PECULIARITIES OF NONLINEAR FERROMAGNETIC RESONANCE

Ya. A. MONOSOV

Institute of Radio Engineering and Electronics, Academy of Sciences, U.S.S.R.

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A theory of NFMR is proposed, which takes account of the inertial mechanism of change of the longitudinal component of the macroscopic magnetization vector. Instability of NFMR is revealed,aand the universality of this phenomenon is established. Immediately beyond the excitation threshold, the instability has harmonic character, with period given by formula (16). Because of magnetostriction, the instability is usually accompanied by lattice oscillations, which give rise to scattering of parametrically excited spin waves. This in turn leads to electromagnetic microwave radiation from the crystal, predominantly of the nature of noise. The results obtained permit satisfactory explanation of a large number of "anomalous" experimental data on NFMR, obtained in the last few years (for example, in references [1-5]).

INTRODUCTION

IN the literature devoted to the experimental study of nonlinear processes in ferrites at microwave frequencies (in a strong radiofrequency field), there have been described over the last few years (cf., for example, [1-6]) "anomalous" results, not explicable within the framework of the existing theory of nonlinear ferromagnetic resonance (NFMR).[7-12]. The following anomalies may be considered the most important:

1) The presence of high-frequency (in the band 0.1 to 10 Mc/sec) oscillations at the fundamental and subsidiary resonances.[1, 2, 5, 6]

2) An unusual form of the characteristics of a ferrite microwave parametric amplifier—an unexpectedly large noise level, and limitation of the amplification with increase of the pumping power.[3, 5, 13]

Besides these, there are a multitude of "smaller" anomalies; for example, dependence of the absorption level and of the high-frequency oscillations on the method of clamping the crystal,[2] discrepancy between theory and experiment in the suppression of subsidiary resonance,[4, 14] noises in the microwave power limiters, and others.

It is significant that the results of the observations are practically independent of the method of connection between the crystal specimen and the cavity in which it is located; apparently, therefore, they are due to processes that go on inside the crystal. It is important to emphasize also that they are observed in the nonlinear, beyond-threshold region, that is, when the radiofrequency field exceeds the threshold for parametric excitation of spin oscillations in the crystal.

From the theory of nonlinear oscillations it is known that phenomena similar to those described in 1) (so-called automodulation) occur in nonlinear systems with an inertial nonlinearity of one of the parameters.[15, 16] In a ferromagnetic crystal, such inertiality can be detected in the change of the longitudinal component of the macroscopic magnetization of the specimen, when the relaxation of the magnetization occurs in the Bloch manner.

We shall try to show that by taking account of this circumstance and of others that accompany it, it is possible to explain at least the majority of the above-mentioned anomalies in the experiments on NFMR.

1. FORMULATION OF THE PROBLEM

The processes being described permit a quasi-classical interpretation, for example within the framework of the method developed by A. Akhiezer and others.[17] The system of equations has the following form:

$$\dot{\mathbf{m}} + \operatorname{div}(\mathbf{M}\dot{u}) = |\gamma|\,[\mathbf{M}\mathbf{H}_e] - \alpha(\mathbf{i}_x, \mathbf{i}_y, \mathbf{i}_z)(\mathbf{M}_0 - \mathbf{M}), \quad (1)^*$$

$^*[\mathbf{M}\mathbf{H}_e] \equiv \mathbf{M} \times \mathbf{H}_e$.

$$\ddot{u} - 2\alpha_a \dot{u} = f / \rho, \tag{2}$$

$$\operatorname{curl} \mathbf{h} = 0, \quad \operatorname{div} \mathbf{b} = 0, \tag{3}$$

$$\mathcal{H} = \int_v \left\{ \frac{1}{2} I \frac{\partial^2 M}{\partial x_i^2} + \frac{1}{8\pi} H^2 + \frac{1}{2} \rho \dot{u}^2 + \frac{1}{2} \lambda_{iklm} u_{ik} u_{lm} + \gamma_{ik}(\mathbf{M}) u_{ik} \right\} dv, \tag{4}$$

Here

$$\mathbf{M} = \mathbf{M}_0 + \mathbf{m}, \quad \mathbf{b} = \mathbf{h} + 4\pi \mathbf{m}, \quad \mathbf{H} = H_0 \mathbf{i}_z + \mathbf{h},$$

$\mathbf{M}_0$ and $\mathbf{m}$ are the constant and alternating components of the vector magnetization, H_0 and $\mathbf{h}$ are the constant (directed along the axis Oz) and alternating components of the magnetic field, h_0 is the magnetic pumping field, u is the elastic displacement, $\mathcal{H}$ is the Hamiltonian of the system. I is a constant proportional to the exchange integral, $\mathbf{H}_e = \partial \mathcal{H} / \partial \mathbf{M} |_{B = \mathrm{const}}$ is the equivalent magnetic field, f is the generalized force, u_{ik} is the strain tensor, ρ is the density of the material, λ_{iklm} is the elastic-constant tensor; α and α_a are the damping constants of the spin oscillations and of the elastic oscillations of the crystal, a_0 is the lattice constant, $\sin\theta = (k_x^2 + k_y^2)/k^2$; $\mathbf{k}$ and $\mathbf{k}_a$ are the wave vectors of the spin and the elastic oscillations.

We shall suppose that the medium is isotropic with respect to elastic and magnetoelastic properties; that is,

$$\gamma_{ik} = \gamma_0 M^2 \delta_{ik} + \gamma_1 M_i M_k,$$

where γ_0 and γ_1 are the magnetostriction constants and δ is the Kronecker symbol.

For simplicity we shall restrict ourselves to the interesting special case of "longitudinal pumping," in which the magnetic microwave field is polarized along the direction of the basic magnetizing field, and spin waves are excited parametrically at angle $\theta = \pi/2$.[9] The results obtained can without difficulty be generalized, for example, to the case of "transverse pumping." Furthermore, we shall disregard interaction between spin and elastic oscillations (hypersonic) of the same frequency. Then instead of the system of equations (1) to (4) we get

$$\ddot{m}_x + \left\{ 2\alpha - c \frac{\dot{\omega}_s}{\omega_H'} - (1 - c) \frac{\dot{m}_z}{M_0} \right\} \dot{m}_x - \omega_k^2 \left\{ 1 + (1 + c) \frac{\omega_s}{\omega_H'} + (1 - c) \frac{m_z - M_0}{M_0} \right\} m_x = 0, \tag{5}$$

$$\dot{m}_z + 2\alpha m_z - 4\pi |\gamma| m_x m_y \sin^2\theta = M_0 \frac{\partial \dot{u}_y}{\partial y}, \tag{6}$$

$$\ddot{u}_y - v^2 \frac{\partial^2 u_y}{\partial y^2} + 2\alpha_a \dot{u}_y = \xi_0 \frac{\partial m_z}{\partial y}, \tag{7}$$

where

$$\omega_H' = |\gamma| [H_0 + H_{\mathrm{exch}} (a_0 k)^2], \quad \omega_k^2 = \omega_H' (\omega_H' + \omega_M \sin^2\theta),$$

$$\omega_M = 4\pi |\gamma| M_0, \quad c = (\omega_H' / \omega_k)^2,$$

$$\omega_s = |\gamma| h_0 \cos(2\omega_0 t + \varphi_s), \quad \xi_0 = (\gamma_0 M_0 + H_0) / \rho.$$

Here it is assumed, without loss of generality, that $k_x = k_{xa} = 0$.

2. AUTOMODULATION IN NONLINEAR FERROMAGNETIC RESONANCE

For easy visualization we consider first the the case $\gamma_0 = 0$, $u = 0$; then magnetostriction is absent in the crystal. There remain[1)] only Eqs. (5) and the homogeneous Eq. (6). Such a system of equations, in its important features, has been studied in the theory of nonlinear oscillations (cf., for example, [15, 16]). In particular, it is shown that an oscillatory solution of the equations in the form

$$m_x = m_{x0} \cos(\omega_0 t + \varphi_0)$$

(where m_{x0} and φ_0 are constants) is, in general, unstable. Instability occurs under the condition

$$\tau_\| / \tau_\perp \neq 0, \tag{8}$$

where for our case

$$\tau_\perp = (\alpha g)^{-1}, \quad \tau_\| = (2\alpha)^{-1}, \quad g = P / P_{\mathrm{cr}} - 1. \tag{9}$$

Here P and P_{cr} are the pumping power and its threshold value.

The larger the ratio (8), the more pronounced the instability is. The value of the parameter g for which instability occurs is smaller, the better the inequality

$$\alpha_H / \beta_H < 1 \tag{10}$$

is fulfilled, where

$$\alpha_H = \frac{1}{8} \alpha \frac{(1 - c)^2}{1 + c} c, \quad \beta_H = \frac{1}{16} \omega_k \frac{(1 - c)^2}{1 + c} (1 - 3c)$$

are coefficients for the active and reactive nonlinearities in the equations (5); they take account of the dissipative and disordering mechanisms for establishment of oscillations in the parametrically excited spin system. For a ferromagnet, this ratio is usually small; thus in our case $|\alpha_H / \beta_H| \cong 2\alpha / \omega_k$.

1)For comparison it is interesting to note that in [7-10] the problem is reduced to the study of equations of the type (5); instead of Eq. (6), only a steady solution in the form $m_z = M_0' - (m_x^2 + m_y^2)/2M_0$ is considered. This automatically excludes additional degrees of freedom from consideration.

Inasmuch as conditions (8) and (10) are usually satisfied in the case of NFMR, it is possible, apparently, to speak of instability of the latter as one of its characteristic, natural properties.

Instability of the stationary state and apparent instability at infinity suppose the existence in the phase space of a limiting cycle, that is a region of automodulation. For $g \ll 1$, the instability should obviously have an approximately harmonic character, and this facilitates the problem of explaining the basic features of this region.

We shall seek a solution of Eq. (5) and (6) in the form

$$m_x = \sum_n m_{xn} \cos(\omega_n t + \varphi_n),$$

$$m_y = \sum_n m_{yn} \sin(\omega_n t + \varphi_n + \Delta\varphi_n), \tag{11}$$

$$m_z = M_0' - \frac{m_x^2 + m_y^2}{2M_0} - m_{z_0} \cos(\Omega t + \varphi_z),$$

where

$$\omega_n = \omega_0 + n\Omega, \quad n = 0,\ 1,\ -1;$$

m_{xn}, m_{yn}, φ_n, and $\Delta\varphi_n$ are constant quantities, and Ω is the automodulation frequency. As a result we obtain the following relations:

$$\frac{\Omega}{\alpha} = \frac{1{,}6}{(1+c)^2} g^{1/2},$$

$$\left(\frac{m_{x0}}{2M_0}\right)^2 = \frac{1}{1-c^2}\frac{\alpha}{\omega_k} g^{1/2},$$

$$\frac{m_{z0}}{2M_0} = \frac{c(1+4c)}{(1+c)^3}\left(\frac{m_{x0}}{2M_0}\right)^2,$$

$$M_0' - M_0 = \frac{4c}{1+c}\left(\frac{m_{x0}}{2M_0}\right)^2, \tag{12}$$

$$\tan \Delta\varphi_n = -\frac{\alpha}{\omega_n}\frac{1-c}{1+c},$$

$$\tan(2\varphi_0 - \varphi_5) = \frac{4(1+c)^2}{(1-c)(1-3c)\sqrt{g}}.$$

Thus in NFMR there arises a complicated spectrum, consisting of equidistant lines in the neighborhood of frequency ω_0, with distance Ω between them, and a line corresponding to Ω. Here Ω depends greatly on the pump power.

A peculiarity of the solution obtained is the indeterminacy of the phase φ_z and accordingly of $\varphi_{\pm 1}$.

3. NONLINEAR MAGNETOELASTIC INTERACTION

For a number of magnetic crystals (among them yttrium garnet), taking Eq. (7) into account leads to nontrivial consequences. It is easy to convince oneself of this by considering that the expressions (12) change slightly in the case $\gamma_0 \neq 0$. This is sufficiently exact if $(\gamma_0\sigma)^2 \ll 1$, where

$$\sigma = 2\frac{M_0}{H_0}\frac{\Omega_M}{[(\Omega_a - \Omega)^2 + (2\alpha_a\Omega)^2]^{1/2}}, \quad \Omega_M = \xi_0 H_0 k_a^2.$$

Then by assuming coordinate-dependent terms in the solutions of (6) and (7) in the form

$$u_y = u_0 \cos(\Omega t + \varphi_z + \varphi_a) \sin k_a y,$$
$$m_z = m_{z0} \cos(\Omega t + \varphi_z) \cos k_a y,$$

it is not difficult to obtain for $(\Omega_a - \Omega)/\alpha_a \gg 1$

$$\frac{u_0 k_a}{2\pi} \cong \frac{\delta l}{l} \cong \frac{1}{2\pi}\frac{m_{z0}}{2M_0}\sigma. \tag{13}$$

Here $\delta l/l$ is the amplitude of the relative elongation of the crystal, and Ω_a is a frequency of acoustical resonance of the crystal specimen.

If we take $\gamma_0 = 10^2$,[13] $\Delta H_k = 0.5$ Oe, $4\pi M_0 = 2 \times 10^3$ G, $H_0 = 10^3$ Oe, $\Omega_a = 1$ Mc/sec, $k_a = 10^2$ cm^{-1}, and $\rho = 5$ g/cm^2, we have $\delta l/l \cong 10^{-6}$. As we see, nonlinear magnetoelastic interaction in NFMR in a crystal with appreciable magnetostriction leads to excitation of quite intense elastic oscillations even when the automodulation frequency does not coincide with a resonance frequency of elastic oscillations. This in its turn significantly changes the properties of the spin-oscillation system, tuned to resonance.

In fact, a change of the longitudinal component of the magnetization of the crystal in time with a small $(\Omega/\omega_0 \ll 1)$ frequency and in space with wavelength $2\pi/k_\alpha$, and of appreciable amplitude $(m_{z0}/\Delta H_k \sim 1)$, is equivalent to the existence of a dynamic magnetic inhomogeneity in the crystal, a sort of magnetic diffraction grating whose lines oscillate periodically. Such an inhomogeneity should have as an effect the scattering of spin waves.

On the basis of these preliminary remarks, we proceed to seek a solution of the system of equations (5) to (7), describing a spatially inhomogeneous automodulation for $g \ll 1$ in the form

$$m_{x,y} = \sum_{n,k} m_{x,y\,n,k} \cos(\omega_n t + \varphi_{n\,x,y}) \cos k_n y,$$

$$m_z = M_0' - \frac{m_x^2 + m_y^2}{2M_0} - \sum_a m_{za} \cos(\Omega t + \varphi_z) \cos k_a y,$$

$$u_y = \sum_\alpha u_{ka} \cos(\Omega t + \varphi_z + \varphi_a) \sin k_a y. \tag{14}$$

In the solution we limit ourselves to the case $\gamma_0\sigma \ll 1$. As a result, on the assumption that the

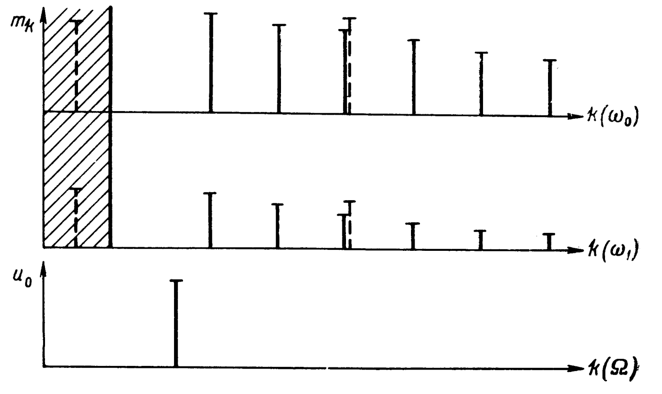

FIG. 1. Scheme illustrating the solution of Eq. (15). The spectrum of spin oscillations is shown at frequencies ω_0 and $\omega_0 + \Omega$, and of the elastic oscillation at frequency Ω. The transformed spectra (for the first lines only) are shown dashed. It is evident that when the wave numbers of the spin and elastic oscillations are close to each other, there is displacement of the spectrum in the region of interaction of electromagnetic waves with the crystal (crosshatched).

change of $m_k(k)$ is slight, we obtain the relations (12) and (13) and an additional condition to determine the change of the wave number k:

$$\sum_l \zeta_{ln} \frac{1}{\delta(k_l - k_r)} \cos \varkappa_{ln} = \sum_a \nu \cos \varkappa_a \sum_r \zeta_{rn'} \cos \varkappa_{rn'}, \quad (15)$$

where k_r are the wave numbers of the parametrically excited spin oscillations, $\kappa_{in} = k_{in}y$, ζ_{rn} is the normalized amplitude of a spin oscillation with wave number k_{rn}: $\zeta_{rn} = m_{xnr}/m_{xn}(k_{r\,min})$, ζ_{ln} is the normalized amplitude of a spin oscillation with altered wave number k_{ln}, and $\nu = m_{za}/m_z(k_{a\,min})$.

Condition (15) can be made fully visualizable if we take into account excitation of only one elastic degree of freedom; that is, instead of the sum over a keep only one term. Such a procedure, of course, is justified when the excitation occurs at a frequency close to an elastic resonance; it is used here only for visualizability. In this case the solution of Eq. (15) is illustrated schematically in Fig. 1.

The most interesting feature of the solution obtained is the large change of the wave vector of the spin oscillations by scattering on the magnetic inhomogeneity $m_z(y)$ and the transformation of a group of spin waves (with $k \sim k_a$) into electromagnetic. Such a transformation should be accompanied by electromagnetic radiation from the crystal. This effect is illustrated also by the ω vs. k diagram (Fig. 2). It can be seen that interaction of a magnon (m) with a photon (pht), directly, is impossible because it would violate the law of conservation of momentum. It becomes possible only when the surplus of momentum is given off by excitation of a phonon (phn).

4. DISCUSSION OF RESULTS

The analysis presented for nonlinear processes in a ferromagnetic crystal, with allowance for the inertial change of the longitudinal component of the macroscopic magnetization, made it possible to establish the phenomenon of instability of NFMR. As is known, inertialess nonlinearity insures, near resonance, a bi-stable state (with large and small amplitudes of oscillation). The above-mentioned inertial change of the magnetization leads to disorganization of the spin-oscillation system, accompanied by periodic jumps from one state to the other. The period of these jumps is large in comparison with the period of the microwave oscillations, and near the excitation threshold it is approximately

$$T = 2\pi\tau (P/P_{cr} - 1)^{-1/2} \quad (16)$$

(where τ is the time of transverse relaxation, and where P and P_{cr} are the pump power and its threshold value).

Very important is the indication by the theory of the universality of this phenomenon in relation to ferromagnetic crystals. It is often easier to observe instability of NFMR than the "subsidiary" absorption effect, and such a method can be recommended for determination of the characteristics of NFMR. It is known that in NFMR many de-

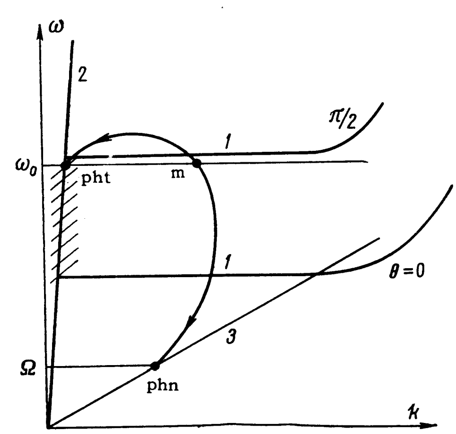

FIG. 2. Energy spectrum of a ferromagnetic crystal: 1, spin curves; 2, electromagnetic straight line (in vacuum); 3, elastic straight line. The region of interaction of electromagnetic waves with the crystal is crosshatched. Shown schematically is the process of interaction of a magnon and a photon, with transfer of the surplus momentum to a phonon: $\Delta k = k_m - k_{pht} = k_{phn}$.

grees of freedom of the spin-oscillation system are excited; their number, at the beginning, increases greatly with increase of the pump power. The effect of this peculiarity on the results obtained above is negligible only in the case $\Omega\tau_{\perp} \gg 1$; this condition, however, is usually not satisfied. Therefore the inertial disorganization of the spin-oscillation system should lead to excitation of additional spin oscillations, which also will make a contribution to the instability of the NFMR. Since the high-frequency oscillations of the magnetization that thus arise are incoherent (as was indicated at the end of Sec. 2), the instability will have noise character.

Naturally, the same character will be possessed by microwave electromagnetic radiation that arises because of scattering of the spin oscillations of automodulational oscillations of the lattice.[6] Since the threshold for parametric excitation of spin oscillations is lower than that for electromagnetic oscillations in ferrite amplifiers,[18] this radiation inevitably occurs and is received as additional (nonthermal) amplifier noise. It is not difficult to show that automodulational high-frequency oscillations of the magnetization cause limitation ("freezing") of the amplification in a ferrite amplifier.

The agreement, at least in important features, of the deductions from the present theory and of a large number of experimental data on NFMR[2)] can evidently be regarded as evidence of the usefulness of the Bloch relaxation mechanism in ferromagnets. It should be mentioned that in the literature there have also been earlier indications of the known virtues of this mechanism (for example,[17, 20]).

In the investigation of nonlinear magnetoelastic interaction, we restricted ourselves for simplicity to the case $\gamma_0\sigma \ll 1$, that is to the case of relatively small coupling. Without going into details, we mention that in the other limiting case, $\gamma_0\sigma \gg 1$, there is a significant dependence of the basic NFMR characteristics, among them the subsidiary absorption, on the elastic and magnetoelastic parameters of the crystal.

2)Among them, ones stimulated by the present theory, for example the observation of high-frequency elastic automodulational oscillations [19].

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