APPEARANCE OF DOMAINS IN "MANY-VALLEY" SEMICONDUCTORS DURING THE PASSAGE OF STRONG CURRENTS

Z. S. GRIBNIKOV, V. A. KOCHELAP, and É. I. RASHBA

Institute for Semiconductors, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor January 31, 1966

J. Exptl. Theoret. Phys. (U.S.S.R.) 51, 266–280 (July, 1966)

When a current passes through a many-valley semiconducting plate, the densities of the electrons in all the valleys differs from the equilibrium values because of the different inclinations of the valleys to the faces of the plate; the effect appears clearly if the intervalley scattering lifetime is considerably longer than the usual relaxation time. The case is considered of strong electric fields E, for which the drift length, L_E, before the intervalley scattering takes place is considerably longer than the diffusion length L. If the plate thickness is $2d \ll L_E$, then the plate splits up into layers (domains) which are parallel to the faces of the plate. Each domain contains, as a rule, only the electrons which belong to one valley; the sequential order of the domains is governed by the inclination of the valleys to the faces of the plate, while the number of domains and their thickness are governed by the ratio of the rates of intervalley scattering in the interior and on the surface. The number of domains is equal to or less than the number of valleys; the electrons not included in the domains are concentrated at the plate surfaces in narrow layers of thickness $\sim L^2/L_E$. If $2d \gg L_E$, then the regions in which the density differs markedly from the equilibrium value extend to a distance of the order of L_E from both surfaces; as a rule, these regions consist of alternating sections of a smooth or abrupt variation in the density, the widths of the sections being respectively of the order of L_E and L^2/L_E. The appearance of the domains alters the electrical conductivity of the plate, gives rise to strong transverse fields which vary rapidly at domain boundaries, etc.

INTRODUCTION

THE electron energy bands of many semiconductors and semimetals (Ge, Si, Bi, and others) have a many-valley structure, i.e., the electron spectrum has many energy minima in momentum space. If the intervalley scattering is so strong that it is comparable with the intravalley scattering, the former alters the values and the temperature dependences of the electrical conductivity and of the galvanomagnetic coefficients; the intervalley scattering may be determined by investigating these effects. However, in many cases, particularly at low temperatures, the intervalley scattering is much weaker than the intravalley scattering and has practically no effect on the transport coefficients. Therefore, if the intervalley relaxation time τ is long, other methods have to be used to measure τ.

These methods are based on the fact that, in the presence of several groups of electrons characterized by a long time τ needed to establish equilibrium between various paths, distributions departing strongly from equilibrium may be established so that the density of electrons in each of the groups will differ considerably from the equilibrium value although the sample as a whole will remain quasi-neutral. This gives rise to a strong acoustoelectric effect,[1,2] to amplification of ultrasound,[3] etc. Since the value of $L = \sqrt{D\tau}$, which represents the diffusion length (D is the diffusion coefficient) is then also large, the departures from the equilibrium distribution should extend over large regions of a semiconductor and should lead to characteristic size effects. The existence of the size effects in the electrical conductivity of many-valley crystals was mentioned in [4] and their existence in the galvanomagnetic phenomena, in [5,6]; the magnetoresistance of Bi in the pre-pinch region was interpreted in [6] on this basis. The photomagnetic effect in many-valley crystals was recently studied in [7].

The case of weak electric fields, when the nonequilibrium corrections to the density are sufficiently small, was considered in [4-6]. However, the estimates given in [4] show that the criteria for the validity of such an approach may not be obeyed even in fields of the order of several V/cm.

Therefore, in the present investigation we shall consider the opposite limiting case of strong fields which disturb strongly the carrier distribution in the valleys practically throughout the whole sample.

The main results of this investigation are as follows. When a current giving rise to a strong electric field passes through a many-valley unipolar semiconducting plate, the electron currents in each of the valleys are directed at an angle to the electric field. The conditions of continuity of these currents in the interior and on the surface give rise to the splitting of the plate into several layers (domains) with boundaries parallel to the surfaces of the plate. Each domain contains, as a rule, only those electrons which belong to one valley, their number being such as to ensure the electrical neutrality. The number of domains is equal to or less than the number of valleys; if there are fewer domains than valleys, the electrons not included in the domains are always concentrated in a thin layer next to one of the surfaces of the plate and the surface electron density is greater than the equilibrium value. The sequential order of the domains is governed by the angles that the principal axes of the electrical conductivity tensors, corresponding to the various valleys, make with the surfaces of the plate. The number of domains and the positions of their boundaries depend on the ratio of the intervalley scattering rates in the interior and on the surfaces of the plate.

The splitting of a crystal, in a strong electric field, into domains with anisotropic electrical conductivity leads to the appearance of effects characteristic of anisotropic semiconductors. As shown in [8-10], narrow strongly enriched layers appear on one of the surfaces in plates made of anisotropic intrinsic semiconductors. Similar enriched layers, but at both surfaces, should appear in plates made of many-valley ambipolar semiconductors, but we shall discuss this in a separate communication.

1. BASIC EQUATIONS

We shall consider only those many-valley semiconductors in which the intervalley scattering time τ is the longest relaxation time, much longer than the characteristic times corresponding to all the intravalley relaxation processes (both in momentum and energy). Then, the electrons in each valley may be ascribed the mobility and diffusion coefficient tensors ($u_{ij}^{(\alpha)}$ and $D_{ij}^{(\alpha)}$); α is the number of the valley. We shall assume that the strong electric fields considered here, which alter markedly the carrier densities in the valleys, do not alter the energy distribution. The equilibrium electron densities in all the valleys are identical and equal to n_0. Though the equilibrium is disturbed, the quasi-neutrality condition is still satisfied:

$$\sum_{\alpha} n_\alpha = \nu n_0 = N, \tag{1}$$

where ν is the number of valleys.

When an electric current flows through an infinite plate of thickness 2d $(-d \le y \le d)$, the following components of the electron currents are important when $E_z = 0$

$$j_x^{(\alpha)} = -D_{xx}^{(\alpha)} n_\alpha E_x - D_{xy}^{(\alpha)} (dn_\alpha / dy + n_\alpha E_y), \tag{2}$$

$$j_y^{(\alpha)} = -D_{yy}^{(\alpha)} (dn_\alpha / dy + n_\alpha E_y) - D_{xy}^{(\alpha)} n_\alpha E_x, \tag{3}$$

where $\mathbf{E}$ is the electric field, measured in units of kT/e. For simplicity, the electrons are assumed to be nondegenerate; all the main conclusions of the present paper remain qualitatively valid in the presence of degeneracy. In a finite sample of rectangular parallelepiped shape, the condition $E_z = 0$ is satisfied approximately by the following ratio of dimensions: $d_z \gg d_x \gg d_y = 2d$. From the condition

$$j_y = \sum_{\alpha} j_y^{(\alpha)} = 0 \tag{4}$$

it follows that

$$E_y = -\left[E_x \sum_{\alpha} D_{xy}^{(\alpha)} n_\alpha + \sum_{\alpha} D_{yy}^{(\alpha)} \frac{dn_\alpha}{dy}\right] \Big/ \sum_{\alpha} D_{yy}^{(\alpha)} n_\alpha. \tag{5}$$

Substituting Eq. (5) for E_y in Eq. (3) and then substituting $j_y^{(\alpha)}$ into the continuity equation

$$\frac{dj_y^{(\alpha)}}{dy} = -\sum_{\beta} \frac{n_\alpha - n_\beta}{\tau_{\alpha\beta}}, \tag{6}$$

where $\tau_{\alpha\beta} = \tau_{\beta\alpha}$ is the relaxation time for transitions between the valleys α and β, we obtain a system of diffusion equations for the densities $n_\alpha(y)$. These equations should be supplemented by the boundary conditions

$$j_y^{(\alpha)}(\pm d) = \pm \sum_{\beta} s_{\alpha\beta}^{\pm} (n_\alpha^{\pm} - n_\beta^{\pm}), \tag{7}$$

where $s_{\alpha\beta}^{\pm} = s_{\beta\alpha}^{\pm}$ are the rates of the intervalley scattering at the surfaces $y = \pm d$, and $n_\alpha^{\pm} = n_\alpha(\pm d)$.

2. TWO-VALLEY SEMICONDUCTOR

As an example, we shall consider a two-valley isotropic semiconductor, cut as shown in Fig. 1. We shall use the notation

$$D = \frac{kT}{e} u \equiv D_{xx}^{(1,2)} = D_{yy}^{(1,2)},$$

$$a \equiv \frac{D_{xy}^{(1)}}{D} = -\frac{D_{xy}^{(2)}}{D} \quad f = \frac{n_1 - n_0}{n_0},$$

$$\tau = \tau_{12}/2, \quad s^{\pm} = 2s_{12}. \tag{8}$$

Then, instead of Eq. (5), we have

$$E_y = -afE_x, \tag{9}$$

and Eqs. (6) and (7) assume the form

$$\frac{d^2f}{d\eta^2} - \mathscr{E}\frac{d(f^2)}{d\eta} - f = 0, \tag{10}$$

$$\left[\frac{df}{d\eta} + \mathscr{E}(1 - f^2) \pm S^{\pm}f\right]_{\eta=\pm\delta} = 0, \tag{11}$$

where

$$\mathscr{E} = E_x/E_L, \quad E_L = 1/aL, \quad L = \sqrt{D\tau},$$

$$S^{\pm} = s^{\pm}L/D. \qquad \eta = y/L, \quad \delta = d/L, \tag{12}$$

Integrating Eq. (2), we obtain the total current through the sample

$$I_x = 2en_0D\left[a(f(\delta) - f(-\delta)) + E_xL\int_{-\delta}^{+\delta} d\eta(1 - a^2f^2)\right]. \tag{13}$$

We shall consider only the solution of Eq. (10) in the limiting nonlinear case of large $\mathscr{E}$; an analysis for intermediate fields and a comparison with the linearized solution are given in the Appendix.

In accordance with the usual procedure, in the case of large $|\mathscr{E}| \gg 1$ we must retain in Eq. (10) the second term, which is proportional to $\mathscr{E}$, as well as the first term because it contains the higher derivative. In this approximation, the first integral of Eq. (10) is

$$df/d\eta - \mathscr{E}f^2 + \mathscr{E}C_1 = 0, \tag{14}$$

and it is evident from Eq. (11) that the constant C_1 is close to unity, i.e., in any case $C_1 \equiv C^2 > 0$ (to make the case definite, we shall assume that $C > 0$). Then, the solution of Eq. (14) becomes

$$f(\eta) = -C\tanh[C\mathscr{E}(\eta - \eta_0)]. \tag{15}$$

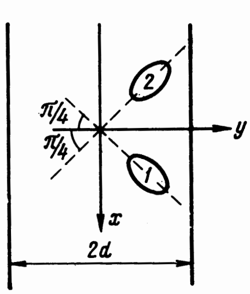

FIG. 1

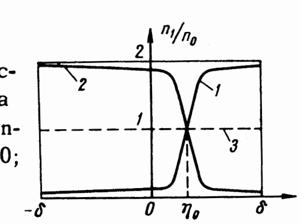

FIG. 2. Distribution of the electron density in the first valley of a plate made of a two-valley semiconductor: curve 1 – $\mathscr{E} < 0$; 2 – $\mathscr{E} > 0$; 3 – $\mathscr{E} = 0$.

The graph of the function (15) consists of two wide plateaus with $f(\eta) \approx \pm C$ and a region $\Delta\eta \approx 1/\mathscr{E}$ wide in which $f(\eta)$ varies rapidly (Fig. 2). Substitution of Eq. (15) into Eq. (10) shows that near η_0 the omitted last term is indeed small compared with the first two terms. However, in the region of the plateau, where the terms containing the derivatives are exponentially small in the approximation of Eq. (15), the last term becomes more important; in this case, we can retain high accuracy and still drop the second derivative so that

$$f(\eta) \approx -C\,\mathrm{sign}[\mathscr{E}(\eta - \eta_0)] - (\eta - \eta_0)/2\mathscr{E}. \tag{16}$$

The first term in Eq. (16) is selected so that the formulas (15) and (16) join smoothly near $|\eta - \eta_0| \approx \ln|2\mathscr{E}|/|\mathscr{E}|$. Since C is close to unity, it follows from Eqs. (15) and (16) that the plate splits into two layers (domains), each of which contains almost exclusively the electrons which belong to one of the valleys; the width of the transition region (domain wall) is of the order of $L/\mathscr{E}$. The field E_y is almost constant within a domain but it decreases rapidly in a domain wall.

In domain walls, where E_y varies rapidly, the condition of quasi-neutrality (1) may not be obeyed; the usual condition of quasi-neutrality has the form, as indicated by Eqs. (9) and (15), $l_D\mathscr{E} \ll L$, where l_D is the Debye length.

In the case considered, $|\mathscr{E}|\delta \gg 1$ and if η_0 is not too close to the edges of the plate, we can use Eq. (16) and omit the diffusion term in Eq. (11); then,

$$\eta_0 = \frac{S^{(+)} - S^{(-)}}{2}, \quad C = \left[1 - \frac{1}{2|\mathscr{E}|}\left(\delta + \frac{S^{(+)} + S^{(-)}}{2}\right)\right]. \tag{17}$$

The physical meaning of this formula for η_0 can be easily understood by writing the equations for the integral balance of carriers of each type in the approximation of the "stepped" carrier distribution; we then obtain

$$L(\delta - \eta_0)/\tau - L(\delta + \eta_0)/\tau + (s^{(+)} - s^{(-)}) = 0, \tag{18}$$

which leads directly to the expression (17) just obtained. Thus, the general distribution of the densities in the form of two domains is obtained even if the intervalley scattering in the interior and at the surface is neglected; however, the position of the boundary between the domains is governed entirely by the integral balance of the scattering acts.

If the difference between the surface scattering rates is large and $|\eta_0| \gtrsim \delta$, determined from Eq. (17), the point $f = 0$ lies at a distance $\sim 1/\mathscr{E}$ from one of these surfaces of the plate. For example, if $S^{(-)} > S^{(+)}$, we must substitute, into the boundary conditions (11), $f(\eta)$ from Eq. (15) if $\eta = -\delta$ and $f(\eta)$ from Eq. (16) if $\eta = +\delta$. Then, for η_0 and C, we obtain the formulas

$$\tanh \mathscr{E}(\delta + \eta_0) = \frac{2\delta + S^{(+)}}{S^{(-)}},$$

$$C = 1 - \frac{1}{2|\mathscr{E}|}(2\delta + S^{(+)}). \qquad (19)$$

In this case, the sample consists only of one domain.

The current–voltage characteristic in strong fields $|\mathscr{E}| \gg 1$, calculated from the formula (13), has the form

$$I_x = 2d\frac{kT}{e}\Sigma_\infty E_x + I_x^0, \qquad (20)$$

where

$$\Sigma_\infty = \sigma(1 - a^2), \quad \sigma = 2eun_0 = euN \qquad (21)$$

(Σ_∞ is the effective electrical conductivity in the limiting nonlinear case), and

$$I_x^0 = 2eD|a|n_0\left(\frac{d}{L}\right)^2\left[1 - \left(\frac{\eta_0}{\delta}\right)^2 + \frac{\tau}{d}(s^{(+)} + s^{(-)})\right] \qquad (22)$$

for a step inside a sample if η_0 is given by the formula (17), or

$$I_x^0 = 4eD|a|n_0\left(\frac{d}{L}\right)^2\left[1 + \frac{\tau s^{(+)}}{d}\right], \qquad (23)$$

if $\eta_0 \approx -\delta$. It is clear from Eqs. (22) and (23) that in both cases $I_x^0 > 0$.

By way of comparison, we note that the effective conductivity in weak fields Σ_0 is given by the formula

$$\Sigma_0 = \sigma\left[1 - ga^2\frac{\tanh\delta}{\delta}\right], \quad g \leqslant 1, \qquad (24)$$

which is analogous to the formula (10) in [4]; here, the parameter g depends on $s^{(\pm)}$ and is given by the formula (11) in the same paper.[1)] Hence, it follows that $\Sigma_0 > \Sigma_\infty$ and the current–voltage characteristic is sublinear; because of the symmetrical distribution of the valleys, $I_x(-E_x) = -I_x(E_x)$.

The change in the spatial distribution of the density due to an increase in E_x is considered in the Appendix.

It is clear from Eq. (9) that an investigation of the dependence $E_y(y)$ allows us to find directly the density distribution $f(y)$. In particular, we can determine the quantities $\Theta^{(\pm)}$:

$$\tan\Theta^{(+)} = \frac{1}{E_x d}\int_0^d E_y(y)\,dy, \quad \tan\Theta^{(-)} = \frac{1}{E_x d}\int_{-d}^0 E_y(y)\,dy, \qquad (25)$$

which are odd functions of E_x. From Eq. (16), it follows that

$$\tan\Theta_\infty^{(\pm)} = \pm|a|\left(1 - \frac{|\eta_0| \pm \eta_0}{\delta}\right)\operatorname{sign} E_x. \qquad (26)$$

In weak fields, $\Theta^{(\pm)} \sim E_x$; for example, for $S^{(\pm)} = 0$

$$\tan\Theta^{(\pm)} = \pm\frac{a^2 u\tau}{d}E_x\left(1 - \frac{1}{\cosh(d/L)}\right). \qquad (27)$$

The appearance of two layers, each of which contains almost exclusively electrons belonging to one valley, leads to a sharp drop in the transverse conductivity, which is now limited by the intervalley scattering rate. Although the value of the resistance to a small current passed along the direction y depends on the conditions at the contacts (in the planes $y = \pm d$), it can be estimated as follows. If we assume that in each of these layers the transverse current consists mainly of the majority carriers, it follows from the integral condition of balance that the boundary between the layers shifts by $j_y\tau/2en_0$ (j_y is the current density along the y axis). This leads to a change in the transverse potential difference, by an amount $j_y\tau a E_x/en_0$, which is equivalent to a transverse conductivity $\sigma E_L/2LE_x$, which is less than the equilibrium transverse conductivity $\sigma/2d$ by a factor L_E/d, where $L_E = LE_x/|E_L|$ is the extended diffusion length.

So far we have considered thin plates with $d \ll L_E$. We shall now deal with a semi-infinite semiconductor occupying the half-space $y > 0$. For a sufficiently large value of y, the distribution

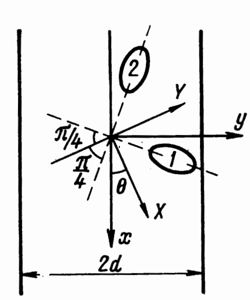

FIG. 3.

[1)]We take this opportunity to note that this formula contains an error 2(L/D) in the numerator should be replaced by (L/2D).

differs little from the equilibrium state, f is small, and we can omit the quadratic term in Eq. (10). Then $f \sim \exp(-\eta)$. This distribution is valid for $|f| \lesssim |2\mathscr{E}|^{-1}$. At low values of η, we can omit the diffusion term and then

$$f(\eta) \approx \operatorname{sign} \mathscr{E} - (S + \eta) / 2\mathscr{E}. \tag{28}$$

Thus, f(y) decreases linearly by a factor of $|2\mathscr{E}|$ in a region $\approx 2L_E$ wide and then decreases rapidly as $\exp(-y/L)$.

In all the cases considered so far, the current-voltage characteristic has remained symmetrical because of the same inclination of the energy ellipsoids with respect to the faces of the plate. If a plate is cut as shown in Fig. 3, then

$$u_{yy}{}^{(1,2)} = u_{xx}{}^{(2,1)} = u(1 \pm a \sin 2\theta),$$
$$u_{xy}{}^{(1,2)} = \pm au \cos 2\theta, \tag{29}$$

and an analysis of the equations shows that all the main conclusions obtained earlier are still valid; in strong fields, for $\theta \neq \pm\pi/4$, the function $f(\eta)$ is retains the form of a step and η_0 is found from Eq. (17). However, the criterion of strong fields becomes more rigorous because of a reduction in u_{xy} by a factor of $\cos 2\theta$ [cf. Eq. (29)] and a corresponding increase in the diffusion field [cf. Eq. (12)]. If $s^{(+)} = s^{(-)}$, rectification is obtained; for example, if one of these velocities is equal to zero, and the other is infinite, then

$$\frac{\Sigma_{+\infty}}{\Sigma_{-\infty}} = \frac{1 - a \sin 2\theta}{1 + a \sin 2\theta}, \quad 0 \leqslant |\theta| < \frac{\pi}{4}. \tag{30}$$

3. MANY-VALLEY SEMICONDUCTOR

We shall now consider a semiconductor with an arbitrary number of valleys ν in a strong field E_x. Since the density of electrons in each valley is limited by the condition (1) if $s_{\alpha\beta} \neq \infty$ and $\tau_{\alpha\beta} \neq 0$, the currents $j_y^{(\alpha)}$ remain finite when $|E_x| \to \infty$. Since the individual terms on the right-hand side of Eq. (3) increase then without limit, the distribution of the densities $n_\alpha(y)$ for $|E_x| \to \infty$ should be found from Eq. (3) by dividing it by E_x and replacing the left hand side with zero. Substituting E_y from Eq. (5), we obtain

$$n_\alpha \sum_\beta D_{yy}^{(\beta)} n_\beta \left\{ (a_\alpha - a_\beta) + \frac{1}{E_x} \frac{d}{dy} \ln\left(\frac{n_\alpha}{n_\beta}\right) \right\} = 0, \tag{31}$$

where $a_\alpha = D_{xy}^{(\alpha)}/D_{yy}^{(\alpha)}$. Because $n_\beta(y)$ is finite, for large values of E_x the last term in Eq. (31) can remain finite only in narrow regions of width $\sim E_x^{-1}$. Therefore, we can almost always omit this term and then Eq. (31) has ν solutions satisfying the condition (1):

$$n_\beta = N\delta_{\alpha\beta}, \quad E_y = -a_\alpha E_x, \quad \alpha = 1, 2, \ldots, \nu. \tag{32}$$

Thus, as in the case of a two-valley semiconductor, a plate can split into domains each of which contains only electrons which belong to one valley; altogether, ν types of domain are possible. A special case is obtained in the presence of "degeneracy," when the coefficients a_β are equal for several valleys; then, one domain contains electrons belonging to several valleys.

We shall now consider the possible sequential order of domains. For this purpose, we shall take a boundary between domains, where some of the derivatives in Eq. (31) are large. Let the boundary be at a point $y_{\alpha\alpha'}$, and on the left (where $y < y_{\alpha\alpha'}$) there is a domain containing electrons from the valley α, and on the right there is a domain containing electrons from the valley α'. Since, in this region, all $n_\beta = \alpha$, α' tend to zero, it follows from Eq. (31) that

$$a_\alpha - a_{\alpha'} + \frac{1}{E_x} \frac{d}{dy} \ln\left(\frac{n_\alpha}{n_{\alpha'}}\right) = 0, \tag{33}$$

and therefore $a_\alpha > a_{\alpha'}$ for $E_x > 0$. Consequently, only those arrangements of domains are possible which make the algebraic values of the anisotropy factors a_α for electrons in these domains decrease from the left to the right. When the field is reversed ($E_x \to -\infty$), the anisotropy factors a_α increase from the left to the right.

We shall select, for $E_x > 0$, an order of numbering of the valleys (and, consequently of the domains) such that the anisotropy factors a_α decrease as the valley number increases:

$$a_1 \geqslant a_2 \geqslant \ldots \geqslant a_{\nu-1} \geqslant a_\nu. \tag{34}$$

The equations for the determination of the thicknesses of individual domains $d_\alpha = y_{\alpha,\alpha+1} - y_{\alpha-1,\alpha}$ can be obtained by integrating Eq. (6) over the sample thickness and using Eq. (7):

$$\sum_\beta \left[N \frac{d_\alpha - d_\beta}{\tau_{\alpha\beta}} + s_{\alpha\beta}{}^{+}(n_\alpha{}^{+} - n_\beta{}^{+}) + s_{\alpha\beta}{}^{-}(n_\alpha{}^{-} - n_\beta{}^{-}) \right] = 0; \tag{35}$$

then,

$$\sum_\beta d_\beta = 2d, \quad \sum_\beta n_\beta{}^{+} = \sum_\beta n_\beta{}^{-} = N. \tag{36}$$

In the "nondegenerate" case, the system (35), (36) gives all d_β and $n_\beta{}^{\pm}$, if we take into account the limitation of $n_\beta{}^{\pm}$, which follows from the sequential order of domains; namely, $n_\alpha{}^{+} \neq 0$ only for a domain α^{+} lying immediately next to the surface

$y = +d$, and also for all cases $\alpha > \alpha^+$ (if $\alpha^+ < \nu$); then $d_\alpha = 0$ for $\alpha^+ < \alpha \le \nu$. Similarly, $n_{\alpha^-} \neq 0$ only for the domain α^-, lying next to the surface $y = -d$, and for all $\alpha < \alpha^-$ (if $\alpha^- > 1$); all $d_\alpha = 0$ when $\alpha < \alpha^-$.

Thus, depending on the relationship between $\tau_{\alpha\beta}$ and $s_{\alpha\beta}{\pm}$ between one and ν domains may exist in a plate. When the number of domains is less than ν, electrons which belong to the valleys with the largest and smallest a_α ($\alpha < \alpha^-$ and $\alpha > \alpha^+$) accumulate only in the surface layers whose thickness is of the order of the compressed diffusion length 1/E.

In the presence of the "degeneracy," the maximum number of domains is equal to the number of different a_β; then each domain contains electrons from all valleys with given a_β. The relationships between their densities, as well as the domain thicknesses, are found from Eqs. (35) and (36).

The distribution of the densities in the domain walls is found from Eq. (33); for example, in the nondegenerate case

$$n_{\alpha'} - n_\alpha = N \tanh\left[\frac{a_\alpha - a_{\alpha'}}{2} E_x (y - y_{\alpha\alpha'})\right]. \qquad (37)$$

The current I_x flowing through a sample is found by integrating Eq. (2) over the thickness; then, in the absence of the "degeneracy"

$$\Sigma_\infty = \frac{I_x}{2dkTE_x/e} = \frac{eN}{2d}\sum_\alpha \left(u_{xx}^{(\alpha)} - a_\alpha u_{xy}^{(\alpha)}\right) d_\alpha. \qquad (38)$$

The value of Σ_∞ depends, through d_α, on the relationship between the intervalley scattering in the interior and on the surface; for $s_{\alpha\beta}{\pm} = 0$, all $d_\alpha = 2d/\nu$ and Eq. (38) simplifies.

4. PLATE OF n-TYPE Ge

The general theory, presented in the preceding section, can be illustrated conveniently by a plate of n-type Ge cut at right angles to a fourfold axis (for example, 010). N-type Ge has four valleys elongated along the threefold axes (Fig. 4). From the symmetry considerations, it follows that all $\tau_{\alpha\beta} = \tau$ and there are only two different rates $s_{\alpha\beta}$—for transitions between neighboring (for example, $1 \rightleftarrows 2$) and opposite (for example, $1 \rightleftarrows 4$) valleys; we shall denote them by s and s′.

We shall determine the dependence of the positions of the domains on the angle φ, which gives the direction of the current with respect to the crystallographic axes (Fig. 4); it is sufficient to consider the interval $0 \le \varphi \le \pi/4$, since all the remaining cases can be obtained from this interval by a suitable transposition of the valleys. Table I

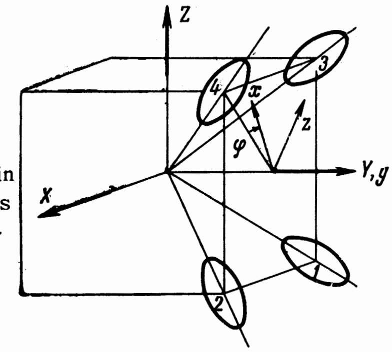

FIG. 4. Ellipsoids of the conductivity tensors in germanium. The XYZ axes are related to the orientation of the plate.

lists $D_{ij}^{(\alpha)}$ and a_α for all the valleys. If we use the notation

$$\mathfrak{d}_\alpha = \frac{d_\alpha}{2d}, \quad \mathfrak{n}_\alpha{}^\pm = \frac{n_\alpha{}^\pm}{N}, \quad \mathfrak{s}_\pm = \frac{s^\pm\tau}{2d}, \quad \mathfrak{s}_\pm{}' = \frac{s'^\pm\tau}{2d}, \qquad (39)$$

then

$$\sum_\alpha \mathfrak{d}_\alpha = \sum_\alpha \mathfrak{n}_\alpha = 1. \qquad (40)$$

In the symmetrical case ($\mathfrak{s}_\pm = \mathfrak{s}, \mathfrak{s}_\pm{}' = \mathfrak{s}'$), we have $\mathfrak{d}_{1,2} = \mathfrak{d}_{4,3}$, $\mathfrak{n}_{1,2}{}^- = \mathfrak{n}_{4,3}{}^+$, $\mathfrak{s}'$ drops out of Eq. (35) and two situations are possible, depending on the value of :

1) $2\mathfrak{s} \le 1$. Then

$$\mathfrak{d}_1 = \frac{1-2\mathfrak{s}}{4}, \quad \mathfrak{d}_2 = \frac{1+2\mathfrak{s}}{4} \quad \mathfrak{n}_1{}^- = \mathfrak{n}_4{}^+ = 1, \qquad (41)$$

the remaining $\mathfrak{n}_\alpha{\pm} = 0$, and the conductivity is

$$\Sigma_\infty = \frac{eNu_t}{u_l + 2u_t}[u_t + 2u_l + 2\mathfrak{s}(u_t - u_l)\cos 2\varphi]. \qquad (42)$$

2) $2\mathfrak{s} \gg 1$. Then

$$\mathfrak{d}_1 = 0, \quad \mathfrak{d}_2 = \frac{1}{2}, \quad \mathfrak{n}_1{}^- = \mathfrak{n}_4{}^+ = \frac{2\mathfrak{s}+1}{4\mathfrak{s}},$$

$$\mathfrak{n}_2{}^- = \mathfrak{n}_3{}^+ = \frac{2\mathfrak{s}-1}{4\mathfrak{s}}, \qquad (43)$$

the remaining $\mathfrak{n}_\alpha{\pm} = 0$, and

$$\Sigma_\infty = \frac{eNu_t}{u_l + 2u_t}[3u_l + 2(u_t - u_l)\cos^2\varphi]. \qquad (44)$$

Table I

	α	
	1	2
$D_{xx}^{(\alpha)}$	$D_t - \frac{2}{3}(D_t - D_l)\cos^2\varphi$	$\frac{2D_l + D_t}{3} + \frac{2}{3}(D_t - D_l)\cos^2\varphi$
$D_{xy}^{(\alpha)}$	$\frac{\sqrt{2}}{3}(D_t - D_l)\cos\varphi$	$\frac{\sqrt{2}}{3}(D_t - D_l)\sin\varphi$
a_α	$\varkappa\cos\varphi$	$\varkappa\sin\varphi$

Note: $D_{xx}^{(3,4)} = D_{xx}^{(2,1)}$, $D_{xy}^{(3,4)} = -D_{xy}^{(2,1)}$, $a_{3,4} = -a_{2,1}$, $D_{yy}^{(\alpha)} = \frac{D_l + 2D_t}{3}$, $\varkappa = \sqrt{2}\,\frac{D_t - D_l}{D_l + 2D_t}$, D_l and D_t are the principal values of the diffusion coefficient tensor for each of the valleys; it is assumed that, as usual, $D_l < D_t$.

Table II

	$2\mathfrak{s}+\mathfrak{s}'\leqslant 1$	$2\mathfrak{s}+\mathfrak{s}'\geqslant 1$, but $\mathfrak{s}+\mathfrak{s}'\leqslant 2$	$\mathfrak{s}+\mathfrak{s}'\geqslant 2$, but $\mathfrak{s}(\mathfrak{s}+\mathfrak{s}')\leqslant 5\mathfrak{s}+\mathfrak{s}'$	$\mathfrak{s}(\mathfrak{s}+\mathfrak{s}')\geqslant$ $\geqslant 5\mathfrak{s}+\mathfrak{s}'$
$\mathfrak{d}_1$	$\frac{1+\mathfrak{s}'}{4}$	$\frac{(1+\mathfrak{s})(\mathfrak{s}+\mathfrak{s}')}{2(3\mathfrak{s}+\mathfrak{s}')}$	$\frac{(1+\mathfrak{s})(\mathfrak{s}+\mathfrak{s}')}{2(3\mathfrak{s}+\mathfrak{s}')}$	1
$\mathfrak{d}_2$	$\frac{1+\mathfrak{s}}{4}$	$\frac{4\mathfrak{s}+(\mathfrak{s}+\mathfrak{s}')^2}{4(3\mathfrak{s}+\mathfrak{s}')}$	$\frac{5\mathfrak{s}+\mathfrak{s}'-\mathfrak{s}(\mathfrak{s}+\mathfrak{s}')}{2(3\mathfrak{s}+\mathfrak{s}')}$	0
$\mathfrak{d}_3$	$\frac{1+\mathfrak{s}}{4}$	$\frac{2-(\mathfrak{s}+\mathfrak{s}')}{4}$	0	0
$\mathfrak{d}_4$	$\frac{1-2\mathfrak{s}-\mathfrak{s}'}{4}$	0	0	0
$\mathfrak{n}_1^+$	0	0	0	$\frac{\mathfrak{s}(\mathfrak{s}+\mathfrak{s}')-\mathfrak{s}'-5\mathfrak{s}}{4\mathfrak{s}(\mathfrak{s}+\mathfrak{s}')}$
$\mathfrak{n}_2^+$	0	0	$\frac{\mathfrak{s}+\mathfrak{s}'-2}{2(\mathfrak{s}+\mathfrak{s}')}$	$\frac{1+\mathfrak{s}}{4\mathfrak{s}}$
$\mathfrak{n}_3^+$	0	$\frac{2\mathfrak{s}+\mathfrak{s}'-1}{3\mathfrak{s}+\mathfrak{s}'}$	$\frac{4\mathfrak{s}+(\mathfrak{s}+\mathfrak{s}')^2}{2(\mathfrak{s}+\mathfrak{s}')(3\mathfrak{s}+\mathfrak{s}')}$	$\frac{1+\mathfrak{s}}{4\mathfrak{s}}$
$\mathfrak{n}_4^+$	1	$\frac{1+\mathfrak{s}}{3\mathfrak{s}+\mathfrak{s}'}$	$\frac{1+\mathfrak{s}}{3\mathfrak{s}+\mathfrak{s}'}$	$\frac{\mathfrak{s}(\mathfrak{s}+\mathfrak{s}')+3\mathfrak{s}-\mathfrak{s}'}{4\mathfrak{s}(\mathfrak{s}+\mathfrak{s}')}$

Note: $\mathfrak{n}_1^- = 1$, $\mathfrak{n}_2^- = \mathfrak{n}_3^- = \mathfrak{n}_4^- = 0$

From Eqs. (42) and (44), it is clear that Σ_∞ depends on $\mathfrak{s}$ only if $\mathfrak{s} < 1/2$; when $\mathfrak{s} = 0$, Σ_∞ loses its angular dependence and

$$\Sigma_\infty(\mathfrak{s}=0) = \sigma(1-\varkappa^2/2), \quad \sigma = {}^1/_3 eN(u_l + 2u_t). \tag{45}$$

In the asymmetrical case, the number of possible situations increases and their criteria become cumbersome. Therefore, we shall consider only the case when $\mathfrak{s}_+, \mathfrak{s}'_+ \neq 0$ (we shall denote them by $\mathfrak{s}$, $\mathfrak{s}'$) and $\mathfrak{s}_- = \mathfrak{s}_- = 0$. The thicknesses of the layers and the carrier densities on the surface are given in Table II. Depending on the values of $\mathfrak{s}$ and $\mathfrak{s}'$, between one and four domains may exist. When $\mathfrak{s}$, $\mathfrak{s}' \to \infty$, the electrons from one valley fill the whole sample, with the exception of a narrow layer at the boundary $y = +d$, whose thickness is of the order of the compressed diffusion length and in which electrons of all the valleys are represented in densities close to the equilibrium value ($\mathfrak{n}_\alpha \to 1/4$).

The degeneracy appears when $\varphi = 0, \pi/4$.

1. For $\varphi = \pi/4$, the current is directed along a fourfold axis,and, as indicated in Table I, $D_{xx}^{(1)} = D_{xx}^{(2)}$, $D_{xy}^{(1)} = D_{xy}^{(2)}$, $a_1 = a_2$. Therefore, electrons from the valleys 1 and 2 behaved in exactly the same way and $n_1 = n_2$ always; the same is true also of electrons from the valleys 3 and 4. The problem reduces to that considered in Sec. 2 for $a = \kappa/\sqrt{2}$ with a corresponding re-definition of τ and $s^\pm$.

2. If $\varphi = 0$, i.e., if the current is directed along a twofold axis, then $a_2 = a_3 = 0$. Therefore, the number of domains cannot exceed three: in addition to the domains containing solely electrons from the valleys 1 and 4, there may exist a domain containing equal amounts of electrons from the valleys 2 and 3; the width of this domain can be calculated, from the formulas given above, as the sum $d_2 + d_3$.

In conclusion, we shall calculate, for the same case $a_2 = a_3 = 0$, the density distribution in a semi-infinite sample ($y > 0$). We shall introduce dimensionless variables which are convenient in such a calculation

$$\chi = \frac{n_1 + n_4 - n_3 - n_2}{N}, \quad \zeta = \frac{n_1 - n_4}{N},$$

$$\xi = \frac{2y}{L}, \quad \mu = \frac{\varkappa E_x L}{2}. \tag{46}$$

Then, from Eqs. (1), (3), (5), and (6) it follows that

$$\frac{d}{d\xi}\left(\frac{d\chi}{d\xi} - \mu\chi\zeta + \mu\zeta\right) = \chi,$$

$$\frac{d}{d\xi}\left(\frac{d\zeta}{d\xi} - \mu\zeta^2 + \mu\frac{\chi}{2}\right) = \zeta. \tag{47}$$

The treatment for an "infinite" sample for high values of μ differs from the treatment for a "finite" sample in that we cannot regard the extended diffusion length to be much longer than the thickness of the sample 2d, but a rigorous opposite inequality must be satisfied. Therefore, in the expressions of Eq. (47) it is necessary to retain the right-hand sides, which describe the intervalley scattering. The diffusion currents, due to the high derivatives in these expressions, should be always neglected for the same reasons as in the case of plates, with the possible exception of narrow regions near certain points. The integrals of the expressions in Eq. (47), without the higher-derivative terms, have the form

$$\zeta^2 = c(\chi+1)^2 - \chi - 1/2,$$

$$-\frac{\xi}{\mu} = 2\zeta - \frac{1}{2}\int\frac{d\chi}{\zeta(\chi)}, \tag{48}$$

where c is a constant of integration.

On the surface of a sample, a boundary condition of the type of Eq. (7) is satisfied and gives rise, in strong fields $|\mu| \gg S, S', 1$, to

$$\chi(0) = 1, \quad \zeta(0) = \pm 1; \tag{49a}$$

the upper sign is used for $\mu > 0$, and the lower – for $\mu < 0$. To make the treatment concrete, we shall consider only the case $\mu > 0$. The results for $\mu < 0$ are obtained by reversing the sign in front of ζ.

On the other hand, when $\xi \to \infty$, the densities tend to equilibrium values so that

$$\chi(\infty) = \zeta(\infty) = 0. \tag{49b}$$

The constants of integration, which satisfy the boundary conditions (49a) and (49b) are, respectively,

$$c_0 = {}^5/_8, \quad c_\infty = {}^1/_2. \tag{50}$$

Near a certain point $\xi = \xi_1$, the solution with the constant c_0 transforms into the solution with c_∞ in a region whose width is of the order of the compressed diffusion length. We shall determine, for $\mu \to \infty$, the limiting values of χ on the left and right of the point ξ_1: $\chi_0(\xi_1)$ and $\chi_\infty(\xi_1)$. For this purpose, we shall require an infinite sample (like a finite sample in Sec. 3) to satisfy the equation for the balance of the intervalley transitions, which follows from the equation of continuity (6). This equation reduces to the requirement of continuity of the electron currents in each of the valleys, from which it follows that

$$\zeta_\infty(\chi_\infty - 1)\big|_{\xi_1} = \zeta_0(\chi_0 - 1)\big|_{\xi_1},$$

$$\zeta_\infty^2 - \frac{\chi_\infty}{2}\bigg|_{\xi_1} = \zeta_0^2 - \frac{\chi_0}{2}\bigg|_{\xi_1}. \tag{51}$$

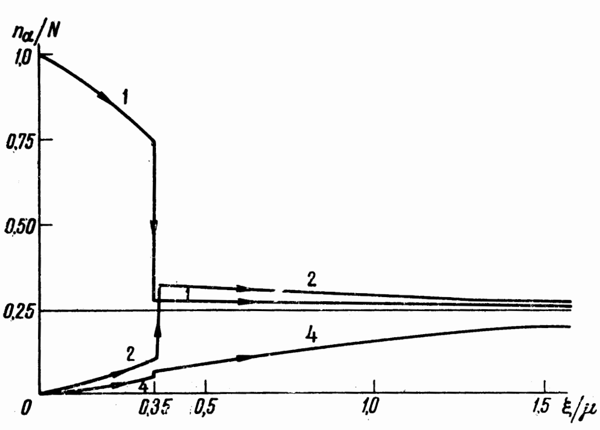

FIG. 5. Distribution of the electron densities in the valleys 1, 2, and 4 in a semi-infinite n-type Ge sample for $\varphi = 0$: 1) n_1/N for $\mu > 0$ and n_4/N for $\mu < 0$; 2) n_2/N; 4) n_4/N for $\mu > 0$ and n_1/N for $\mu < 0$.

An analysis of Eq. (48) with the boundary conditions (49a) and (49b) and the continuity conditions (51), shows that $\chi(\xi)$ and $\zeta(\xi)$ are monotonic in the continuous regions, $\zeta(\xi) > 0$ everywhere, and $\chi(\xi)$ is positive for $\xi < \xi_1$ and negative for $\xi > \xi_1$. The solution of Eq. (51), using the first of the two expressions in Eq. (48), as well as Eq. (50), gives

$$\zeta_0(\xi_1) = 1/\sqrt{2}, \quad \chi_0(\xi_1) = \frac{3}{5}; \quad \zeta_\infty(\xi_1) = \frac{\sqrt{2{,}6} - 1}{2\sqrt{2}} \cong 0{,}217,$$

$$\chi_\infty(\xi_1) = -\frac{\sqrt{2{,}6} - 1}{\sqrt{2}} \cong -0.306 \tag{52}$$

and allows us to determine the coordinate $\xi_1 \approx 0.35\mu$ and the solutions $\zeta(\xi)$ and $\chi(\xi)$ over the whole range $(0, \infty)$.

The dependences of n_1, n_2, and n_4 on ξ/μ, obtained from these solutions, are plotted in Fig. 5.

Thus, an infinite sample should also have a domain wall separating a surface domain from the sample interior, the width of the surface domain being proportional to the field E_x, and there should be a strong departure from the equilibrium densities of electrons in the valleys.

Naturally, in the general case of a semi-infinite many-valleyed semiconductor, there should also be a domain structure in a surface layer of thickness $\sim\mu$.

CONCLUSIONS

We shall now consider the problem of the theoretical criteria. The first limitation is that the intervalley time τ should be considerably longer than the intravalley relaxation time t. As far as we know, at present only the values of τ for Ge and Bi are known. As already mentioned in [4], it follows from the data of Weinreich et al.[1] that this criterion is satisfied by Ge at temperatures close to 30°K and that L is sufficiently large to observe a redistribution of carriers between domains even in fields of the order of several V/cm. If we assume that the intervalley scattering time for Bi is of the order of the electron–hole recombination time, the ratio τ/t is found to be about 30 at 4°K (according to [11]; then $\tau \approx 2 \times 10^{-8}$ sec) and about 5000 at 77° K (according to [6]). In these estimates, the value of t was determined from the mobility; the use of the energy relaxation time t_ξ might change somewhat the values of t. However, we can show that the heating of carriers by an electric field, accompanied by a severalfold increase in their average energy, does not invalidate the pattern of splitting of a sample into domains, as discussed above, provided the heating is not accompanied by a sharp drop in τ.

The second criterion is related to the value of the intervalley surface scattering rate s. We can assume that in Eq. (3) all $j_y^{(\alpha)} \approx 0$ only for fields E_x such that

$$|E_x| \gg s / |D_{xy}^{(\alpha)}| \quad (53)$$

for all α. This criterion can be satisfied even in the range of fields which do not heat carriers, if

$$s \ll |D_{xy}^{(\alpha)}| E_c, \quad (54)$$

where E_c is a characteristic field corresponding to the heating of carriers; for the usual values of the parameters, the right-hand side of Eq. (54) is of the order of 10^6–10^7 cm/sec. The criterion (54), expressed in terms of the relaxation times, has the form

$$S = sL / D_{yy} \ll \sqrt{\tau / t_\varepsilon}. \quad (55)$$

To determine the domain dimensions, we need to know the absolute values of S. If there is no surface band curvature, then the maximum possible value of s can be estimated from the wall current on the surface. For example, for a two-valley semiconductor with equiprobable surface scattering into both valleys, $s_{max} \approx v/8$, where v is the average thermal velocity of electrons. Hence, it follows that

$$S_{max} = s_{max} L / D_{yy} \sim {}^1/_5 \sqrt{\tau / t}. \quad (56)$$

It is clear from Eq. (56) that the value of S_{max} for $\tau/t \approx 10^2$ is only slightly greater than unity. The condition $S \ll 1$ is easily obtained by an upward bending of the bands near the surface.

We have assumed earlier that $E_z = 0$. However, as in the galvanomagnetic experiments, in addition to $E_z = 0$, another limiting case is possible: $I_z = 0$, which is obtained when $d_x \gg d_z \gg d_y$. In this case, the whole qualitative pattern of the effects and the equations for the balance of carriers [including Eq. (35) for the domain thickness] are retained, but the formulas for the field E_y and for the currents become different. When E_z vanishes, both cases—in terms of the symmetry considerations—are naturally identical; in particular, this happens in the situations considered in Sec. 2 as well as in Sec. 4 for $\varphi = 0, \pi/4$.

In conclusion, we shall list again the effects which appear in many-valley unipolar semiconductors in strong electric fields and give rise to the splitting of plates into domains.

1. The nonlinearity of the current-voltage characteristic.

2. The appearance, in plates of cubic crystals, of a transverse electric field and, in the case of the inequality of the surface intervalley scattering rates s^+ and s^-, a transverse emf. For a two-valley sample, this emf is

$$\Psi_\perp = 2|aE_x| d \frac{S^+ - S^-}{2\delta} \quad \text{when} \quad |S^+ - S^-| < 2\delta,$$
$$\Psi_\perp = \pm 2|aE_x| d \quad \text{when} \quad |S^+ - S^-| > 2\delta. \quad (57)$$

3. A considerable rise in the transverse resistivity.

4. The rectification of the current in the case of unequal rates s^+ and s^- and unequal slopes of the valleys with respect to the faces of a plate.

5. Anisotropy of the conductivity of a plate which depends on the velocities $s_{\alpha\beta}\pm$.

6. The partial or even complete emptying of some electron valleys for a suitable selection of the values of $s_{\alpha\beta}\pm$.

The splitting of semiconductors into domains may be the cause of a number of other effects which we have not considered in this paper. In particular, it may alter greatly the piezoresistance coefficients which, in a many-valley semiconductor, are governed to a considerable degree by electron transitions to a valley "vacated" as a consequence of deformation. This mechanism of the piezoresistance does not apply when such a transition has already taken place under the influence of a field.

A considerable change in the galvanomagnetic properties of a sample is also possible.

APPENDIX

We shall use $F = \mathscr{E} f$. Then, Eq. (10) becomes

$$\frac{d^2F}{d\eta^2} - \frac{d(F^2)}{d\eta} - F = 0, \quad (A.1)$$

and its first integral is

$$F^2 + c = P - {}^1/_2 \ln|1 + 2P|, \quad P = dF / d\eta. \quad (A.2)$$

This function is plotted in Fig. 6; it has two branches, which approach at the point of discontinuity. We shall now determine which parts of the plot correspond to the integral curves of the bound-

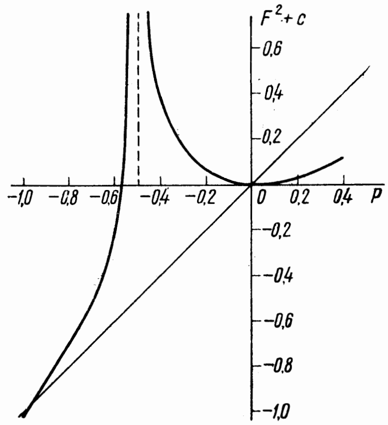

FIG. 6. Plot of the function $F^2 + C = f(P)$ as given by formula (A.2).

ary problem considered for various $\mathscr{E}$. For simplicity, we shall assume that $S^{\pm} = 0$; then, Eq. (11) becomes

$$P + \mathscr{E}^2 - F^2 = 0 \text{ when } \eta = \pm\delta. \qquad \text{(A.3)}$$

In weak fields, the right-hand side of Eq. (A.2) can be expanded in powers of P, including only the quadratic terms, and F^2 can be neglected in Eq. (A.3). Then,

$$F = -\mathscr{E}^2\frac{\sinh\eta}{\cosh\delta}, \quad P = -\mathscr{E}^2\frac{\cosh\eta}{\cosh\delta}, \qquad \text{(A.4)}$$

i.e., the solution corresponds to the region $P < 0$ of the right-hand branch (Fig. 6). The dependence $F(\eta)$ is superlinear.

By analogy with Sec. 2, we can easily see that in strong fields P varies from $-1/2$, which corresponds to the plateau regions, to $-\mathscr{E}^2$ in the region of the most rapid variation of $F(\eta)$; therefore, the solution lies on the left-hand branch in Fig. 6. The dependence $F(\eta)$ is now sublinear.

The conditions for a transition when $\mathscr{E}$ increases, from a solution corresponding to one branch to a solution corresponding to the other branch are interesting. This transition may be continuous, if in the critical region $P(\eta)$ is constant over the whole sample and equal to $-1/2$. We can easily check that $F(\eta) = -\eta/2$ does indeed satisfy the equations (A.1) and (A.3) for

$$\mathscr{E}_{cr}^2 = 1/2 + (\delta/2)^2. \qquad \text{(A.5)}$$

The effective electrical conductivity in this field is

$$\Sigma_{cr} = \sigma\left(1 - a^2\frac{1+\delta^2/6}{1+\delta^2/2}\right); \qquad \text{(A.6)}$$

for all values of δ we have $\Sigma_\infty < \Sigma_{cr} < \Sigma_0$. Similarly

$$\tan\Theta_{cr}^{(\pm)} = \pm|a|\frac{\delta}{\sqrt{2+\delta^2}}\operatorname{sign}E_x. \qquad \text{(A.7)}$$

[1] G. Weinreich, T. M. Sanders, Jr., and H. G. White, Phys. Rev. **114**, 33 (1959).

[2] V. L. Gurevich and A. L. Éfros, JETP **44**, 2131 (1963), Soviet Phys. JETP **17**, 1432 (1963).

[3] M. Pomerantz, Phys. Rev. Letters **13**, 308 (1964).

[4] É. I. Rashba, JETP **48**, 1427 (1965), Soviet Phys. JETP **21**, 954 (1965).

[5] S. H. Koenig, Helv. Phys. Acta **34**, 765 (1961).

[6] S. Tosima and T. Hattori, J. Phys. Soc. Japan **19**, 2022 (1964) [see also: T. Hattori and M. C. Steele, J. Phys. Soc. Japan **18**, 1294 (1963); S. Tosima and R. Hirota, J. Phys. Soc. Japan **19**, 468 (1964)].

[7] R. N. Zitter, Phys. Rev. **139**, A2021 (1965).

[8] É. I. Rashba, FTT **6**, 3247 (1964), Soviet Phys. Solid State **6**, 2597 (1965).

[9] I. I. Boĭko, I. P. Zhad'ko, É. I. Rashba, and V. A. Romanov, FTT **7**, 2239 (1965), Soviet Phys. Solid State **7**, 1806 (1966).

[10] É. I. Rashba, V. A. Romanov, I. I. Boĭko, and I. P. Zhad'ko, Phys. Status Solidi **16**, 43 (1966).

[11] R. N. Zitter, Phys. Rev. Letters **14**, 14 (1965).

Translated by A. Tybulewicz
33