HYPERSOUND ABSORPTION IN QUARTZ AND RUBY CRYSTALS

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Submitted to JETP editor February 25, 1966

J. Exptl. Theoret. Phys. (U.S.S.R.) **51**, 383–393 (August, 1966)

Detailed measurements have been completed of the frequency-temperature dependence of the absorption coefficients of both longitudinal and transverse hypersonic waves in the direction of the binary X axis of a quartz crystal in a wide range of temperatures from 10 to 300°K at a frequency of 10^9 Hz and from 10 to 40°K at 9.4×10^9 and 4×10^{10} Hz. Absorption of a longitudinal hypersonic wave has been measured along the trigonal Z axis of quartz and ruby at 10^9 Hz and 9.4×10^9 Hz. It has been found that the three-phonon scattering process of longitudinal and transverse thermal phonons, respectively, is responsible for the hypersonic absorption at low temperatures. This process can be used to explain the fan-like shape of the frequency-temperature dependences of the hypersonic absorption coefficient.

1. INTRODUCTION

WITH the development of methods of excitation of hypersonic waves in solids,[1-3] the problem naturally arises of the investigations of the conditions of their propagation in crystals—measurement of the velocity and absorption coefficient. Most interest attaches to measurement of the absorption of hypersound at different frequencies over a wide temperature range, from helium to room temperature. These measurements make it possible to obtain information on phonon-phonon interactions in solids, allow us to extend the acoustic methods of investigation, using monochromatic waves, to that region in which up to now all information has been obtained from experiments with noncoherent thermal phonons—on the basis of measurements of thermal conductivity, etc.

The absorption of hypersonic waves has been studied in a number of researches. Results have been obtained[2, 4] by measurement of the temperature dependence of the absorption coefficient at low hypersonic frequencies. Measurements have been carried out at high frequencies.[5, 6] Isolated data from these researches, referring to various portions of the hypersonic range, have been obtained under heterogeneous conditions, and do not allow us to make a completely definite conclusion as to the mechanism responsible for the absorption of hypersonic waves in dielectric crystals. In this connection, the detailed measurement of the absorption coefficients of longitudinal and transverse hypersonic waves over the greatest range of frequencies available at the present time and over a wide range of temperatures in various single crystals, made under identical conditions of experiment, are of great importance.

Quartz and ruby crystals which possess trigonal syngony are of definite interest in these investigations. High quality samples of these crystals can be obtained, with a comparatively small number of defects; this allows us to separate the low frequency absorption, which is associated with the interaction of hypersonic waves with the thermal vibrations of the crystalline lattice, from absorption resulting from scattering from crystal defects.

We describe here the results of an investigation of the absorption of longitudinal and transverse hypersonic waves in a single crystal of natural α quartz, at frequencies 10^9, 10^{10} and 4×10^{10} Hz, and of longitudinal waves in an artificial red ruby crystal at 10^9 and 10^{10} Hz, over a wide range of temperatures. The agreement between theory and experiment are discussed in detail with the aim of making clear the mechanism determining the absorption of hypersonic waves in dielectric crystals at various temperatures.

2. SPECIAL FEATURES OF THE EXPERIMENTAL METHOD

The longitudinal and transverse hypersonic waves were excited along a twofold axis of symmetry—the X axis of the quartz—by means of a slowed-down electromagnetic wave surface, which made it possible to investigate the absorption of both longitudinal and transverse waves with the same sample, at different hypersonic frequencies and over a wide range of temperatures. The choice of the direction of the X axis was associated with the fact that pure longitudinal and transverse hypersonic waves, the phase and group velocities of which are in the same direction, can be propagated along this axis.

The system for the excitation of the longitudinal and transverse hypersound in quartz along the X axis at 10^9 Hz is a coil in which a slow electromagnetic surface wave with a slow-wave coefficient 10^2 is propagated. The hypersound was excited in the same direction at 10^{10} and 4×10^{10} Hz by comb-like periodic structures. The periodic structure with the surface wave was placed on the bounding plane of the quartz, perpendicular to the X axis.

The direction of propagation of the surface electromagnetic wave in the structure was chosen along the Y axis of the quartz at all frequencies. In this case, longitudinal and hypersonic waves were simultaneously excited in the quartz. The excitation of the hypersound took place in a pulsed regime (pulse length 0.3 microsecond). As a consequence of the different velocities of propagation of the waves, this made it possible to measure the absorption coefficients of the longitudinal and transverse hypersound independently, since separate echo-signal sequences correspond to each pulse.[3]

The specimens of natural single-crystal quartz, in which the absorption of the longitudinal and transverse hypersonic waves were studied, were high quality crystals of the "ekstra" brand and had the shape of a parallelepiped with dimensions $15 \times 20 \times 10$ mm, with edges oriented along the crystallographic directions X, Y, and Z. The faces of the samples perpendicular to the X axis were made optically flat and parallel (the roughness on these planes did not exceed 0.05 micron; the angle of nonparallelism was less than 2″). The deviations of the edges of the parallelepiped from the X, Y, and Z axes were measured by means of a URS-50 x-ray goniometer and did not exceed 10–20′.

The high accuracy of machining of the bounding faces of the quartz (perpendicular to X axis) is necessary in order to satisfy the condition $kl\varphi < 1$ (where $\mathbf{k}$ is the wave vector of the hypersound, l the length of the specimen along the Y axis, and φ the angular departure from parallelism of the bounding faces). Under these conditions, the phase front of the hypersonic wave essentially does not change direction after repeated reflections from the bounding faces of the crystal. This makes it possible to obtain a rather large value for the coefficient of the double transformation of the electromagnetic wave into the hypersound and back, and to obtain a large number of echo-signals, corresponding to hypersonic waves of different polarizations. Thus, at frequencies of 10^9 and 10^{10} Hz, 20–30 echo-pulses were observed, corresponding to the longitudinal and transverse hypersonic waves, which made possible reliable measurements of the temperature dependence of the absorption coefficient of the hypersound.

The method of excitation of the longitudinal hypersonic wave along a threefold symmetry axis—the Z axis in quartz and ruby—differed from that described above. In this case, a resonator method was used, described previously in [7]. The hypersound, excited in the X-cut piezoquartz, was transmitted into crystals of Z quartz or ruby. The acoustic hypersonic contact was made by means of a thin film (~ 0.1 micron) of vacuum grease or ceresin.

In the excitation of the hypersound in the quartz and its transmission into the crystal under study, difficulties arise in the determination of the absorption coefficient in this crystal, because losses in the piezoquartz transformer must be added to the acoustical losses in the specimen. The error of the measurement (because of the losses in the piezoquartz) can increase significantly in that region of temperature where the absorption in the quartz is much higher than in the crystal under study (for example, in a single-crystal ruby).

The influence of the acoustic losses in the piezoquartz on the measurement of the absorption coefficient was eliminated in the following way. The piezoquartz transformer—an X-cut disc of thickness 1 mm, attached to the studied specimen of Z-quartz or ruby—was placed in cavity resonators of the 30 and 3 cm bands so that the plane of the piezoquartz on which the acoustic contact is made with the test sample was in the electric field of the resonator.

The longitudinal hypersonic wave was excited directly on the bounding face of the Z-quartz or ruby. Therefore, the absorption coefficient, measured by the ratio of the echo-signals corresponding to the hypersonic waves passing through different acoustic path-lengths, was determined only by the losses in the test sample and did not depend on the absorption in the transformer. Losses of hypersound in the reflection from the contact layer and from the boundary of the crystal with the gaseous helium were small, as a consequence of the large difference in the acoustical resistance, and did not have an effect on the measurement of the temperature dependence of the absorption coefficient.

The samples of Z-quartz and pink ruby (concentration of Cr_2O_3—0.05%) were single crystals of high optical homogeneity and had the shape of rods of diameter 2.6 mm and length 18 mm with the geometric axis directed along the trigonal axis of the crystal. The ends of the rod were polished to optical flatness and were parallel (the roughness ≈0.1 micron, the angle of nonparallelism less than 5″). The departure of the geometric axis of the rod from the crystallographic Z direction was less than 10′ for quartz (by measurements on an x-ray goniometer) and 0.5°–1° for ruby (by optical measurements).

The nonexponential character of the change in amplitude of the echo-signals in successive pulses did not allow us to make measurements of the absolute value of the absorption coefficient of the hypersound. Therefore, just as in the works of Jacobsen and Bömmel,[2, 5] the change in the absorption coefficient was measured as the temperature was raised above the temperature of liquid helium.

3. RESULTS OF MEASUREMENT

A significant change in the absorption of a longitudinal hypersonic wave in the direction of the X axis of quartz begins at a temperature around 17–18° K (Fig. 1); thereafter, it increases sharply with increase in temperature, reaching a value of ≈ 2 db/cm at a temperature of 40° K for a frequency of 10^9 Hz. Beginning with the temperature 60° K, the value of the absorption coefficient α changes comparatively little right up to room temperature. Whereas at 60° K the absorption at this frequency amounts to 3 db/cm, at room temperature it is equal to 4.5 db/cm. At higher frequencies, 9.4×10^9 and 4×10^{10} Hz, a sharp increase in absorption to a value of 10 db/cm takes place even at a temperature of 28–30° K, which limits the temperature interval in which measurements of the the absorption coefficient of hypersound can be carried out.

A fan-shaped spreading of the curves in the temperature range 18–20° K is characteristic of the frequency-temperature dependence of the absorption coefficient of longitudinal hypersound in the direction of the X axis. It is important to note that the temperature dependence of the absorption for frequencies of 9.4×10^9 and 4×10^{10} Hz is very strong at the beginning of the interval, close to T^7, but it then weakens to T^6–T^5 at temperatures 28–

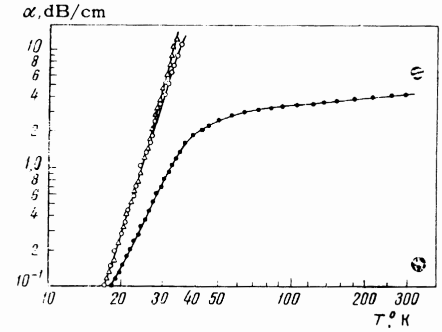

FIG. 1. Absorption of a longitudinal hypersonic wave propagating along the X axis of quartz: ● – for a frequency of 10^9 Hz, ○ – 9.4×10^9Hz, △ – 4×10^{10} Hz.

30° K. It also follows from these measurements that at the beginning of the indicated temperature range the frequency dependence of the absorption is practically nonexistent at frequencies 9.4×10^9–4×10^{10} Hz. This dependence appears only upon increase of the temperature to 30° K.

A similar fan-shaped character of the curves appears in the absorption of longitudinal hypersound in the direction of the threefold symmetry axis of Z-quartz (Fig. 2). The temperature dependence followed a T^7 law at 9.4×10^9 Hz and at the temperature range 18–30° K, and a T^4 law at 10^9 Hz and temperatures 20–25° K. The gently-sloping high-temperature plot (for $T > 60°$ K) at 10^9 Hz is similar to the plot of the absorption coefficient in the direction of the X axis.

Let us consider the absorption of transverse hypersonic waves in quartz (Fig. 3). The variation of the frequency-temperature curves of the absorption of fast transverse waves ($v_{t1} = 5.05 \times 10^5$ cm/sec) in the direction of the X axis has the same character as the absorption of the longitudinal wave propagating in the same direction. However, two important differences can be noted. In the high temperature region ($T > 60°$ K), the absorption coefficient of the transverse wave for a

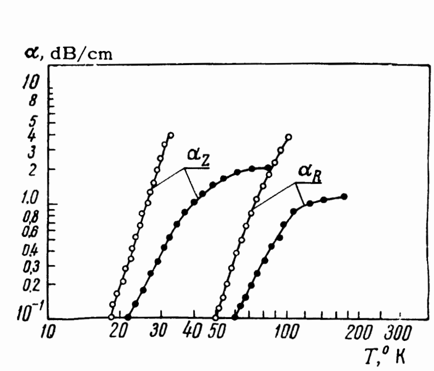

FIG. 2. Absorption of a longitudinal hypersonic wave propagating along the Z axis of quartz (α_Z) and ruby (α_R): ● – for a frequency of 10^9 Hz, ○ – 9.4×10^9 Hz.

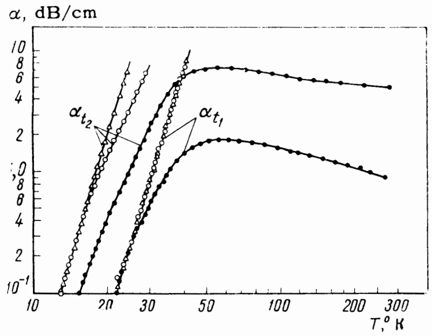

FIG. 3. Absorption of fast (α_{t_1}) and slow (α_{t_2}) transverse hypersonic waves along the X axis of quartz: ● – at frequency of 10^9 Hz, ○ – 9.4×10^9 Hz, △ – 4×10^{10} Hz.

frequency of 10^9 Hz is much less than the absorption coefficient for the longitudinal wave. Furthermore, upon increase in temperature, the absorption does not increase as in the case of the longitudinal wave but decreases slowly from 2 dB/cm at 50° K to 0.8 dB/cm at room temperature (290° K). Another significant difference is that the curves of the frequency-temperature dependence in the region of temperatures 18–30° K and frequencies 10^9–4×10^{10} Hz are shifted to the right in comparison with the corresponding curves for the absorption of the longitudinal wave.

The curves of the frequency-temperature dependence of the absorption coefficient (Fig. 3) for the slow transverse wave ($V_{t2} = 3.5 \times 10^5$ cm/sec) begin at temperatures much lower than the corresponding curves of the longitudinal absorption and the fast transverse wave absorption. For a frequency of 10^9 Hz, the temperature dependence follows a T^6–T^5 law, while the value of the absorption coefficient exceeds the corresponding value for the longitudinal and fast transverse waves. At high frequencies of 9.4×10^9 and 4×10^{10} Hz there is a much steeper temperature dependence: T^7. It must be noted that the absorption curves corresponding to the frequencies 9.4×10^9 and 4×10^{10} Hz are practically identical at the beginning of the temperature interval.

A comparison of the curves of Figs. 1 and 3 shows that upon increase in temperature the slow transverse wave is absorbed initially, then the longitudinal, and finally the fast transverse wave. This fact is well observed in the measurements; in the succession of echo pulses, the pulses corresponding to the slow transverse wave disappear upon increase in temperature, then those of the longitudinal wave. Pulses of the fast transverse wave are the last to be absorbed.

Let us consider the absorption of a longitudinal hypersonic wave in ruby (Fig. 2). Measurement of the absorption coefficient of hypersound in ruby (Al_2O_3 with impurity of 0.05% Cr_2O_3) was carried out at frequencies of 10^9 and 9.4×10^9 Hz in the direction of the trigonal Z axis (the optic axis). For ruby, just as for quartz, one can distinguish three regions with different temperature dependence for the absorption coefficient.

The region of low temperatures, where the temperature dependence of absorption is small, the total absorption is determined by the crystal defects. This region extends to a temperature of ≈ 50° K. The region of strong temperature dependence of the absorption is T = 50–100° K. The high temperature region is T > 100° K, where the absorption changes slightly with temperature. The difference of the frequency-temperature absorption curves from the corresponding curves for quartz lies in two areas: a) the region of strong temperature dependence for the ruby is located at much higher temperatures than the corresponding region in quartz; b) the temperature dependence of the absorption of the longitudinal wave is less steep (T^4–T^5) than the dependence of the absorption of the longitudinal wave in quartz. In contrast with quartz, the most pronounced frequency dependence of the absorption coefficient occurs at 10^9 and 10^{10} Hz. The value of the absorption coefficient of the longitudinal wave in ruby in the high temperature region, amounts to ≈ 1 db/cm at 10^9 Hz, which is much less than the absorption of the longitudinal wave in quartz in the same temperature range.

4. DISCUSSION OF THE RESULTS

1. First we shall consider the absorption of hypersonic waves in the high temperature region, where the absorption coefficient changes comparatively little with temperature. The absorption of the longitudinal hypersonic wave in this region agrees qualitatively with the theory of Akhiezer,[8] according to which the sound absorption at $\omega_0\tau \ll 1$ ($\omega_0/2\pi$ is the sound frequency, τ_0 the relaxation time of the phonons) is connected with a change in the distribution function of the phonon gas in the presence of the sound wave. This mechanism is seen to be very strong at 10^9 Hz,[2] much stronger than the classical mechanism of thermal conductivity, which brings about the inverse process of heat transfer between the regions of compression and expansion in the hypersonic wave.

It is more difficult to explain the absorption of the transverse hypersonic wave in the high temperature region. The effect of thermal conductivity evidently does not take place in this case. The comparatively weak temperature dependence of the absorption for transverse waves agrees with the Akhiezer theory; a quantitative comparison, however, is difficult because of the absence of data on the anharmonic coefficients corresponding to deformations in the transverse wave. Attention is called to the clearly expressed maximum for the absorption of the transverse wave in the region of temperatures of 60 °K and the subsequent monotonic decrease in the absorption with increase in temperature.

The presence of such a maximum is obviously not connected with the Akhiezer mechanism and indicates that in the present case, along with this mechanism, other processes of absorption of the transverse hypersound play a significant role, especially scattering by dislocations.[9]

2. We consider the absorption of hypersound at low temperatures, in the region with the strong temperature dependence of the absorption coefficient. In this region $\omega_0 \tau_0 > 1$, and the absorption of the hypersound can be regarded as the scattering of hypersonic quanta by phonons. In this connection, we represent the external hypersonic wave in the form of a flux of N_0 phonons with wave vector $\mathbf{k}_0$, polarization p_0, and frequency ω_0, and we regard its absorption as a decrease in the number of quanta N_0 as the result of transitions for which the occupation numbers of the thermal phonons change.

By considering the normal three-phonon processes of interaction of phonons, which are most important at low temperatures, we shall start out from the Hamiltonian which, in accord with [10], has the form

$$\mathcal{H} = \frac{1}{3!} \sum_{\mathbf{k}_0 p_0 \mathbf{k}_1 p_1 \mathbf{k}_2 p_2} \delta_{0,\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_0} (\overset{+}{a}_{\mathbf{k}_0 p_0} - a_{-\mathbf{k}_0 p_0}) (\overset{+}{a}_{\mathbf{k}_1 p_1} - a_{-\mathbf{k}_1 p_1}) \times (\overset{+}{a}_{\mathbf{k}_2 p_2} - a_{-\mathbf{k}_2 p_2}) F_{\mathbf{k}_0 p_0 \mathbf{k}_1 p_1 \mathbf{k}_2 p_2}; \tag{1}$$

Here a^+, a are the phonon annihilation and creation operators; $\mathbf{k}_0$, $\mathbf{k}_1$, $\mathbf{k}_2$ and p_0, p_1, p_2 are the wave vectors and polarizations of the interacting phonons; $\delta_{0,\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_0}$ is a function that differs from zero when $\mathbf{k}_0 + \mathbf{k}_1 + \mathbf{k}_2 = 0$ (the condition of conservation of momentum); F is a scalar which, within the framework of the theory of an elastic continuum, is determined by the expression

$$F_{\mathbf{k}_0 p_0 \mathbf{k}_1 p_1 \mathbf{k}_2 p_2} = \left(\frac{\hbar^3}{8V\rho_0^3} \right)^{1/2} \left[\frac{k_0 k_1 k_2}{v_{\mathbf{k}_0 p_0} v_{\mathbf{k}_1 p_1} v_{\mathbf{k}_2 p_2}} \right]^{1/2} \times \sum_{\alpha\beta\gamma\delta\rho\sigma} e^{\alpha}_{\mathbf{k}_0 p_0} \overset{0}{u}_{\beta} e^{\gamma}_{\mathbf{k}_1 p_1} \overset{1}{u}_{\delta} e^{\rho}_{\mathbf{k}_2 p_2} \overset{2}{u}_{\sigma} A^{\beta\delta\sigma}_{\alpha\gamma\rho}, \tag{2}$$

where V, ρ_0 are the volume and density of the solid, v the velocity of the phonons, e and u the direction cosines of the wave vectors and the polarizations of the phonons, and $A^{\beta\delta\sigma}_{\alpha\gamma\rho}$ the anharmonic coefficients of the crystal.

The conditions for the conservation of energy and momentum of the interacting phonons materially limit the number of processes leading to absorption of the external hypersonic quantum. In a crystal which possesses dispersion of the sound velocity, the absorption of the transverse hypersound can take place, as a consequence of the conservation rules, only as the result of scattering by the longitudinal thermal phonons. This process, which we tentatively denote as $t + l = l$, was first considered by Landau and Rumer[11] for the case of an isotropic body. The longitudinal hypersonic waves are absorbed as the result of the decay of the external longitudinal hypersonic quantum by means of the longitudinal and transverse thermal phonons $l = l + t$.[12]

The condition of symmetry of the trigonal crystal (in particular, quartz and ruby), as Herring has shown,[13] also solves another absorption process of the longitudinal sound—the result of scattering by transverse thermal phonons which are propagated close to the line of creation of the phonon spectrum $l + t' = t''$.

On the basis of (1), (2), and the conservation conditions, we obtain the following expression for the probabilities of the considered three-phonon processes of hypersound absorption:

$$P_t^{ll} = Q_1 \frac{N_0}{v_t v_l^8} \omega_0 T^4, \qquad P_l^{tl} = Q_2 \frac{N_0}{v_l^5 v_t^4} \omega_0^4 T,$$

$$P_l^{t't''} = Q_3 \frac{N_0}{v_l^2 v_t^7} \omega_0^3 T^2. \tag{3}$$

Here P_t^{ll}, P_l^{tl}, $P_l^{t't''}$ are, respectively, the probabilities of processes $t + l = l$, $l = t + l$, $l + t' = t''$; v_t and v_l are the velocities of the transverse and longitudinal hypersonic waves; T is the temperature, $\hbar\omega_0 \ll kT$ and $T \ll \Theta$; Θ is the Debye temperature; Q_1, Q_2, Q_3 are the quadratic combinations of the anharmonic coefficients $A^{\beta\delta\sigma}_{\alpha\gamma\rho}$, computed on the basis of (2) and the conservation conditions.

We compare the probabilities of the absorption of hypersound computed by (3). The symmetry conditions of trigonal crystals do not cause any one of the coefficients $A^{\beta\delta\sigma}_{\alpha\gamma\rho}$ to vanish, and there is no reason for supposing that the quantities Q_1, Q_2, Q_3 [which enter in (3)] differ by more than an order of magnitude.[14] Taking this into account, we can estimate the ratio of the probabilities of absorption of hypersound in quartz and ruby with accuracy to within an order of magnitude:

$$a = \frac{P_l^{ll}}{P_t^{ll}} \approx \left(\frac{v_l}{v_t}\right)^3 \left(\frac{\hbar\omega_0}{kT}\right)^3,$$

$$b = \frac{P_l^{t't''}}{P_t^{ll}} \approx \left(\frac{v_l}{v_t}\right)^2 \left(\frac{\hbar\omega_0}{kT}\right)^2. \tag{4}$$

For the region of temperatures $T = 15-40\,^\circ K$ of interest to us and frequency $\omega_0 = 2\pi \times 10^{10}$ Hz we have, in the case of quartz: $a \approx 10^{-5}$, $b \approx 10^{-3}$.

Consequently, the absorption of the longitudinal hypersonic wave ought to be several orders of magnitude smaller than the absorption of the transverse wave. However, measurements both by ourselves and in [2-5] show that the absorption of the longitudinal and transverse waves in the temperature ranges under consideration are of the same order of magnitude. No less marked is the difference in the frequency-temperature dependences of the hypersonic absorption. This gives us a basis for assuming that one and the same process, different from those considered above, is responsible for the absorption of the longitudinal and transverse hypersonic waves.

In the calculation of the probabilities of the processes (3), it was assumed that the lifetime of the interacting phonons is practically unlimited. This circumstance is taken into consideration by the fact that a δ-function of the difference of the energy of the interacting phonons enters into the corresponding integral for the determination of P.[11] Let us consider the absorption of hypersound as the result of phonon-phonon collisions, assuming that the time τ is finite, and is equal to the mean relaxation time τ_0. We assume here that $\omega_0\tau_0 > 1$.

Calculation shows that the introduction of a finite τ_0 into consideration (in place of the δ-function, we have used another function with a finite maximum and which falls off continuously to zero) leads only to a small correction in the expression for the probabilities (3)—of the order of $(\omega_0\tau_0)^{-1}$. However, an important consequence of the finite τ_0, as Simons[15] has shown and, more rigorously, Maris,[16] is the resolution of such processes of interaction of the phonons which, as $\tau_0 \to \infty$, are forbidden by the conservation conditions—in particular, the interaction of parallel longitudinal phonons.[15]

Let us consider the absorption of hypersound in the interaction of phonons of the same polarization. The conservation of energy can be written in the form

$$|\omega_2(\mathbf{k}_2, p_2) - \omega_1(\mathbf{k}_1, p_1) - \omega_0(\mathbf{k}_0, p_0)| \leqslant 1/\tau_0. \tag{5}$$

Since $\hbar\omega_0 \ll kT$, then the essential role in the sound absorption is played only by phonons with frequencies ω_1, $\omega_2 \gg \omega_0$. Taking this into account, we have

$$\left|\cos\gamma - \frac{v_0}{u_1}\right| \leqslant \frac{v_0}{u_1}\,\frac{1}{\omega_0\tau_0}, \tag{6}$$

where

$$v_0 = \frac{\omega_0}{k_0}\mathbf{e}, \qquad \mathbf{u}_1 = \frac{\partial\omega_1}{\partial\mathbf{k}_1}, \qquad \cos\gamma = \left(\mathbf{e}, \frac{\partial\omega_1/\partial\mathbf{k}_1}{u_1}\right),$$

$\mathbf{e}$ is a unit vector with the direction of the phase front of the hypersonic wave.

Account of dispersion leads to the following expression for the cosine of the angle of the interacting phonons of the same polarization:

$$\cos\gamma \geqslant 1 - \frac{1}{\omega_0\tau_0} + \frac{1}{2}\left(\frac{\pi}{2}\,\frac{\omega_1}{\omega_m}\right)^2, \tag{7}$$

where ω_1 is the frequency of the phonon interacting with the external hypersonic phonon, and ω_m is the limiting frequency. The condition (7) shows that the quanta of the external hypersound interact with the phonons of the same polarization and the frequency of these phonons does not exceed $\omega_p = 2\sqrt{2}\,\omega_m/\pi\sqrt{\omega_0\tau_0}$.

Taking it into account that the angle between the direction of the interacting phonons is small, $0 \leq \gamma \leq (2/\omega_0\tau_0)^{1/2}$, and that for longitudinal phonons only one term of the sum (2) is significant, with the coefficient A_{111}^{111}, we obtain on the basis of (1), (2), and (7), an expression for the probability of the absorption of the longitudinal hypersonic wave in scattering by the longitudinal thermal phonons (the process $l + l = l$):

$$P_l^{ll} = \frac{2\pi N_0 k^4 (A_{111}^{111})^2}{8V\rho_0{}^3\hbar^2 v_l^9}\,\omega_0 T^4 \int_0^{x_p} \frac{x^4 e^x dx}{(e^x - 1)^2},$$

$$x_p{}^l = \frac{2\sqrt{2}\hbar\omega_m{}^l}{\pi kT(\omega_0\tau_0{}^l)^{1/2}}, \tag{8}$$

where ω_m^l and τ_0^l are the limiting frequency and the relaxation time for the longitudinal phonons. An expression analogous to (8) can be obtained for the absorption of the transverse hypersound in scattering by transverse thermal phonons (the process $t + t = t$):

$$P_t{}^{tt} = \frac{2\pi N_0 k^4}{8V\rho_0{}^3\hbar^2 v_t{}^9} Q^2 \omega_0 T^4 \int_0^{x_p^t} \frac{x^4 e^x\,dx}{(e^x - 1)^2},$$

$$x_p{}^t = \frac{2\sqrt{2}\hbar\omega_m^t}{\pi kT(\omega_0\tau_0{}^t)^{1/2}}, \tag{9}$$

where Q is the linear combination of the anharmonic coefficients A_{111}^{222} and A_{111}^{333} (the transverse wave is propagated along the X axis of the quartz or ruby); ω_m^t and τ_0^t are the limiting frequency and the relaxation time at the transverse phonons.

In view of the absence of data on the coefficients $A_{\alpha\gamma\rho}^{\beta\delta\sigma}$ and of exact values of ω_m and τ_0, we can get only qualitative comparison of theory with experiment.

Let us consider the absorption of a longitudinal hypersonic wave in quartz at low temperatures in the range 15–40°K, where the strong temperature dependence of the hypersonic absorption is observed. The frequency-temperature dependence of the absorption for the process $l + l = l$ follows from (8):

$$P_l^{ll} \sim \omega T^4 \zeta(x_p{}^l), \quad x_p{}^l = -\frac{2\sqrt{2}\hbar\omega_m{}^l}{\pi k T(\omega_0\tau_0{}^l)^{1/2}} = A\frac{T^{3/2}}{\sqrt{\omega_0}},$$

$$A = 2\sqrt{2}\hbar\omega_m{}^l/\pi k\sqrt{\alpha^l}, \quad \tau_0{}^l = \alpha^l T^{-5}, \quad \alpha^l = \mathrm{const}, \quad (10)$$

where the integral in (8) is expressed as a function of the upper limit by $\zeta(x_p^l)$. The choice of the temperature dependence for τ_0^l in (10) of the form T^{-5} means that the lifetime of the dominant thermal phonons with energy $\hbar\omega \approx kT$, with which the hypersonic quanta chiefly collide, is determined by the normal three-phonon processes of scattering with a characteristic temperature dependence of the probability, according to (3), given by $\omega_0^\alpha T^{5-\alpha}$ (where $\alpha = 1, 3, 4$). Then, for $\omega_0 = kT/\hbar$ we get $P \sim T^{+5}$, and consequently, $\tau_0 \sim T^{-5}$. Such a dependence also agrees with the measurements of the specific heat and the thermal conductivity[17] in the temperature range of interest to us.

The dependence of P_l^{ll} on the frequency and temperature, given by (10), permits a satisfactory description of the character of the frequency-temperature curves of the absorption of longitudinal hypersound in quartz and ruby. Thus, for example, at 20°K, according to [17], $\tau_0 \approx 5\times10^{-9}$ sec, $\Theta = \hbar\omega_m/k = 300$°K and for a frequency $\omega_0 = 2\pi \times 10^{10}$ Hz, we have $x_p^l \approx 1$. The function $\zeta(x_p^l)$ can be approximated here as $(x_p^l)^2$. We then get $P_l^{ll} \sim T^7$, which does not depend on the frequency.

With increase in temperature x_p^l increases and for $x_p^l > 10$ the function $\zeta(x_p^l)$ is practically independent of its argument, and consequently $P_l^{ll} \sim \omega_0 T^4$, that is, a frequency dependence appears but the temperature dependence becomes weaker. Furthermore, one can, for the given type of wave and crystal, choose a value of the parameter A in (10) so as to reconcile the experimental and theoretical curves of the frequency-temperature dependence of the absorption of hypersound. Figure 4 illustrates such an alignment for the absorption co-

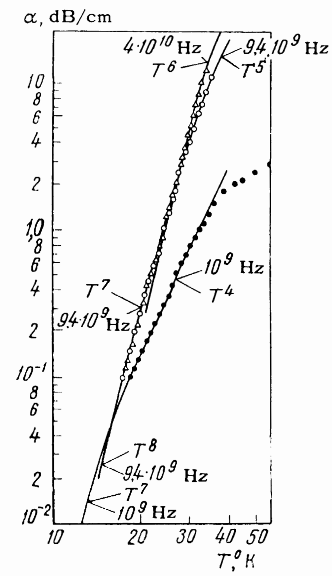

FIG. 4. Matching of experimental data with the theoretical curve for the absorption of a longitudinal wave along the X axis of quartz: △, ○, ● – experiment, the continuous curve, theory.

efficient of longitudinal hypersonic waves with frequencies 10^9, 9.4×10^9, and 4×10^{10} Hz, which are propagated in the direction of the binary X axis of the quartz.

The absorption in the ruby is considered in a similar fashion. The shift of the region with strong temperature dependence of the hypersonic absorption in the ruby in the direction of higher temperatures is associated with the large value of the Debye temperature ($\Theta = 980$°K). The values for the parameter A in the case of longitudinal hypersound in quartz and ruby are shown in the table.

A comparison of the probabilities of the absorption of the longitudinal hypersound in the process $l + l = l$ for $x_p \approx 1$ with processes of the type $l + t' = t''$ and $l = t + l$ under the same assumptions as for (4) gives

$$P_l{}^{l,\,l} \gg P_l{}^{t,\,l},\, P_l{}^{t',\,t''}.$$

We consider the absorption of transverse hypersonic waves. The frequency-temperature dependence of the probability, because of the scattering

Type of wave	Crystal (direction of propagation)	$A\cdot10^{-3}$, $\sec^{-1/2}\cdot\deg^{-3/2}$
Longitudinal	quartz, X axis	8
Fast transverse	quartz, X axis	2
Slow transverse	quartz, X axis	18
Longitudinal	quartz, Z axis	6
Longitudinal	ruby, Z axis	2

t + t = t, is similar to the dependence for the longitudinal hypersound, and permits us to describe satisfactorily the experimental results of the absorption of the fast and slow transverse waves in the direction of the binary X axis of quartz in a temperature range 15–40° K, where the parameter $x_p^t \geq 1$. The corresponding values of the parameter A are shown in the table. It is easy to show also that $P_t^{tt} \gg P_t^{ll}$, since $v_l > v_t$. However, it is important to note that the process of the absorption of transverse waves t + t = t is dominant in the trigonal crystals of quartz, ruby, or other crystals in which the anharmonic coefficients A_{111}^{222} and A_{111}^{333} are different from zero. In cubic crystals in which these coefficients are equal to zero, the process of the type t + t = t is absent and the absorption will be determined by the process of Landau and Rumer, $t + l = l$, with the corresponding frequency-temperature dependence $\omega_0 T^4$, which agrees with the recent measurements of the absorption of hypersound in cubic crystals.[18]

The process of Landau and Rumer will be dominant among three-phonon processes in the absorption of transverse hypersound in an arbitrary crystal for very low temperatures, where $x_p^t \ll 1$ or $T \ll (\pi^2 \omega_0 \alpha^t / 8\Theta)^{1/3}$. The probability of scattering of one type of phonons is materially reduced since $\xi(x_p^t) \ll 1$. Under these conditions, $l + t' = t''$, the role of the Herring process also increases. However, the experimental discovery of such procprocesses is made difficult by the fact that the absorption of the hypersound at low temperatures (T < 5—10°K for quartz and T < 30—40°K for ruby) is due principally to scattering by the crystal defects.

It is necessary to note also that the measurement of the frequency-temperature dependence of the hypersonic absorption in the interaction with thermal phonons of the same polarization makes it possible to estimate the relaxation times τ_0 and τ_0^t for the dominant phonons at low temperatures.

In conclusion, we express our gratitude to V. N. Balabanov for help in completing the measurements, to V. N. Glazunov for activity in preparation of the crystalline specimens. We are also grateful to É. A. Kaner and V. G. Bar'yakhtar for useful discussions of the results of the research.

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Translated by R. T. Beyer
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