DEVELOPMENT OF AN OPTICAL WAVEGUIDE IN THE PROPAGATION OF LIGHT IN A NONLINEAR MEDIUM

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The factors governing the efficiency of waveguide propagation of a real laser beam in a medium whose refractive index depends on the intensity of the propagating wave are discussed. The effects due to the finite duration of the laser pulse are analyzed in detail. The evolution of the optical waveguide in time and space is investigated theoretically in the quasioptical approximation by taking into account the inertia of the nonlinear properties of the medium. The development ("growth") rate of the optical waveguide is calculated. In the general case it is found to depend on the coordinate. The results can be used to calculate the length of the optical waveguide produced in a nonlinear medium by a laser pulse and to estimate the fraction of light energy moving along the waveguide (the efficiency of the self-trapping process).

1. PHENOMENA of waveguide propagation and self-focusing of powerful light beams in a medium whose refractive index depends on the intensity of the propagating wave are, as shown by recent theoretical[1-7] and experimental (for example, [8-10]) investigations, of exceedingly great interest for nonlinear optics. Allowance for the variation of the form of the phase front and the form and diameter of the light beam, due to self-focusing, is essential in many studies of the behavior of a powerful laser beam in a material medium (see [6, 10]). These effects can, in particular, explain certain anomalies observed in stimulated scattering (see [6, 9, 10]).

It must be noted that although the cited theoretical papers contain detailed studies of many aspects of self-focusing and waveguide propagation, the factors that determine the energy efficiency of these processes still remain unclear. The latter is a particularly vital question in view of the fact that the available experimental data offer evidence of a relatively low energy yield: according to the data given in [8, 10], the fraction of the laser-pulse energy which becomes self-focused in a linear medium is relatively small. This can be attributed to two factors:

a) The inhomogeneity of the distribution of the intensity over the transverse section of the laser beam, causing the latter not to be self-focused as a unit but to become stratified.

b) The finite relaxation time of nonlinear polarization, which causes the rate of development ("growth") of the optical waveguide to differ from the speed of light; obviously the role of this factor is particularly important in the self-focusing of short laser pulses.

For a quantitative investigation of effects connected with the finite duration of the pulse it is obviously necessary, unlike in [1-6], to solve the nonstationary problem.

2. The subject of the present paper is a theoretical investigation of nonstationary waveguide propagation of a laser pulse of finite duration τ_p in a nonlinear medium whose inertial nonlinearity properties can be characterized by a nonlinear polarization relaxation time τ. We note that a qualitative discussion of this problem was recently presented by Zel'dovich and Raĭzer;[7] the expression given by them for the rate of "growth" of the optical waveguide:

$$u = z_{\text{dif}} / \tau, \text{ where } z_{\text{dif}} = kr_0^2$$

is the so-called diffraction length and r_0 is the radius of the produced waveguide, which will be shown later to be valid in the Fresnel zone and for sufficiently large τ.

3. Assume that at the instant $t = 0$ a cylindrical light beam, whose power is equal to the critical power (see [3, 5, 6]) necessary for waveguide propagation, enters the nonlinear medium at $z = 0$. If the channel diameter amounts to dozens of wavelengths and more, then the process of wave propagation is described approximately by the quasi-optics equations (see [5, 6]):

$$\frac{2}{v}\frac{\partial s}{\partial t}+2\frac{\partial s}{\partial z}+\left(\frac{\partial s}{\partial r}\right)^2=\frac{n_2}{n_0}p+\frac{1}{k^2A}\left(\frac{\partial^2 A}{\partial r^2}+\frac{1}{r}\frac{\partial A}{\partial r}\right),$$

$$\frac{1}{v}\frac{\partial A}{\partial t}+\frac{\partial A}{\partial z}+\frac{\partial s}{\partial r}\frac{\partial A}{\partial r}+\frac{A}{2}\left(\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}\right)=0, \quad (1)$$

where n_2p is the nonlinear part in the refractive index. We assume that the time dependence of the polarization p is determined by a relaxation equation of the form[1)]

$$\tau\partial p / \partial t + p = A^2. \quad (1a)$$

In Eqs. (1) and (1a), A is the wave amplitude and s is a correction to the eikonal of the plane wave, i.e.,

$$\mathbf{E} = \mathbf{e}A \exp\{i(\omega t - kz - ks)\},$$

$v = \omega/k$ is the wave propagation velocity in the linear medium, and τ is the inertia time of the nonlinear response of the medium.

We introduce in place of t a new independent variable $\xi = t - z/v$; then we have in place of (1) the system

$$2\frac{\partial s}{\partial z}+\left(\frac{\partial s}{\partial r}\right)^2=\frac{n_2}{n_0}p+\frac{1}{k^2A}\left(\frac{\partial^2 A}{\partial r^2}+\frac{1}{r}\frac{\partial A}{\partial r}\right),$$

$$\frac{\partial A}{\partial z}+\frac{\partial s}{\partial r}\frac{\partial A}{\partial r}+\frac{A}{2}\left(\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}\right)=0,$$

$$\tau\partial p/\partial\xi + p = A^2, \quad (2)$$

the first two equations of which have the same form as in the case of the stationary problem (see [6]); the inertia of the nonlinear polarization is taken into account by the third equation.

We seek the solution of the system (2) in the form of a spherical wave with variable radius of curvature:

$$s = {}^1\!/_2\,\beta(z,\xi)r^2 + \varphi(z,\xi),$$

$$A^2 = \frac{E_0^2(\xi)}{f^2(z,\xi)}\exp\left[-\frac{r^2}{r_0^2 f^2(z,\xi)}\right] \quad (3)$$

with boundary conditions at z = 0:

$$\beta(0,t)=0, \quad \varphi(0,t)=0, \quad f(0,t)=1;$$

$$E_0^2(t)=\begin{cases}E_0^2 & \text{for } 0\leqslant t\leqslant \tau_p\\ 0 & \text{for } t<0\end{cases}. \quad (4)$$

Confining ourselves to the section of the beam near the axis, we find the equation for the function $f(z, \xi)$ that characterizes the variation of the width of the beam and its amplitude:

$$\frac{1}{f}\frac{\partial^2 f}{\partial z^2}=\frac{1}{k^2r_0^4f^4}-\frac{n_2E_0^2}{n_0r_0^2}\frac{1}{\tau}\int_0^\xi\frac{1}{f^4}\exp\left(\frac{\eta-\xi}{\tau}\right)d\eta. \quad (5)$$

Equation (5) differs from the analogous equation of the stationary theory of waveguide propagation of light in having a time-dependent integral in the nonlinear term; it is easy to see that the role of the nonstationary processes is determined by the relation between ξ and τ.

Let us verify first that (5) describes the limiting cases of a beam propagating in a linear medium, and of the stationary self-trapping beam considered in [2,3].

4. The nonlinearity of the medium does not influence the propagation of the beam when $\xi \ll \tau$. The last term in (5) can be neglected here and the function

$$f^2(z) = (z/z_{\text{dif}})^2 + 1 \quad (6)$$

describes the smearing of the beam due to the diffraction divergence; $A^2 \sim f^{-2}$. The foregoing means that the frontal part of the laser pulse, corresponding to $\xi \ll \tau$ (if the pulse duration $\tau_p \ll \tau$, then this pertains to the entire pulse), does not become self-trapped in a medium with inertial nonlinearity.

It is also easy to see that the stationary mode of the waveguide propagation of light, wherein the function f does not depend on z, is attained only for sufficiently large ξ. Indeed, let $\partial f/\partial z = 0$. Then, recognizing that the power of the beam, in accordance with the conditions of our problem, is equal to the critical power ($n_2E_0^2/n_0 = 1/k^2r_0^2$), we arrive at the equation

$$\frac{1}{f^4}=\frac{1}{\tau}\int_0^\xi\frac{1}{f^4}\exp\left(\frac{\eta-\xi}{\tau}\right)d\eta, \quad (7)$$

which is satisfied if f = 1 and $\xi \to \infty$.

5. To analyze the phenomena in the region where the optical waveguide is formed, we take account of the fact that, by virtue of the arguments presented above, the function f (the beam width) depends little on the variable ξ; therefore we can take it out of the integral sign in (5). Then, solving the ordinary differential equation for f, we obtain

$$f^2(z,\xi) = (z/z_{\text{dif}})^2\exp(-\xi/\tau)+1; \quad 0\leqslant\xi\leqslant\tau_p \quad (8)$$

(we note that the solution (8) describes also the limiting cases indicated above).

Using (8), we can determine the rate of "growth" of the waveguide from the condition

1)If the dependence of the refractive index on the wave intensity is connected with the Kerr effect, then p is proportional to the corresponding component of the anisotropy tensor.

$f = \text{const}$. For simplicity we put $f^2 = 2$, corresponding to $z = z_{dif}$ when $\xi = 0$. By virtue of (a) the equality $f^2 = 2$ is conserved if the following relation is satisfied

$$t - z/v = 2\tau \ln (z/z_{dif}). \quad (9)$$

Relation (9) pertains to the region $z \geq z_{dif}$ and connects the length of the optical waveguide z with the time of its development t. For the rate of growth of the waveguide u we obtain from (9)

$$1/u = 1/v + 2\tau/z. \quad (10)$$

According to (10), when $\tau = 0$ the rate of growth of the waveguide is equal to the speed of light ($u = v$); when $\tau \neq 0$ we have $u < v$, and the rate of growth decreases with increasing τ.

We note that the velocity u, determined from (10), coincides with the velocity given in [7] only when $z = z_{dif}$ and $2\tau/z_{dif} \gg 1/v$. The difference between the rate of growth and the speed of light causes the length of the optical waveguide produced within the time of the laser pulse to be smaller than the distance traversed by the wave, and consequently only part of the laser-pulse energy is self-trapped (see the figure). Using (9) and (10) we obtain for the length of the optical waveguide L, produced after a time τ_p, the equation

$$L + 2\tau v \ln (L/z_{dif}) = \tau_p v, \quad (11)$$

and for the energy efficiency η of the self-trapping process, which is equal to the ratio of the light energy transported along the waveguide to the total energy of the light pulse, we get

$$\eta = 1 - 2(\tau/\tau_p) \ln (z/z_{dif}); \quad z \geqslant z_{dif}. \quad (12)$$

By virtue of the difference between the velocities u and v at $z = z_{cr} = z_{dif} \exp (\tau_p/2\tau)$, the optical waveguide vanishes—the wave becomes detached from the waveguide. Formulas (8)–(12) yield quantitative estimates for the different experimental situations. As follows from the foregoing calculation, the role of the inertial effects is determined by the values of τ, τ/τ_p, and z/z_{dif}.

If the mechanism responsible for the waveguide propagation of the light is the high-frequency Kerr effect (the orientation of the anisotropic molecules in the light field), then for typical low-viscosity liquids $\tau \approx 10^{-12}$ sec and the inertia effects are significant at sufficiently small $z_{dif} \sim \lambda$ ($r_0 \sim \lambda$). If the waveguide propagation is due to electrostriction, then Eq. (1a) must generally speaking be replaced by the wave equation for sound pressure, but an approximate estimate can be obtained by putting in (1a) $\tau = r_0/v_s$, where v_s is the speed of sound. In this case the inertia effects are quite appreciable; it can be assumed that these are precisely the cause of the experimentally observed[10] strong correlation of self-focusing properties of liquids with the Kerr-effect constant.

In conclusion we note that by using Eq. (5) we can analyze also the more general case when a wave with non-plane phase front ($\beta(0, t) = 1/R$) and a power different from critical enters into a nonlinear medium.

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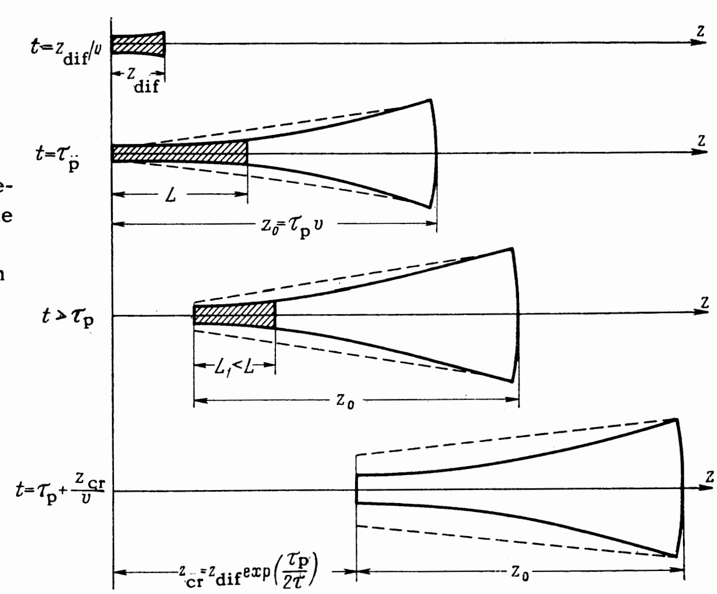

Phase of development of the self-trapped optical waveguide by a powerful light wave. The waveguide part of the beam is shaded; the dashed lines show the shape of the beam in the absence of nonlinearity. The maximum length of the waveguide L is attained at the instant of termination of the pulse $t = \tau_p$. After $t > t_p$, the waveguide becomes detached from the boundary $z = 0$ and its length decreases as the wave propagates, since $u < v$. When $z = z_{cr}$ the waveguide vanishes.

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