$, and τ^+ and τ^- are the times of scattering with emission and absorption of a phonon, respectively. Since $\tau^- \sim e^u\tau^+ \gg \tau^+$, it is obvious that $E_0^- \ll E_0^+$.

Let us consider the motion of the electrons in momentum space (Fig. 1) after application of the field. For simplicity we assume that when E = 0 all the electrons are in a sphere of radius equal to the thermal momentum $p_T \ll p_0$. Inside the sphere p_0 (passive region) the electrons can only absorb phonons, and outside the sphere (active region) they can also emit phonons. When the field E is turned on, the electrons begin to move along the straight-line trajectories indicated in the figure. If $E \ll E_0^-$, then the majority of the electrons will absorb a phonon without reaching the active region. The absorption is accompanied by "instantaneous" emission; the compound scattering which is produced thereby[5,6] is almost elastic, and the steady-state distribution will have a small degree of anisotropy.

In the opposite case, $E \gg E_0^-$, most electrons will reach the boundary of the active region in a time $\tau_E = p_0/eE \ll \tau^-$ without being able to absorb a phonon. Before phonon emission can take place, the electrons will penetrate deep into the active

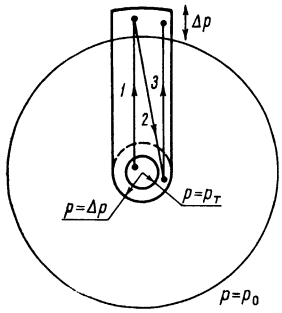

FIG. 1. Motion of electron in momentum space at $E_0^- \ll E \ll E_0^+$. 1 – Trajectory of initially accelerated electron; 2 – transition connected with phonon emission; 3 – trajectory in the second acceleration cycle.

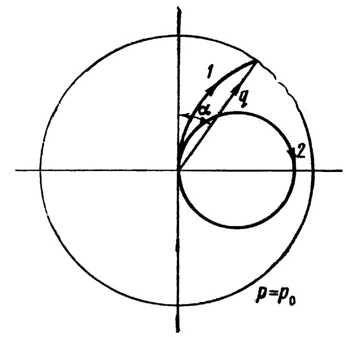

FIG. 2. Motion of electron in the presence of a magnetic field (for simplicity we put $\Delta p = 0$ and $T = 0$): 1 – Trajectory for $H < H_c$; 2 – Trajectory for $H > H_c$.

region by an amount $\Delta p \sim eE\tau^+$. If $E \gg E_0^+$, then $\Delta p \gg p_0$ and the phonon emission will occur at electron energies $\epsilon \gg \omega_0$. This means that scattering by optical phonons will be almost elastic and the distribution will have a small degree of anisotropy. However, if $E_0^- \ll E \ll E_0^+$, then $\Delta p \ll p_0$, and therefore the electron will return to a sphere of radius Δp after emitting a phonon, and then the acceleration cycle will be repeated. It is obvious that the distribution established thereby will differ noticeably from zero only in a narrow cylinder along the field, with height p_0 and radius $\Delta p \ll p_0$. Such a distribution can be approximately described by (1.2); an exception is the sphere Δp, corresponding to the interval $\epsilon \lesssim \Delta\epsilon$.

Electron acceleration cycles interrupted by phonon emission were already considered by Shockley[7]. Bray and Pinson indicate[8] that this should lead to an increase in the anisotropy of the distribution. The arguments presented above had as their purpose to emphasize that a needle-like distribution can actually be realized only when the collision times in the passive and active regions are essentially different and when the field is bounded not only from below but also from above.

It is easy to predict the change of the picture in the presence of a transverse magnetic field H (Fig. 2). The electron trajectory bends (for a parabolic band—along the arc of a circle of radius mcE/H); in this case, the trajectory reaches the limit of the active region when $H < H_C$ and closes in the passive region when $H \gg H_C$ (for a parabolic band $H_C = 2(c/v_0)E$, where $mv_0 = p_0$). In fields $H < H_C$ the distribution function differs noticeably from zero only near the bent trajectory, and can therefore be written in the form

$$f(\mathbf{p}) = f_0(\varepsilon)\cdot 4\pi\delta(\varphi)\,\delta(\cos\vartheta - \cos\vartheta(\varepsilon)), \qquad (1.4)$$

if the equation of the trajectory in polar coordinates $\epsilon, \vartheta, \varphi$ with polar axis parallel to E is

$$\varphi = 0, \quad \vartheta = \vartheta(\varepsilon). \qquad (1.5)$$

In fields $H > H_C$ the electron rotates along a trajectory that is closed in the passive region, until it absorbs a phonon. The resultant compound scattering "spills" the distribution in the entire passive region. In other words, a radical rearrangement of the distribution takes place when $H = H_C$.

2. In accordance with the qualitative consideration presented above, we write down the kinetic equation for the fields $E_0^- \ll E \ll E_0^+$ and $H < H_C$ in the form

$$F(\mathbf{p}) + B(\mathbf{p}) = 0 \qquad (p < p_0), \qquad (2.1)$$

$$F(\mathbf{p}) - A(\mathbf{p}) = 0 \qquad (p > p_0). \qquad (2.2)$$

Here F is the field term and A and B are the influx and outflow terms. Substituting (1.4) in F, we can readily show that

$$F(\mathbf{p}) = -\frac{1}{g(\varepsilon)}\frac{d}{d\varepsilon}\left(g(\varepsilon)f_0(\varepsilon)\dot{\varepsilon}_F(\varepsilon)\right) \times 4\pi\delta(\varphi)\,\delta(\cos\vartheta - \cos\vartheta(\varepsilon)). \qquad (2.3)$$

We have introduced here the speed of the representative point along the energy axis under the influence of the field:

$$\dot{\varepsilon}_F(\varepsilon) = eEv(\varepsilon)\cos\vartheta(\varepsilon), \qquad (2.4)$$

and $g(\epsilon)$ and $v(\epsilon)$ are the state density and the electron velocity.

The outflow term can be written in the form

$$A(\mathbf{p}) = f(\mathbf{p})/\tau_0(\varepsilon), \qquad (2.5)$$

where $\tau_0(\epsilon)$ is the lifetime (relative to phonon emission)[9]. From (2.3) and (2.5) we see that (2.2) is indeed satisfied by the angular dependence (1.4); cancelling it out, we obtain the equation

$$-\frac{1}{g}\frac{d}{d\varepsilon}(gf_0\dot{\varepsilon}_F) = \frac{f_0}{\tau_0}. \qquad (2.6)$$

Introducing the distribution on the energy axis

$$n(\varepsilon) = g(\varepsilon)f_0(\varepsilon) \qquad (2.7)$$

and the mean free path on it

$$\lambda_F(\varepsilon) = \dot{\varepsilon}_F(\varepsilon)\tau_0(\varepsilon), \qquad (2.8)$$

we obtain from (2.6)

$$n(\varepsilon)=C\frac{1}{v(\varepsilon)\cos\vartheta(\varepsilon)}\exp\left\{-\int_{\omega_0}\frac{d\varepsilon'}{\lambda_F(\varepsilon')}\right\},\ \varepsilon>\omega_0. \quad (2.9)$$

In accord with the meaning of the employed approximation, we have

$$B(\mathbf{p})=I_0\delta(\mathbf{p}), \quad (2.10)$$

where I_0 is the number of electrons returning to the passive region per unit time:

$$I_0=\int_{p>p_0}(d\mathbf{p})A(\mathbf{p}). \quad (2.11)$$

Calculating I_0 with the aid of (2.5) and (2.9), we get $I_0 = CeE$. Substituting now (2.10) in (2.1) and stipulating continuity of $n(\epsilon)$ at $\epsilon = \omega_0$, we obtain

$$n(\varepsilon)=C/v(\varepsilon)\cos\vartheta(\varepsilon),\qquad \varepsilon<\omega_0. \quad (2.12)$$

If $n(\epsilon)$ is singular when $\epsilon\to 0$, then the solution is meaningful only if the singularity is integrable, for only in this case is there a small number of electrons in the disregarded region $\epsilon \lesssim \Delta\epsilon$.

The constant C is obtained from the density normalization condition

$$\int_0^\infty d\varepsilon\, n(\varepsilon)=n. \quad (2.13)$$

Since, by assumption, the number of electrons in the active region is small, we can integrate up to ω_0 in (2.13). Substituting here (2.12) and using (2.4), we get

$$C=n/\tau_F eE, \quad (2.14)$$

where

$$\tau_F=\int_0^{\omega_0}\frac{d\varepsilon}{\dot\varepsilon_F(\varepsilon)} \quad (2.15)$$

is the time in which the electron reaches the limit of the active region.

The solutions for $n(\epsilon)$ are physically almost obvious: (2.9) is the equation for "survival" on the energy axis, and (2.12) is determined simply by the time which the electron spends in the interval $d\epsilon$ of the energy axis while moving with velocity $\dot\epsilon_F$ under the influence of the fields.

For a parabolic band, the derivation can be carried through to conclusion. We introduce the dimensionless quantities

$$y=\varepsilon/\omega_0,\qquad h=H/H_c. \quad (2.16)$$

We then find

$$\tau_F\equiv\tau_{EH}=\tau_E\frac{\arcsin h}{h},\qquad \tau_E=\frac{p_0}{eE}. \quad (2.17)$$

It is important to note that turning on the magnetic field, as seen from (2.17), changes the time τ_F little, and therefore the criteria obtained for E when $H = 0$ are sufficient also when $H \neq 0$.

The lifetime $\tau_0(\epsilon)$ can be expanded for ϵ close to ω_0; we then obtain the same result for deformation and polarization optical scattering; this result is conveniently represented in the form

$$\tau_0(\varepsilon)=\tau_0(y-1)^{-1/2}. \quad (2.18)$$

Substituting this in (2.9) we obtain the distribution

$$n(\varepsilon)=\frac{n}{2\omega_0}\frac{h}{\arcsin h}\times\begin{cases}[y(1-h^2y)]^{-1/2}, & y<1\\ (1-h^2)^{-1/2}\exp\left[-\frac{1}{w}\frac{2}{3}(y-1)^{3/2}\right], & y>1\end{cases}, \quad (2.19)$$

where the small parameter is

$$w=\frac{\tau_0}{\tau_E}\sqrt{1-h^2}=\frac{eEv_0\tau_0}{\omega_0}\sqrt{1-h^2}. \quad (2.20)$$

From (2.19) we find that the relative number of electrons in the active region is $w^{2/3}$, that is, it is really small. When $H = 0$, (2.19) coincides with the result of Gunn[10].

3. The flow of current in the regime considered above is accompanied by generation of optical phonons, which are spontaneously emitted by the electrons in the active region. Unlike the generation of phonons with distributions with small or moderate anisotropy[11], in our case all the produced phonons are practically monochromatic with respect to magnitude and direction of the momentum **q**, which is equal to p_0 and makes an angle $\alpha = \sin^{-1} h$ with the direction of **E**. The number of phonons generated per unit time is I_0. Using (2.14) we find that $I_0 = n/\tau_F$, i.e., the frequency of generation of phonons by each electron is simply equal to the acceleration-cycle repetition frequency. Of course, it must be assumed that the number of produced phonons is sufficiently small to be able to neglect scattering by them. With increasing H, the phonon flux is deflected away from the direction of **E**, with practically no change in intensity, since τ_F is practically independent of H. When $H = H_c$ we have $\alpha = \pi/2$. When $H > H_c$ the distribution "spills" and the generation of monochromatic phonons ceases.

With the aid of (2.19) we can calculate the components of the current (longitudinal-dissipative and transverse-Hall currents):

$$j_\parallel=j_0\frac{h}{\arcsin h},\qquad j_\perp=j_0\left(\frac{1}{h}-\frac{\sqrt{1-h^2}}{\arcsin h}\right), \quad (3.1)$$

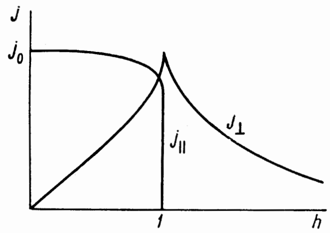

FIG. 3. Longitudinal and transverse currents (relative to E) vs. the magnetic field at a fixed electric field.

where $g_0 = env_0/2$ is the saturation current flowing when $H = 0$.

Formulas (3.1) pertain only to the case $H < H_c$. When $H > H_c$ it is essential to allow for phonon absorption, without which dissipative current is utterly impossible. If we nevertheless neglect the currents containing the factor e^{-u}, then we can assume that when $H > H_c$ we get

$$j_{\parallel} = 0, \quad j_{\perp} = j_0/h. \tag{3.2}$$

Thus, the change in distribution at $H = H_c$ leads at this point to a maximum of the Hall current and to a sharp decrease of the dissipative current (Fig. 3).

4. Let us consider the conditions under which the model investigated above can be realized experimentally. It must be borne in mind first that, in low-temperature experiments, scattering in the passive region is determined usually not by optical phonons, but by acoustic phonons or impurities. Accordingly, the meaning of τ^- should be modified.

Let us discuss first the experiments (see [8]) carried out with heavy holes, $m = 0.3\, m_0$, in germanium at nitrogen temperatures and in fields up to 2,000 V/cm. At $E = 0$ the observed mobility $\mu = 5 \times 10^4$ cm^2/V-sec corresponds to $\tau^- = 8 \times 10^{-12}$ sec. Under the same conditions $\tau^+ \sim \tau_0 = 10^{-12}$ sec. Assuming that $\omega_0 = 6 \times 10^{13}$ sec^{-1} (400°K), we obtain $E_0^- = 400$ V/cm and $E_0^+ = 3{,}000$ V/cm. Thus, the field E actually lies in the interval between E_0^- and E_0^+, but the gap between them is not large enough, and this is precisely why the anisotropy did not reach in these experiments the limiting value, although it was quite noticeable.

The conditions will be more favorable at hydrogen temperatures and for deep acceptors which are farther than ω_0 from the bottom of the band. These will be "frozen out" and scattering by the ionized impurities will not take place. Scattering by acoustic phonons at $T = 20°$, according to [12], gives $\mu = 5 \times 10^5$ cm^2/V-sec, that is, $E_0^- = 40$ V/cm. On the other hand, impact ionization will likewise be insignificant, for in the region $\epsilon < \omega_0$ the number of electrons is small. A slight carrier density, $n \sim 10^{12}$–10^{13} cm^{-3}, can be produced by infrared illumination. Under these conditions, the unaccounted for interelectron collisions correspond to $\tau^{ee} \sim 10^{-10}$ sec and are insignificant.

The period of phonon generation in fields $E = 500$ V/cm is $\tau_E \sim 10^{-12}$ sec, which is comparable with the lifetime of the optical phonon, $\tau_F \sim 10^{-12}$ sec, as estimated from the dispersion of infrared absorption [11]. The singularities of galvanomagnetic effects and the vanishing of generation in this case correspond to $H_c = 5$ kOe. We note also that quantization in the magnetic field is insignificant here, since $H = H_c$ corresponds to a cyclotron frequency $\omega_H = 0.5 \times 10^{12}$ sec^{-1}, whereas the average electron energy is $\epsilon \sim \omega_0$ and is much larger than ω_H.

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