NONLINEAR THEORY OF WAVE PROPAGATION IN SEMICONDUCTORS

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Propagation of electromagnetic waves in a semiconductor is studied by taking into account nonlinear effects due to heating of the electrons by the field. Nonlinear anomalous and normal skin effects are considered. The character of the field attenuation and also the dependence of the effective electron temperatures on the frequency of the incident field and its amplitude are studied for both the resonance and nonresonance cases. It is shown that the effective temperature in the resonance case exceeds that in the nonresonance one. It is found that the attenuation depth of the electron temperature is greater than the attenuation depth of the field in the anomalous case and of the same order of magnitude in the normal case. The dependence of the surface impedance on the amplitude and frequency of the incident electromagnetic field and the stationary magnetic field is found. The specific interaction of electromagnetic waves due to heating of the electron gas is considered. It is shown that propagation of small-amplitude waves may change considerably in the presence of a large-amplitude wave.

NONLINEAR effects connected with the heating of the carrier gas by the delayed transfer of energy from the carriers to the lattice take place in semiconductors situated in relatively weak electric fields[1]. The influence of this circumstance on the propagation of electromagnetic waves in a semiconductor was considered by one of the authors in[2], where it was shown that the heating greatly influences the characteristics of the electromagnetic waves propagating in the semiconductor. The connection between the electric current and the electric field producing it was assumed in this case to be local, that is, normal skin effect was assumed to take place. There is undisputed interest in an investigation of the anomalous skin effect under conditions when the electron gas is heated by the electric field.

The terms normal and anomalous skin effect must be specially defined in nonlinear theory. Nonlinear propagation of electromagnetic waves involves two parameters with the dimensions of length: the mean free path l_m, connected with the transfer of momentum, and the mean free path l_e connected with the transfer of energy [1]. In the case of quasielastic scattering by the phonons, the only scattering that will be considered here, we have $\hbar\omega_\eta/\epsilon \ll 1$ [1] and $l_m \ll l_e$. Here ω_η is the frequency of the phonons on which the energy relaxation takes place, and ϵ is the carrier energy.

The normal skin effect takes place if the field penetration depth L greatly exceeds the two mean free paths. In our case these conditions can be written as follows: $L \gg l_e, l_m$. In the case of the anomalous skin effect, the depth of field penetration L is comparable with or smaller than the mean free path. In the linear theory there is only one such length—l_m; this is why in weak fields there is only the anomalous skin effect, determined by the relation $L \lesssim l_m$. In the nonlinear theory there are two anomalous skin effects, for $L \lesssim l_e$ and $L \lesssim l_m$. We note, however, that in the "second" anomalous skin effect ($L \lesssim l_m$) the principal role is played not by effects connected with the heating of the carrier gas, but by the so-called striction effects[3]. It is essential that the "second" skin effect in semiconductors is not realizable in practice, owing to the small mean free path connected with the momentum transfer, and will therefore not be considered here. Thus, in this paper we shall take anomalous skin effect to mean an electromagnetic propagation process for which the inequality $l_m \ll L \lesssim l_e$ is satisfied, that is, we shall investigate the "first" anomalous skin effect.

Having made these preliminary remarks, let us formulate the subject of the present communication. We investigate in this article the propagation of electromagnetic waves in a semiconductor for both the anomalous and the normal skin effect. For the case of the normal skin effect we obtain a number of new results, compared with [2]. The anomalous skin effect for nonlinear propagation is considered in the present paper for the first time.

1. FUNDAMENTAL EQUATIONS OF THE PROBLEM

The system of equations defining the problem consists of the kinetic equation for the distribution function and of Maxwell's equations. Assuming that the collisions between the electrons and phonons are quasielastic, the carrier distribution function can be sought in the form [4]

$$\Phi(\mathbf{p},\mathbf{r},t) = f(\varepsilon,\mathbf{r},t) + (\chi(\varepsilon,\mathbf{r},t),\ \mathbf{p}/p). \tag{1.1}$$

Here $\mathbf{p}$ is the quasimomentum of the carrier, $\mathbf{r}$ its coordinates, t the time, and ϵ the carrier energy connected with the quasimomentum by the relation $\epsilon = p^2/2m$, where m is the effective mass of the carrier. We shall henceforth consider electrons, although all the deductions pertain, of course, also to holes.

By means of the usual methods (see, for example, [1,4]) we can show that the second term of (1.1) is much smaller than the first, and obtain for f and χ the following equations:

$$\frac{\partial f}{\partial t}+\frac{\mathbf{p}}{3m}\nabla_r\chi+\frac{e}{3}\frac{1}{n(\varepsilon)}\frac{\partial}{\partial\varepsilon}\left\{\frac{p}{m}n(\varepsilon)\chi\mathbf{E}\right\}$$
$$=\frac{1}{n(\varepsilon)}\frac{\partial}{\partial\varepsilon}\left\{n(\varepsilon)A(\varepsilon)\left[\frac{1}{T}f(\varepsilon)(1-f(\varepsilon))+\frac{\partial f}{\partial\varepsilon}\right]\right\}$$
$$+S\{f,f\},$$

$$\frac{\partial\chi}{\partial t}-\omega_H[\mathbf{h}\chi]+\nu(\varepsilon)\chi=\frac{p}{m}\nabla_r f+e\mathbf{E}\frac{p}{m}\frac{\partial f}{\partial\varepsilon}. \tag{1.2*}$$

Here $n(\epsilon) = 4\sqrt{2}\pi m^{3/2}\epsilon^{1/2}$ is the state density, $\mathbf{E}$ the electric field, $\omega_H = |e|H/mc$ is the Larmor frequency, H the external constant magnetic field, $\mathbf{h} = \mathbf{H}/H$, and $A(\epsilon)$ and $\nu(\epsilon)$ are given by

$$A(\varepsilon)=\frac{T}{\pi}\sqrt{\frac{2m}{\varepsilon}}\int_0^{2p}d\eta\,\eta W_\eta\hbar\omega_\eta,$$

$$\nu(\varepsilon)=\frac{\pi T}{p\varepsilon}\int_0^{2p}d\eta\,\eta^3W_\eta(\hbar\omega_\eta)^{-1}, \tag{1.3}$$

where T is the lattice temperature, η the quasimomentum of the phonon, and W_η the probability of transition of the electron from the state $\mathbf{p}$ to the state $\mathbf{p} \pm \eta$ when the electron is scattered by the phonon.

The quantity $A(\epsilon)/\epsilon^2$ coincides, apart from a constant factor, with the frequency $\nu_e(\epsilon)$ of the collisions connected with the energy transfer, and $\nu(\epsilon)$ is the collision frequency connected with the momentum transfer [1)]. In the presence of several mechanisms for the transfer of energy and momentum we have

$$A(\varepsilon)=\sum_k A_k(\varepsilon),\qquad \nu(\varepsilon)=\sum_l \nu_l(\varepsilon). \tag{1.4}$$

The summation over k is carried out here over all the energy-transfer mechanisms, while the summation over l is for the momentum transfer. We note that $\nu_e \sim (\hbar\omega_\eta/\epsilon)^2\nu(\epsilon)$, $S\{f, f\}$ is the collision integral describing the electron-electron collisions and having an order of magnitude $\nu_{ee}(\epsilon)f(\epsilon)$ ($\nu_{ee}(\epsilon)$—frequency of interelectron collisions). Formulas for $\nu_{ee}(\epsilon)$ for nondegenerate and degenerate electron gases are given in [5,8].

We assume satisfaction of the inequality

$$\nu_{ee}(\varepsilon)\gg\nu_э(\varepsilon) \tag{1.5}$$

(for estimates it is necessary to put in this inequality $\epsilon = \bar{\epsilon}$, where $\bar{\epsilon}$ is of the order of the Fermi energy ϵ_0 for a fully degenerate gas and of the electron temperature for a nondegenerate gas). If the electron gas is nondegenerate and the energy transfer is via scattering by acoustic phonons, then the inequality (1.5) goes over into the criterion of Fröhlich and Paranjape [5].

It is well known (see [4], for example) that if inequality (1.5) is satisfied, then f is a Fermi function

$$f(\varepsilon,\mathbf{r},t)=\left(1+\exp\frac{\varepsilon-\mu(\mathbf{r},t)}{\Theta(\mathbf{r},t)}\right)^{-1}. \tag{1.6}$$

The chemical potential μ and the electron temperature Θ are determined here from two balance equations, one obtained by integrating the first equation of the system (1.2) over the momenta, and the other by multiplying this equation by the energy and integrating it over the momenta [4].

In the derivation of the balance equations we shall assume that the characteristic distance L, over which the field changes, is much larger than the Debye radius $d \sim (\bar{\epsilon}/4\pi e^2N)^{1/2}$. It is well known from plasma theory that when this inequality is satisfied the plasma can be regarded as quasineutral. For a semiconductor with carriers of the

*$[\mathbf{h}\chi] \equiv \mathbf{h}\times\chi$.

1)If the scattering is from an impurity, then W_η in the second formula of (1.3) is the transition probability connected with the electron-impurity collision.

same sign this means that the electron charge density at any point is equal to the equilibrium density, if we disregard processes such as impact ionization, change of the recombination coefficient in the field, etc. It follows therefore that the concentration of the electrons does not depend on the coordinates. In addition, we assume that the frequency of the incident field is $\omega \gg \nu_e$. As shown by Ginzburg and Gurevich [4], under this assumption, in the zeroth approximation in ν_e/ω, we can neglect the derivative of f with respect to time in the first equation of (1.2). From this circumstance it follows that the temperature, together with the chemical potential (which is a function of the temperature and of the concentration), likewise does not depend on the time. The connection between the chemical potential, the temperature, and the concentration is given by the normalization condition

$$\frac{2}{h^3}\int f(\varepsilon, \mathbf{r})n(\varepsilon)d\varepsilon = N. \tag{1.7}$$

Let us consider an electromagnetic monochromatic wave incident on a half space z > 0 filled with a semiconductor or a semimetal. The wave is normally incident on the interface between the vacuum and the semiconductor z = 0. (It was shown by Ginzburg and Gurevich [4] that, under the assumptions made, monochromatic waves propagate in the semiconductor.) The electric field **E** can be represented as the sum of a constant field $\mathbf{E}_c$ connected with the gradient of the electron temperature and an alternating field $\sim e^{-i\omega t}$. The amplitude of this field will be henceforth denoted by **E**. This will not lead to any misunderstanding in the future. Accordingly χ in the second equation of (1.2) can be represented as the sum of two quantities: χ_c, which does not depend on the time, and $\chi_v e^{-i\omega t}$. With the aid of χ we can express the electric current j and the heat flux **Q** by means of the following formulas [6]:

$$\mathbf{j} = \frac{16\pi me}{3h^3}\int_0^\infty \varepsilon\chi(\varepsilon)\,d\varepsilon, \quad \mathbf{Q} = \frac{16\pi m}{3h^3}\int_0^\infty \varepsilon^2\chi(\varepsilon)\,d\varepsilon. \tag{1.8}$$

Substituting the value of χ obtained from the second equation of (1.2) into the first equation of the same system, and then integrating over all the momenta with and without multiplication by ϵ, we obtain a system of two transport equations:

$$\operatorname{div}\mathbf{j}_c = 0, \quad \operatorname{div}\mathbf{Q}_c = \overline{B}_{ik}E_iE_k^* - A(\Theta)(\Theta/T - 1). \tag{1.9}$$

We have introduced here the following notation (see [6]):

$$\mathbf{j}_c = e^2J_{10}\mathbf{E}_c' - eJ_{11}\nabla\ln\Theta + [e^2J_{20}\mathbf{E}_c' - eJ_{21}\nabla\ln\Theta, \mathbf{h}] + \mathbf{h}(e^2J_{30}\mathbf{E}_c' - eJ_{31}\nabla\ln\Theta, \mathbf{h}),$$

$$\mathbf{Q}_c = eJ_{11}\mathbf{E}_c' - J_{12}\nabla\ln\Theta + [eJ_{21}\mathbf{E}_c' - J_{22}\nabla\ln\Theta, \mathbf{h}] + \mathbf{h}(eJ_{31}\mathbf{E}_c' - J_{32}\nabla\ln\Theta, \mathbf{h}),$$

$$\mathbf{E}_c' = \mathbf{E}_c - \nabla\frac{\mu}{\Theta},$$

$$J_{lj} = -\frac{16\sqrt{2}\,\pi\omega_H^{l-1}m^{1/2}}{3h^3}\int_0^\infty \frac{d\varepsilon\,\varepsilon^{j+3/2}}{\nu^l[1+(\omega_H/\nu)^2]}\frac{df}{d\varepsilon}; \tag{1.10}$$

$$A(\Theta) = \frac{32m^3T}{h^3}\int_0^\infty \hbar\omega_{2p}W_{2p}f(\varepsilon)\,d\varepsilon,$$

$$\overline{B}_{ik} = -\frac{4e^2}{3m}\int_0^\infty d\varepsilon\,n(\varepsilon)\,\varepsilon B_{ik}(\varepsilon)\frac{df}{d\varepsilon},$$

$$B_{ik}(\varepsilon) = \frac{\nu(\varepsilon)}{(\omega^2 - \omega_H^2 + \nu^2(\varepsilon))^2 + 4\omega^2\nu^2(\varepsilon)} \times \Big\{(\omega^2 + \omega_H^2 + \nu^2(\varepsilon))\delta_{ik} + \frac{\omega_H^2}{\omega^2 + \nu^2(\varepsilon)} \times (\omega_H^2 - 3\omega^2 + \nu^2(\varepsilon))h_ih_k + 2i\omega\,\omega_H h_l e_{lik}\Big\}, \tag{1.11}$$

e_{lik} is a completely antisymmetrical unit tensor and j_c and $\mathbf{Q}_c$ are the densities of the electric current and of the heat flux, which are connected with the thermal emf. In the one-dimensional case, which is considered now, j_c, Q_c, Θ, and other quantities depend only on c. The integrals J_{li} and $\overline{B}_{ik}$ can be calculated in general form only for a fully degenerate Fermi gas. The corresponding formulas are quite simple, but will not be presented here, and we confine ourselves only to calculation of $A(\Theta)$.

For acoustic and optical phonons, in either a deformation or a polarization interaction with the lattice, the dependence of ω_{2p} and W_{2p} on the electron energy ϵ is determined by the following formulas [7]:

$$\hbar\omega_{2p} = \hbar\omega_T(\varepsilon/T)^{k_1}, \quad W_{2p} = W_T(\varepsilon/T)^{k_2}. \tag{1.12}$$

Here $\hbar\omega_T$ and W_T are the values of $\hbar\omega_{2p}$ and W_{2p} at $p = \sqrt{2T/m}$, $k_1 = 0$ for optical and $k_1 = 1/2$ for acoustic phonons, and the values of k_2 are given in [2].

Using this circumstance, we can obtain for $A(\Theta)$ the following formula:

$$A(\Theta) = A_0(\Theta/T)^{r-1}. \tag{1.13}$$

For a fully degenerate electron gas r = 1, and

$$A_0 = \frac{32m^3T\varepsilon_0\hbar\omega_TW_T}{(k_1 + k_2 + 1)\hbar^3}\left(\frac{\varepsilon_0}{T}\right)^{k_1+k_2},$$

while for nondegenerate gas $r = k_1 + k_2 + 1/2$ and

$$A_0 = \frac{4\sqrt{2}\Gamma(r + 1/2)}{\pi^{3/2}}Nm^{3/2}T^{1/2}\hbar\omega_TW_T.$$

Similarly we can write an expression for $\nu(\epsilon)$: for a nondegenerate gas $\nu(\epsilon) = \nu_0(T)(T/\epsilon)^q$, where

$$\nu_0(T) = \frac{2^{2(k_2-k_1)+5/2}\pi(mT)^{3/2}W_T}{(k_2-k_1+1)\hbar\omega_T}, \quad q = 1/2 + k_1 - k_2. \quad (1.14)$$

The values of r and q were presented for several cases earlier [2].

We now derive the energy balance equation. To this end it is necessary to eliminate the static field E_C from the expression for the heat flux Q_C. To do so, we use the boundary conditions for the static current. We assume that there is no static current in the xy plane. This is the customary supplementary condition for the investigation of isothermal thermomagnetic effects[6]. The z component of the current is likewise equal to zero. This follows from the first equation of (1.9) and from the absence of current flow through the vacuum-semiconductor interface. Thus, to determine E_C we have the condition $j_C = 0$. The thermomagnetic field E_C obtained in this manner is an interesting study in itself. In this communication we shall not consider it.

Substituting E_C in the second expression of (1.10) and taking into account the one-dimensionality of the problem, we obtained for Q_z the relation

$$Q_z(v) = -T\varkappa(v)\,dv/dz. \quad (1.15)$$

We have introduced here a dimensionless electron temperature $v = \Theta/T$ and $\kappa(v)$ is the coefficient of electronic thermal conductivity. For a nondegenerate gas, the thermal conductivity coefficient was calculated in[6]. In the absence of a magnetic field we can write the following expression for $\kappa(v)$:

$$\varkappa(v) = \lambda_0 v^{1+q}, \quad \lambda_0 = \left(\frac{5}{2}+q\right)\frac{uNT}{e},$$

$$u = \frac{4}{3\sqrt{\pi}}\Gamma\left(\frac{5}{2}+q\right)\frac{e}{m\nu_0}, \quad (1.16)$$

λ_0 is the heat conductivity in the absence of an electromagnetic field, and u is the mobility of the electrons in a weak electric field.

In a strong magnetic field

$$\varkappa(v) = \lambda_H^{\pm} v^{1\pm q}, \quad \lambda_H^{+} = \lambda_0\cos^2\varphi, \quad \varphi \neq \pi/2;$$

$$\lambda_H^{-} = \frac{C_q(5/2 - q + q^2)}{5/2 + q}\lambda_0\left(\frac{uH}{c}\right)^{-2}, \quad \varphi = \frac{\pi}{2},$$

$$C_q = \frac{16}{9\pi}\Gamma\left(\frac{5}{2}+q\right)\Gamma\left(\frac{5}{2}-q\right). \quad (1.17)$$

Here λ_H is the thermal conductivity in the absence of an electromagnetic field, the plus sign pertains to $\varphi \neq \pi/2$, and the minus sign to $\varphi = \pi/2$ (φ is the angle between the magnetic field and the z axis). The criteria for the weak and strong magnetic field are the inequalities:

$$\left(\frac{\omega_H}{\nu}\right)^2 \sim \left(\frac{uH}{mc}v^q\right)^2 \ll 1, \quad \left(\frac{\omega_H}{\nu}\right)^2 \sim \left(\frac{uH}{mc}v^q\right)^2 \gg 1.$$

When $\varphi = 0$, Eq. (1.17) goes over into (1.16).

For a completely degenerate electron gas

$$\varkappa(v) = \frac{\pi^2 NT}{3\nu(\varepsilon_0)m} \times v\left[1 + \frac{e^2H^2}{\nu^2(\varepsilon_0)m^2c^2}\cos^2\varphi\right] \Big/ \left[1 + \frac{e^2H^2}{\nu^2(\varepsilon_0)m^2c^2}\right]$$

$$\nu(\varepsilon_0) = \nu_0(T/\varepsilon_0)^q. \quad (1.18)$$

Finally, we can rewrite the energy balance equation in the form

$$T\frac{d}{dz}\varkappa(v)\frac{dv}{dz} + \bar{B}_{ik}(v)E_iE_k^* = A_0 v^{r-1}(v-1). \quad (1.19)$$

It is necessary to add the boundary conditions to this differential equation. We choose them in the form

$$\left.\frac{dv}{dz}\right|_{z=0} = 0, \quad v|_{z\to\infty} \to 1. \quad (1.20)$$

The first boundary condition corresponds to the vanishing of the heat flux through the surface z = 0, and the second to the equality of the electron and lattice temperatures at infinity.

In addition to the heat-conduction equation, the complete system of equation contains Maxwell's equations, which in our case are

$$\frac{d^2E_x}{dz^2} + \frac{\omega^2}{c^2}[AE_x - iBE_y] = 0,$$

$$\frac{d^2E_y}{dz^2} + \frac{\omega^2}{c^2}[iBE_x + CE_y] = 0,$$

$$E_z = -\frac{\sin\varphi}{\varepsilon_{11}\sin^2\varphi + \varepsilon_{33}\cos^2\varphi}\{\varepsilon_{12}E_x + (\varepsilon_{33}-\varepsilon_{11})E_y\cos\varphi\},$$

$$A = \varepsilon_{11} + \frac{\varepsilon_{12}^2\sin^2\varphi}{\varepsilon_{11}\sin^2\varphi + \varepsilon_{33}\cos^2\varphi},$$

$$iB = \frac{\varepsilon_{12}\varepsilon_{33}\cos\varphi}{\varepsilon_{11}\sin^2\varphi + \varepsilon_{33}\cos^2\varphi}$$

$$C = \frac{\varepsilon_{11}\varepsilon_{33}}{\varepsilon_{11}\sin^2\varphi + \varepsilon_{33}\cos^2\varphi} \quad (1.21)$$

By ϵ_{11}, ϵ_{12}, and ϵ_{33} we denote the components of the dielectric tensor in a coordinate frame in which the axis 33 is directed along the magnetic field:

$$\varepsilon_{11} = 1 - \frac{32\sqrt{2}\pi^2 m^{1/2} e^2}{3\omega}\int_0^\infty \frac{(\omega + i\nu(\varepsilon))\varepsilon^{3/2}}{\omega_H^2 - (\omega + i\nu(\varepsilon))^2}\frac{df}{d\varepsilon}d\varepsilon,$$

$$\varepsilon_{12} = \frac{32\sqrt{2}\pi^2 m^{1/2} e^2 \omega_H i}{3\omega}\int_0^\infty \frac{\varepsilon^{3/2}}{\omega_H^2 - (\omega + i\nu(\varepsilon))^2}\frac{df}{d\varepsilon}d\varepsilon,$$

$$\varepsilon_{33} = 1 - \frac{32\sqrt{2}\pi^2 m^{1/2} e^2}{3\omega}\int_0^\infty \frac{\varepsilon^{3/2}}{\omega + i\nu(\varepsilon)}\frac{df}{d\varepsilon}d\varepsilon. \tag{1.22}$$

The boundary conditions for Maxwell's equations have, as usual, the form

$$E_{x,y}|_{z=+0} = E_{x,y}|_{z=-0}; \quad \left.\frac{\partial E_{x,y}}{\partial z}\right|_{z=+0} = \left.\frac{\partial E_{x,y}}{\partial z}\right|_{z=-0};$$

$$\lim_{z\to\infty} \mathbf{E} \to 0. \tag{1.23}$$

If a plane wave $E^{(0)}(\exp(i\omega z/c) + P\exp(-i\omega z/c))$ is incident on a half-space filled with a dielectric, then it follows from the boundary conditions (1.23) that the expressions for the reflection coefficient P and for the reflection coefficient R are

$$R = \frac{E(+0)}{E(-0)}, \quad R = \frac{2\zeta}{1+\zeta}; \quad P = \frac{\zeta - 1}{\zeta + 1};$$

$$\zeta^{-1} = -\frac{ic}{\omega}\frac{1}{E(+0)}\frac{\partial E(+0)}{\partial z}. \tag{1.24}$$

When $\zeta \ll 1$ we have

$$R = 2\zeta, \quad P = -1 + 2\zeta. \tag{1.25}$$

Let us note one more circumstance. Maxwell's equations are not linear, by virtue of the dependence of A, B, and C on the temperature. For semimetals and degenerate semiconductors, however, this is not the case, for accurate to $(\Theta/\epsilon_0)^2 \ll 1$, $df_0/d\epsilon = -\delta(\epsilon - \epsilon_0)$, and, as can be seen from (1.21), A, B, and C are not functions of the temperature, and the fields are described by the formulas of the linear theory.

In concluding this section, let us make one more remark. The Fermi distribution function (1.6), with parameters determined by the transport equations (1.9), is the first term in the expansion of the distribution function when inequality (1.5) is satisfied. However, even if this inequality is not satisfied, in this case the results obtained for the macroscopic quantities are sufficiently accurate. This circumstance was discussed many times in the literature. We shall return to it in the third section.

2. ANOMALOUS SKIN EFFECT

It will be convenient to rewrite the energy balance equation (1.19) in the form

$$d^2w/dz^2 - \delta^2 Q(w) = -P_{ik}(w)E_i E_k^*;$$

$$w = \int_0^v \varkappa(v)\,dv \Big/ \int_0^1 \varkappa(v)\,dv, \quad \delta^2 = A_0 \Big/ T\int_0^1 \varkappa(v)\,dv,$$

$$Q(w) = [v(w)]^{r-1}\{v(w) - 1\},$$

$$P_{ik} = \bar{B}_{ik}[v(w)] \Big/ T\int_0^1 \varkappa(v)\,dv. \tag{2.1}$$

The boundary conditions for w follow from the boundary conditions for v and are

$$\left.\frac{dw}{dz}\right|_{z=0} = 0, \quad w|_{z=\infty} = 1. \tag{2.2}$$

From the expression for δ it follows that $\delta \sim 1/l_e$ when $\varphi = \pi/2$ and $\delta \sim uH/cl_e$ when $\varphi = \pi/2$ and $uH/c \gg 1$. It is interesting to note that when $uH/c \gg 1$ and $\varphi = \pi/2$, the value of δ increases rapidly and the conditions for the anomaly of the skin effect become more stringent. We shall consider a strongly anomalous skin effect ($L \ll \delta^{-1}$).

To solve (2.1) we can use in this case the method of successive approximations, neglecting the right-hand side in the zeroth approximation, and then regarding it as a perturbation. The physical meaning of this procedure consists in the following. It follows from (2.1) that the characteristic distance over which the temperature decreases is $1/\delta$. By virtue of the large anomaly of the skin effect, the field attenuates much more rapidly. Thus, the right side of (2.1) plays the role of surface sources and can be neglected when solving the balance equation in the zeroth approximation. It must be taken into account in the next approximation in order to satisfy the boundary conditions on the plane z = 0. When solving Maxwell's equations (1.21) in the direct vicinity of the boundary, the quantity w can be replaced by w_0, since w remains practically unchanged over distances of the order of L; this changes Maxwell's equations into a system with constant coefficients, which can be readily solved. To find the distribution of the field over the entire space, we can use the WKB method, owing to the slow change of w.

By virtue of the foregoing, the solution of (2.1) will be sought in the form

$$w = w' + w'', \; w'' \ll w', \tag{2.3}$$

where w′ is a solution of (2.1) without the right-hand side. This solution is written in the form

$$-\sqrt{2}\,\delta z = \int_{w_0'}^{w'} dw' \left[\int_1^{w'} dw'\,Q(w')\right]^{-1/2}. \tag{2.4}$$

In the derivation of (2.4) we took into account the boundary conditions at $+\infty$.

For w″ we obtain, accurate to quantities of order $\sim\delta/\xi$ ($\xi \sim 1/L$; $\delta/\xi \ll 1$),

$$d^2w''/dz^2 = -P_{ik}(w_0')E_iE_k^*. \tag{2.5}$$

As follows from the foregoing, the field near the boundary

$$E_i = E_{0i}\exp\{in(w_0')\omega z/c - \xi(w_0')z\}, \tag{2.6}$$

where $n(w_0')$ is the refractive index and $\xi(w_0')$ is the attenuation index. Substituting (2.5) in (2.6) we obtain

$$w'' = -\frac{P_{ik}(w_0')E_{i0}, E_{k0}^*}{4\xi^2(w_0')}e^{-2\xi(w_0')z}. \tag{2.7}$$

From the boundary condition for w at z = 0 we obtain an equation for the determination of w_0':

$$\frac{P_{ik}(w_0')E_{i0}E_{k0}^*}{2\xi(w_0')} = \sqrt{2}\,\delta\left[\int_1^{w_0'}Q(w')\,dw'\right]^{1/2}. \tag{2.8}$$

Formulas (2.3), (2.4), (2.7), and (2.8) completely solve the problem of determining w, and consequently also the dimensionless temperature.

We note that E_{i0} and E_{k0} in formula (2.7) must be expressed in terms of the amplitudes of the incident field E_i^0. This can be readily done with the aid of the formula for the refractive index $R(w_0')$. In addition, to be able to write down many formulas compactly, it is convenient to express all the E_i^0 in terms of one of them, say E_x^0 [2]. We shall henceforth omit the subscript x. In order to find the temperature as a function of w, we shall use formula (2.1). Taking (1.17) into account, we obtain

$$w = v^{2\pm q}. \tag{2.9}$$

For all the known cases of scattering, $2 \pm q > 0$. This inequality will henceforth be assumed satisfied, as well as the inequality $r \pm q > 0$, which ensures the absence of runaway[9].

We proceed to study the dependence of the temperature and of the fields on the coordinates. We consider first the temperature. Formulas (2.4), (2.7), and (2.9) determine completely the dependence of the temperature on the coordinate z. However, we are unable to calculate the integral in formula (2.4) for arbitrary r and q. We shall therefore consider two regions of the semiconductor: a region directly adjacent to the surface z = 0, when we confine ourselves to the case $v(z) \gg 1$, and a region deep inside the semiconductor, where the inequality $v - 1 \ll 1$ is satisfied.

If we assume that $v(z) \gg 1$, then the integral in (2.4) can be readily evaluated, and we obtain for the temperature the expression

$$v = v_0'\{[1-(2\pm q-r)\delta(v_0')z]\}^{2/(2\pm q-r)} - \frac{\delta(v_0')}{\xi(v_0')}e^{-2\xi(v_0')z} \tag{2.10}$$

where

$$\delta(v_0') = [2(2\pm q)(2\pm q-r)]^{-1/2}\delta v_0'^{(r-2\mp q)/2}.$$

The second term in the curly brackets is much smaller than the first, in accordance with the assumption that the skin effect is strongly anomalous ($\delta(v_0')/\xi(v_0') \ll 1$), and will be neglected. This neglect is equivalent to assuming that v_0' coincides with the temperature v_0 on the surface of the sample, so that no distinction will be made between the two. We note, however, that in the investigations of thermomagnetic effects, when the derivative of the temperature plays an important role, the second term in the curly brackets of formula (2.1) is significant.

Let now $v - 1 \ll 1$. This corresponds to that region of the sample, where the temperature of the electron gas is close to the lattice temperature. The values of v for large z can be readily obtained from (2.4) with the aid of (2.9) by separating the singularity of the integral, in analogy with the procedure used in the calculation of the self-action factor in[2]. For large z, we have the following formula for v:

$$v(z) = 1 + S_v\exp\{-\delta z/(2\pm q)^{1/2}\}, \tag{2.11}$$

where S_v is the self-action factor for a temperature and is determined from the expression

$$S_v = (v_0 - 1)\exp\left\{\frac{1}{\sqrt{2}}\int_{v_0}^{1}\left[v^{1\pm q}\left(\frac{v^{r+2\pm q}-1}{r+2\pm q} - \frac{v^{r+1\pm q}-1}{r+1\pm q}\right)^{-1/2} - \frac{\sqrt{2}}{v-1}\right]dv\right\}. \tag{2.12}$$

This factor behaves in different fashion, depending on the relation between r and q. A simple investigation shows that when $v_0 \gg 1$

$$S_v \sim \exp\left\{-\frac{2^{1/2}(r+2\pm q)^{1/2}}{2\pm q-r}v_0^{(2\pm q-r)/2}\right\} \ll 1$$

$$\text{for } 2\pm q-r > 0,$$

$$S_v \sim 1 \text{ for } 2\pm q-r < 0. \tag{2.13}$$

Formulas (2.10)–(2.13) go over into the formulas for a degenerate electron gas by replacing ν_0 with $\nu_0(\epsilon_0) = T_0(T/\epsilon_0)^q$, and putting q = 0 and r = 1 everywhere except in $\nu(\epsilon_0)$.

Let us consider now the electric field in a semiconductor. Once the temperature has been determined, Maxwell's equations (1.22) become, by virtue of (1.21), linear equations with coefficients that depend on z. As already indicated, owing to the inequality $\delta(v_0) \ll \xi(v_0)$, the temperature varies slowly as a function of the coordinates compared with the field, as a result of which we can use the WKB method for the solution of Maxwell's equations. The solution can be written up in general form, but then it becomes meaningful only after a number of simplifications, when definite limitations are imposed on the frequencies and on the magnetic fields.

We note that since we obtained the temperature as a function of the coordinate only in the regions where $v(z) \gg 1$ and $v(z) - 1 \ll 1$, the field is likewise defined only in these regions. In the region $v(z) \gg 1$, the explicit form of the field depends essentially on the character of the initial assumptions. For large z, in the region with $v(z) - 1 \ll 1$, we have

$$E(z) = RE^0 S_E \exp\left\{i\frac{\omega}{c}nz - \xi_0 z\right\}, \tag{2.14}$$

where S_E is the so called self-action factor [4], ξ_0 is the attenuation of the wave in the linear theory, and n is the refractive index in the linear theory. The self-action coefficient will be calculated below under various assumptions.

We shall consider three cases:

I. The following relations are satisfied

$$\frac{\nu}{\omega} \ll 1^{\,2)}, \quad \frac{\nu}{|\omega - \omega_H|} \ll 1, \quad \frac{\nu}{|\omega - \Omega_{1,2}|} \ll 1;$$

$\Omega_{1,2}$ are the magnetoplasma resonance frequencies[3)]. Physically this case corresponds to high frequencies. In addition, we assume that the frequencies are far from resonance.

Expanding A, B, and C in (1.21) in powers of ν and solving the resultant equations by the WKB method, we obtain for the field the formula

$$E(z) = RE^0 \exp\left\{i\frac{\omega}{c}nz - \xi_0 \int_0 v^{-q} dz\right\}. \tag{2.15}$$

In the region where $v(z) \gg 1$, we have

$$E(z) = RE^0 \exp\left\{i\frac{\omega}{c}nz + (2 \pm q - r - 2q)^{-1}\frac{\xi_0 v_0^{-q}}{\delta(v_0)}\right.$$
$$\left.\times\left[(1 - (2 \pm q - r)\delta(v_0)z)^{(2\pm q - r - 2q)/(2\pm q - r)} - 1\right]\right\}. \tag{2.16}$$

[2)]In longitudinal propagation this inequality can be replaced by $\nu/\omega_H \ll 1$.

[3)]The magnetoplasma resonance frequencies are given by the formula (see [18,2]):

$\Omega_{1,2}^2 = {}^1\!/_2(\omega_0^2 + \omega_H^2) \pm {}^1\!/_2[(\omega_0^2 + \omega_H^2)^2 - 4\omega_0^2\omega_H^2\cos^2\varphi]^{1/2}$.

Let us find the self-action multiplier. When $z \to \infty$ the integral in the exponential function in (2.15) also tends to infinity, since v tends to unity as $z \to \infty$. We separate the diverging part of this integral. To this end we rewrite (2.15) in the form

$$E(z) = RE^0 \exp\left\{i\frac{\omega}{c}nz - \xi_0 z - \xi_0\int_0^z (v^{-q} - 1)\,dz\right\}. \tag{2.15'}$$

For large z, the upper limit in the integral of (2.15) can be set equal to infinity, since this integral converges. Comparing (2.15) at $z = \infty$ with (2.14), we obtain for S_E the formula

$$S_E = \exp\left\{-\xi_0\int_0^\infty (v^{-q} - 1)\,dz\right\}. \tag{2.17}$$

It will be convenient for what follows to change over in (2.16) from integration with respect to z to integration with respect to v. This is readily done with the aid of (2.4) and (2.9). Ultimately we get

$$S_E = \exp\left\{\frac{(2 \pm q)^{1/2}\xi_0}{2^{1/2}\delta}\int_{v_0}^1 v^{1\pm q}(v^{-q} - 1)\right.$$
$$\left.\times\left(\frac{v^{r\pm q+2} - 1}{r \pm q + 2} - \frac{v^{r+1\pm q} - 1}{r + 1 \pm q}\right)^{-1/2} dv\right\}. \tag{2.17'}$$

From (2.17) we see that $S_E < 1$ when $q < 0$ and $S_E > 1$ when $q > 0$.

We write out the asymptotic expression for S_E with $v_0 \gg 1$:

$$S_E \sim \exp\left\{-\frac{2^{1/2}(2 \pm q)^{1/2}(r + 2 \pm q)^{1/2}}{2 \pm q - r - 2q}\frac{\xi_0}{\delta}v_0^{(2\pm q - 2q - r)/2}\right\} \ll 1$$

when $q < 0$ and $2 \pm q - 2q - r > 0$;

$$S_E \sim \exp\left\{\frac{2^{1/2}(2 \pm q)^{1/2}(r + 2 \pm q)^{1/2}}{2 \pm q - r - 2q}\frac{\xi_0}{\delta}v_0^{(2\pm q - r)/2}\right\} \gg 1 \tag{2.18}$$

when $2 > 0$ and $2 \pm q - r > 0$;

$$S_E \sim e^{\alpha\xi_0/\delta}, \quad \alpha\begin{cases} < 0, & q < 0 \\ > 0, & q > 0 \end{cases}, \quad |\alpha| \sim 1$$

when $2 \pm q - 2q - r < 0$ and $2 \pm q - r < 0$.

We present formulas for v_0 and ξ_0 when $v_0 \gg 1$:

$$v_0 = \left[\frac{(2 \pm q + r)^{1/2}(2 \pm q)^{1/2}\psi(\omega, \omega_H)\,|E_0|^2}{2\sqrt{2}\;\delta\lambda_H{}^{\pm}T}\right]^{2/(2\pm q + r)}. \tag{2.19}$$

In the general case, it is meaningless to write out the unwieldy expressions for $\Psi(\omega, \omega_H)$ and ξ_0.

Let us consider the particular cases when there is no magnetic field and of a longitudinally propagating helical wave ($\omega \ll \omega_H$; $\varphi = 0$). When $\omega_H = 0$ we have

$$\xi_0 = \frac{2\Gamma(^5/_2 - q)}{3\sqrt{\pi}} \frac{\omega_0^2 \nu_0}{c\omega^2[1-\omega_0^2/\omega^2]^{1/2}}, \quad n = \left(1 - \frac{\omega_0^2}{\omega^2}\right)^{\pm 1/2}$$

$$v_0 = \left\{\left[\frac{2\sqrt{2}(2+q+r)^{1/2}(2+q)^{1/2}}{\pi}\right]\right.$$

$$\left.\left[\frac{(1-\omega_0^2/\omega^2)^{1/2}}{[1+(1-\omega_0^2/\omega^2)^{1/2}]^2} \frac{c|E^0|^2}{T\lambda_0\delta_0}\right]\right\}^{2/(2+q+r)}. \quad (2.20)$$

In the case of helical waves we obtain for ξ_0 and v_0 the formulas

$$\xi_0 = \frac{2\Gamma(^5/_2 - q)}{3\sqrt{\pi}} \frac{\omega_0 \nu_0 \omega^{1/2}}{\omega_H^{3/2} c}, \quad n = \frac{\omega_0}{(\omega\omega_H)^{1/2}},$$

$$v_0 = \left[\frac{2\sqrt{2}(2+q+r)^{1/2}(2+q)^{1/2}}{\pi} \frac{\omega^{1/2}\omega_H^{1/2} c|E^0|^2}{\omega_0\delta_0\lambda_0 T}\right]^{2/(2+q+r)}, \quad (2.21)$$

δ_0 is the value of δ when H = 0.

II. Assume now that one of the following two relations is satisfied:

$$\omega = \omega_H, \quad \nu/\omega_H \ll 1, \quad \varphi = 0, \quad \frac{\omega_0^2 v^q}{\omega_H \nu_0} \ll 1$$

or

$$\omega_H = 0, \quad \frac{\nu}{\omega} \gg 1, \quad \frac{\omega_0^2 v^q}{\omega \nu_0} \ll 1.$$

Physically, the first corresponds to cyclotron resonance and the second to a low-frequency field. In both cases the displacement current is much larger than the conduction current. In view of the fact that the character of the derivation is the same here as in the first case, we write down the results immediately

$$E = RE^0 \exp\left\{i\frac{\omega}{c}z - \xi_0 \int_0 v^q dz\right\}; \quad (2.22)$$

when $v(z) \gg 1$ we have

$$E = RE^0 \exp\left\{i\frac{\omega}{c}z + \frac{1}{2+3q-r}\frac{\xi_0 v_0^q}{\delta(v_0)}[(1-(2+q+r)\right.$$

$$\left.\times\,\delta(v_0)z)^{(2+3q-r)/(2+q-r)} - 1]\right\}. \quad (2.23)$$

The self-action factor is

$$S_E = \exp\left\{\frac{(2+q)^{1/2}}{2^{1/2}} \frac{\xi_0}{\delta} \int_{v_0}^{1} v^{1+q}(v^q - 1)\right.$$

$$\left.\times\left(\frac{v^{r+2+q}-1}{r+2+q} - \frac{v^{r+1+q}-1}{r+1+q}\right)^{-1/2} dv\right\},$$

$$S_E > 1 \text{ for } q < 1, \quad S_E < 1 \text{ for } q > 0. \quad (2.24)$$

The asymptotic expression for S_E when v_0 is large is

$$S_E \sim \exp\left\{-\frac{2^{1/2}(2+q)^{1/2}(r+2+q)^{1/2}}{2+3q-r}\frac{\xi_0}{\delta} v_0^{(2+3q-r)/2}\right\} \ll 1$$

for $q > 0,\ 2+3q-r > 0$;

$$S_E \sim \exp\left\{\frac{2^{1/2}(2+q)^{1/2}(r+2+q)^{1/2}}{2+q-r}\frac{\xi_0}{\delta} v_0^{(2+q-r)/2}\right\} \gg 1 \quad (2.25)$$

for $q < 0,\ 2+q-r > 0$;

$$S_E \sim e^{\alpha\xi_0/\delta}, \quad |\alpha| \sim 1, \quad \alpha \begin{cases} <0, & q>0 \\ >0, & <0 \end{cases},$$

for $2+3q-r < 0,\ 2+q-r < 0$.

ξ_0, v_0, and n are determined by the following formulas:

$$\xi_0 = \frac{2\Gamma(^5/_2 + q)}{3\sqrt{\pi}} \frac{\omega_0^2}{c\nu_0}; \quad n = 1;$$

$$v_0 = \left(\frac{(2+r+q)^{1/2}(2+q)^{1/2}}{2^{1/2}\pi} \frac{c|E^0|^2}{\delta_0\lambda_0 T}\right)^{2/(2+q+r)}. \quad (2.26)$$

The coefficients of reflection and refraction for cases I and II are:

$$P = \frac{1-n}{1+n} + \frac{2i\xi_0}{(1+n)^2} v_0^{\mp q},$$

$$R = \frac{2}{1+n} + \frac{2i\xi_0}{(1+n)^2} v^{\mp q}. \quad (2.27)$$

The upper sign in (2.27) corresponds to the first case, and the lower to the second.

III. Let us assume that one of the following three relations is satisfied:

$$\omega = \Omega_{1,2}, \quad \nu/\Omega_{12} \ll 1;$$
$$\omega = \omega_H, \quad \nu/\omega_H \ll 1, \quad \omega_0^2 v^q/\omega_H\nu_0 \gg 1, \quad \varphi = 0;$$
$$\nu/\omega \gg 1, \quad \omega_0^2 v^q/\omega\nu_0 \gg 1.$$

We deal here with magnetoplasma and cyclotron resonances, and also with low-frequency waves. The displacement current is assumed negligibly small. The field has the following form:

$$E = 2\zeta E^0 \left(\frac{v_0}{v}\right)^{q/4} \exp\left\{-(1-i)\xi_0 \int_0^z v^{q/2} dz\right\}. \quad (2.28)$$

In the region where $v(z) \ll 1$, we can readily obtain for the field the expression

$$E = \frac{2\zeta E^0}{[1-(2\pm q-r)\delta(v_0)z]^{q/2(2\pm q-r)}}$$

$$\times \exp\left\{-\frac{1-i}{2\pm q+q-r}\frac{\xi_0 v_0^{q/2}}{\delta(v_0)}.\right.$$

$$\times[(1-(2\pm q-r)\delta(v_0)z)^{(2\pm q+q-r)/(2\pm q-r)}-1]\Big\}. \quad (2.29)$$

The self-action factor is

$$S_E = v_0^{q/4}\exp\Big\{\frac{(1-i)(2\pm q)^{1/2}}{2^{1/2}}\frac{\xi_0}{\delta}\int_{v_0}^{1}(v^{q/2}-1)$$
$$\times v^{1\pm q}\Big(\frac{v^{r+2\pm q}-1}{r+2\pm q}-\frac{v^{r+1\pm q}-1}{r+1\pm q}\Big)^{-1/2}dv\Big\}. \quad (2.30)$$

We write out the asymptotic expression for S_E when v_0 is large:

$$S_E \sim \exp\Big\{-\frac{2^{1/2}(1-i)(2\pm q)^{1/2}(r+2\pm q)^{1/2}}{2\pm q+q-r}\frac{\xi_0}{\delta}v_0^{2\pm q+q-r}\Big\}$$

for $q>0,\ 2\pm q+q-r>0$;

$$S_E \sim \exp\Big\{\frac{2^{1/2}(1-i)(2\pm q)^{1/2}(r+2\pm q)^{1/2}}{2\pm q-r}\frac{\xi_0}{\delta}v_0^{2\pm q-r}\Big\} \quad (2.31)$$

for $q<0,\ 2\pm q-r>0$;

$$S_E \sim v_0^{q/4}e^{\alpha(1-i)\xi_0/\delta},\qquad |\alpha|\sim 1,$$

$$\alpha\begin{cases}<0, & q>0 \quad \text{for} \quad 2\pm q+q-r<0\\ >0, & q<0 \quad \text{for} \quad 2\pm q-r<0\end{cases}.$$

We note that if the last two limitations on the frequencies are satisfied, then it is necessary to retain the plus sign in front of $\pm q$. If magnetoplasma resonance takes place, then we obtain for v_0 the formula

$$v_0=\Big[\frac{(2\pm q)^{1/2}(2\pm q+r)^{1/2}}{2^{3/4}}\frac{L(\omega_H)\,|E^0|^2}{\delta\lambda_\pm^H\,T}\Big]^{2/(2+r\pm q+q)}. \quad (2.32)$$

The function $L(\omega_H)$ will not be given, in view of its complexity.

The formulas greatly simplify when $\varphi=\pi/2$:

$$v_0=\Big\{\frac{4(2-q)^{1/2}(2-q+r)^{1/2}\Gamma^2(5/2+q)}{3\pi^{1/4}\Gamma^{1/2}(5/2-q)[C_q(5/2-q+q^2)/(5/2+q)]^{1/2}}$$
$$\times\frac{[(\omega_0^2+2\omega_H^2)(\omega_0^2+\omega_H^2)^{1/2}]^{1/2}c}{\delta\lambda_0\omega_0\nu_0^{1/2}T}\Big\}^{2/(2+r)},$$

$$\xi_0=\frac{1}{c}\Big[\frac{3\pi^{1/2}}{\Gamma(5/2-q)}\frac{\omega_0^2\omega_H^2(\omega_H^2+\omega_0^2)^{1/2}}{\nu_0(\omega_0^2+2\omega_H^2)}\Big]^{1/2}. \quad (2.33)$$

When the second relation is satisfied we obtain for the temperature and for ξ_0

$$v_0=\Big[\Big(\frac{6(1+q)(r+q+2)}{\Gamma(5/2+q)\pi^{3/2}}\Big)^{1/2}\frac{\omega_H\nu_0^{1/2}c\,|E^0|^2}{\omega_0\delta\lambda_0T}\Big]^{1/(1+q\pm r/2)},$$

$$\xi_0=\Big(\frac{2\Gamma(5/2+q}{3\pi^{1/2}}\Big)^{1/2}\frac{\omega_0\omega_H^{1/2}}{\nu_0^{1/2}c}. \quad (2.34)$$

The results for the third relation, in the case of circular polarization of the incident wave, are obtained by replacing ω_H with ω. We present also a formula for the impedance

$$\zeta=\frac{1+i}{2}\frac{\omega}{c\xi_0}v_0^{-q/2}\ll 1. \quad (2.35)$$

We note that by virtue of the inequalities imposed on r and q, the dependence of v_0 on E^0 is in all cases such that v_0 increases with increasing E^0.

3. NORMAL SKIN EFFECT AND WAVE INTERACTION

The normal skin effect for semiconductors with low carrier density was considered earlier [2]. We shall investigate here the case of large density, when the criterion (1.5) is satisfied and the distribution function can be regarded as Maxwellian.

In the case of normal skin effect, $\xi(v_0)\ll\delta(v_0)$, we can neglect in the balance equation (1.19) the terms with the derivative with respect to the coordinate, obtaining:

$$\bar{B}_{ik}(v)E_iE_k^*=A_0v^{r-1}(v-1). \quad (3.1)$$

This algebraic equation must be solved simultaneously with the system of Maxwell's equations (1.21). To obtain the results in compact form let us consider the same particular cases as in Sec. 2.

$$\text{I.}\quad \frac{\nu}{\omega}\ \text{or}\ \frac{\nu}{\omega_H},\ \frac{\nu}{|\omega-\omega_H|},\ \frac{\nu}{|\omega-\Omega_{1,2}|}\ll 1.$$

When these inequalities are satisfied, the solution is best sought in the form

$$E(z)=u(z)e^{-i\omega nz/c}. \quad (3.2)$$

Expanding A, B, and C in (1.21) in powers of ν, we obtain for μ_k the following equation (see also [12]):

$$du_k/dz+\xi_0v^{-q}u_k=0. \quad (3.3)$$

The u_k for different k are interelated linearly by virtue of Maxwell's equations. In this connection, we shall henceforth use only one component of the vector u, say u_x, which under the assumptions made will be denoted by u.

Expanding $\bar{B}_{ik}$ in powers of ν, we obtain for v the expression

$$M(\omega,\omega_H)v^{-q}u^2=A_0v^{r-1}(v-1). \quad (3.4)$$

Eliminating u from (3.3) and (3.4), we get an equation for v:

$$\{(r+q)v^q-(r+q-1)v^{q-1}\}dv/dz$$
$$+2\xi_0(v-1)=0. \qquad (3.5)$$

The solution of this equation takes the form

$$\int_{v_0}^{v}\frac{[(r+q)v-(r+q-1)]v^q}{v(v-1)}dv=-2\xi_0 z, \qquad (3.6)$$

where $v_0 = v(+0)$. The value of v_0 is connected with u_0 by Eq. (3.4), wherein u_0 must be expressed in terms of the amplitude of the incident field u^0 with the aid of (2.13) and (2.16). For $v_0 \gg 1$ we get

$$v_0=\left(\frac{4M_1(\omega,\omega_H)\,|E^0|^2}{A_0(1+n)^2}\right)^{1/(r+q)}. \qquad (3.7)$$

In the general case the expressions for M_1 and $n(\omega, \omega_H)$ are rather cumbersome. For the important case of helical waves ($\omega_H \gg \omega, \nu$) with longitudinal propagation we get

$$v_0=\left(\frac{16\Gamma(^5/_2-q)}{3\pi^{1/2}}\,\frac{\nu_0\omega\,|u^0|^2}{\omega_H A_0}\right)^{1/(r+q)}, \qquad (3.7')$$

n and ξ_0 are described by formulas (2.23). Equation (3.6) defines v as an implicit function of z, after which the field is obtained from (2.24).

Let us assume that the electron-gas temperature is high compared with the lattice temperature. Then we have for the region where $v(z) \gg 1$

$$E=RE^0e^{-i\omega nz/c}\left(1-\frac{2q}{r+q}\xi_0 v_0^{-q}z\right)^{(r+q)/2q},$$

$$v=v_0\left(1-\frac{2q}{r+q}\xi_0 v_0^{-q}z\right)^{1/q}. \qquad (3.8)$$

In the region where $v - 1 \ll 1$ we have

$$E=RE^0S_E\,e^{i\omega nz/c-\xi_0 z},\qquad v=1+\frac{4MS_E^2|E^0|^2}{(1+n)^2A_0}e^{-2\xi_0 z}. \qquad (3.9)$$

The self-action coefficient S_E is given here by

$$S_E=\exp\left\{\frac{1}{2}\int_1^{v_0}\frac{[(r+q)v-(r+q-1)](v^q-1)}{v(v-1)}dv\right\}, \qquad (3.10)$$

with

$$S_E\sim\begin{cases}\exp\left[\dfrac{r+q}{2q}v_0^q\right]\gg 1 & \text{for } q>0,\\ v_0^{-(r+q)/2}\ll 1 & \text{for } q<0.\end{cases} \qquad (3.11)$$

II. We can investigate analogously the case when any one of the following two systems of equations is satisfied, either

$$\omega=\omega_H,\quad \varphi=0,\ \omega_0^2v^q/\omega_H\nu_0\ll 1$$

or

$$\omega_H=0,\quad \omega_0^2v^q/\omega\nu_0\ll 1.$$

All the results are obtained here by replacing q in (3.8)—(3.11) by −q. With this, we have for v_0

$$v_0=\left(\frac{4\Gamma(^5/_2+q)}{3\pi^{1/2}}\,\frac{\omega_0^2|u^0|^2}{\nu_0A_0}\right)^{1/(r-q)} \qquad (3.12)$$

if the first system of relations is satisfied. If the second system is satisfied, the factor 4 in the parentheses should be replaced by 2. The polarization of the incident field is assumed linear. The reflection and refraction coefficients for the last two cases are described by the formulas of (2.27).

III. We proceed to investigate the resonances in the low-frequency case. Assume that one of the following three systems is satisfied:

$$\omega=\Omega_{1,2},\ \nu/\Omega_{1,2}\ll 1;$$
$$\omega=\omega_H,\quad \nu/\omega_H\ll 1,\quad \varphi=0,\quad \omega_0^2v^q/\omega_H\nu\gg 1;$$
$$\omega_H=0,\quad \omega\ll\nu,\quad \omega_0^2v^q/\omega\nu_0\gg 1.$$

Solving the equation for the temperature under any of the three foregoing assumptions and expanding A, B, and C in (1.21) in terms of the corresponding small parameter in the region $v(z) \gg 1$, we obtain for the field the equation

$$\frac{d^2F_{1,2}}{dz^2}+2i\xi_0^2\left|\frac{F_{1,2}}{F_{1,2}^0}\right|^{2q/(r-q)}F_{1,2}=0, \qquad (3.13)$$

where $F_{1,2} = E_x - k_{1,2}E_y$ are the normal waves, and $F^0_{1,2}$ the values of $F_{1,2}$ for the incident wave. The coefficients $k_{1,2}$ have different values for each of the cases. The method of solving equations of this type is described in detail in[2], and we present only the solution:

$$F=2\zeta F^0\left(1+\frac{q}{[(2r-q)(r-q)]^{1/2}}\frac{\omega}{c|\zeta|}z\right)^{\Omega},$$

$$\Omega=-\frac{r-q}{q}+\frac{i}{q}[r(r-q)]^{1/2};$$

$$v=v_0\left(1+\frac{q}{[(2r-q)(r-q)]^{1/2}}\frac{\omega}{c|\zeta|}z\right)^{-2/q}. \qquad (3.14)$$

Here ζ is the impedance, which is connected with the coefficients of reflection and refraction by formulas (2.20). The indices (1, 2) will be omitted where there is no danger of misunderstanding. We present the expressions for $k_{1,2}$, ζ and v_0 for the different cases:

In the case of magnetoplasma resonance

$$k_{1,2}=\frac{i}{b}\left[\frac{a-g}{2}\mp\left(\frac{(a-g)^2}{4}+b^2\right)^{1/2}\right];$$

$$a=-\frac{\varepsilon_{12}''^2\sin^2\varphi}{\varepsilon_{11}''\sin^2\varphi+\varepsilon_{33}''\cos^2\varphi},$$

$$b=\frac{\varepsilon_{12}''\varepsilon_{33}'\cos\varphi}{\varepsilon_{11}''\sin^2\varphi+\varepsilon_{33}''\cos^2\varphi}$$

$$g=\frac{\varepsilon_{11}'\varepsilon_{33}'}{\varepsilon_{11}''\sin^2\varphi+\varepsilon_{33}''\cos^2\varphi}. \qquad (3.15)$$

ϵ'_{ik} and ϵ''_{ik} are the real and imaginary parts of the dielectric tensor at v = 1.

For the impedance and for v_0 we obtain*

$$\zeta=2^{-(r+q)/2r}\left(\frac{r}{r-q}\right)^{(r-q)/4r}\xi_0^{-(r-q)/r}|\gamma|^{-q/r}$$

$$\times\left(\frac{2\Gamma(^5/_2-q)}{3\pi^{1/2}}\frac{\omega_0^2\nu_0|F^0|^2}{W(\omega)A_0}\right)^{-q/2r}$$

$$\times\exp\left\{-i\,\mathrm{arctg}\left(\frac{r-q}{r}\right)^{1/2}\right\}, \qquad (3.16)$$

where

$$v_0=2^{1/r}\left(\frac{r}{r-q}\right)^{1/2r}\xi_0^{-2/r}|\gamma|^{2/r}$$

$$\times\left(\frac{2\Gamma(^5/_2-q)}{3\pi^{1/2}}\frac{\omega_0^2\nu_0|F^0|^2}{W(\omega)A_0}\right)^{1/r}$$

$$W(\omega)=\frac{\omega^2(\omega^2-\omega_H^2)^2}{\omega^4+\omega_H^4\cos^2\varphi+\omega^2\omega_H^2(1-3\cos^2\varphi)}$$

$$\xi_0^2=a+g\pm\left[(a-g)^2+\frac{b^2}{4}\right]^{1/2},$$

$$\gamma_{12}=\pm\frac{\sin\varphi\, b\,[i\varepsilon_{12}''k_{21}+(\varepsilon_{33}'-\varepsilon_{11}')\cos\varphi]}{(\varepsilon_{11}''\sin^2\varphi+\varepsilon_{33}''\cos^2\varphi)[(a-g)^2+4b^2]^{1/2}}$$

In the case of magnetoplasma resonance, the nonzero z-component of the electric field, E_z, is given by

$$E_z=\left(\frac{r}{r-q}\right)^{1/2}\frac{\gamma\zeta F_0}{|\zeta|^2\xi_0^2}\left(1+\frac{q}{[(2r-q)(r-q)]^{1/2}}\frac{\omega}{c|\zeta|}z\right)^{\Omega'},$$

$$\Omega'=-\frac{r+q}{q}+\frac{i[r(r-q)]^{1/2}}{q}. \qquad (3.17)$$

It is of interest to present formulas for ζ and v_0 in the case of transverse propagation ($\varphi=\pi/2$):

$$\zeta=2^{(r+q)/2r}\pi^{q/2r}\left[\frac{\Gamma(^5/_2-q)}{3\sqrt{\pi}}\right]^{1/r}\left(\frac{r}{r-q}\right)^{(r-q)/4r}\left(\frac{\omega_H\nu_0}{\omega_0^2}\right)^{1/2}$$

$$\times\left(1+\frac{\omega_0^2}{2\omega_H^2}\right)^{1/2}\left(1+\frac{\omega_0^2}{\omega_H^2}\right)^{(r-q)/4r}\left(\frac{\omega_0}{\omega_H}\right)^{1/2r}$$

$$\times\left[\frac{\omega_0|F^0|^2}{A_0}\right]^{-q/2r}\cdot\exp\left\{-i\,\mathrm{arctg}\left(\frac{r-q}{r}\right)^{1/2}\right\},$$

$$v_0=2^{1/r}\left(\frac{r}{r-q}\right)^{1/2r}\left(1+\frac{\omega_0^2}{\omega_H^2}\right)^{1/2r}\left[\frac{2\Gamma(^5/_2-q)}{3\pi^{3/2}}\right]^{1/r}$$

$$\times\left[\frac{\omega_H|F^0|}{A_0}\right]^{1/r}. \qquad (3.16')$$

In the case of cyclotron resonance k = i (the second value of k corresponds to a nonresonant wave). The formulas for the impedance and v_0 are

$$\zeta=2^{-q/r}\left(\frac{r}{r-q}\right)^{(r-q)/4r}\left(\frac{4\Gamma(^5/_2+q)\,\omega_0^2}{3\pi^{1/2}\omega_H\nu_0}\right)^{-1/2}\left[\frac{\omega_H|E^0|^2}{4\pi A_0}\right]^{-q/2r}$$

$$\times\exp\left\{-i\,\mathrm{arctg}\left(\frac{r-q}{q}\right)^{1/2}\right\},$$

$$v_0=\left(\frac{r}{r-q}\right)^{1/2r}\left[\frac{\omega_H|E^0|^2}{4\pi A_0}\right]^{1/r}. \qquad (3.18)$$

We have used here the fact that $F^0=(1-i)E^0$ (see[2]).

Finally, if the third system of relations is satisfied, then the formulas for the corresponding quantities are obtained from (3.18) by replacing ω_H with ω and $|E^0|^2$ with $\frac{1}{2}(1+|k|^2)|E^0|^2$ (k is the polarization coefficient of the incident field).

Comparing (3.8) and (3.14)–(3.18)[4)] with the analogous formulas of [2] we can readily verify that the difference lies in inessential constants of the order of unity (due allowance must be made in the comparison for the difference in notation). This confirms once more that the electron-temperature approximation is sufficiently accurate even if the inequality (1.5) is not satisfied. It can be assumed that this statement holds for the anomalous skin effect, too.

In conclusion let us discuss a specific electromagnetic-wave interaction connected with heating of the electron gas. Assume that two waves propagate in a semiconductor, with frequencies ω_1 and ω_2, and with one amplitude E_1 much larger than the other, E_2. As follows from [4], no combination harmonics appear in the zeroth approximation in the small parameter $\nu/\Omega_{1,2}$. The wave interaction is manifest in the fact that the second wave propagates in a gas heated by the first. This influences the phase and the attenuation of the second wave.

The field of the second wave is determined by Maxwell's equations (1.21) with a dielectric tensor that depends on the electron-gas temperature, which is determined in turn by the parameters of

*arctg ≡ $\tan^{-1}$.

4)There are errors in formulas (4.6), (4.8), (4.19), (4.22) and (4.23) of [2], which correspond to our formulas (3.15), (3.16), (3.17) and (3.16′).

the second wave (assumed known). In the case of the anomalous field effect we can use for the field of the second wave, subject to satisfaction of the appropriate inequalities, the formulas derived in Sec. 2, in which we must substitute v_0 corresponding to the first wave and n and ξ_0 corresponding to the second. In the normal skin effect the situation is somewhat more complicated, since the dependence of the temperature on the coordinates and on other parameters varies with the different requirements imposed on the frequency ω_1.

Let us consider several examples which, while not covering all the possible cases, suffice to explain the situation.

Assume that the conditions designated by the Roman numeral I are satisfied for the frequency ω_2. If ω_1 satisfies the same relations, then the formulas for E_2 are

$$E_2 = R_2 E_2^0 e^{i\omega_2 n_2/c}\left(1-\frac{2q}{r+q}\xi_{10}v_0^{-q}z\right)^{(r+q)\xi_{20}/2q\xi_{10}} \quad (3.19)$$

for $v(z) \gg 1$;

$$E_2 = R_2 E_2^0 S_{E_2}\exp\left\{-\left(i\frac{\omega_2}{c}n_2 z-\xi_{20}z\right)\right\},$$

$$S_{E_2}\sim\begin{cases}\exp\left\{\dfrac{r+q}{2q}\dfrac{\xi_{20}}{\xi_{10}}v_{10}^{q}\right\}\gg 1, & q>0\\ v_0^{-(r+q)/2}\dfrac{\xi_{20}}{\xi_{10}}\ll 1, & q<0\end{cases}$$

for $v(z) - 1 \ll 1$.

On the other hand, if ω_1 satisfies conditions III, we obtain for E_2 in the region $v(z) \gg 1$

$$E_2 = 2\zeta_2 E_2^0\exp\left\{-\frac{[(2r-q)(r-q)]^{1/2}}{3q}\frac{c}{\omega_1}\frac{\xi_{20}v_0^{-q}}{|\zeta_1|}\times\left[\left(1+\frac{q}{[(2r-q)(r-q)]^{1/2}}\frac{\omega_1}{c|\zeta_1|}z\right)^3-1\right]\right\}. \quad (3.20)$$

Let now ω_2 satisfy relations III. If ω_1 satisfies relations I, we have for E_2

$$E_2 = 2\zeta_2 E_2^0\left(1-\frac{2q}{r+q}\xi_0 v_0^{-q}z\right)^{-1/4q}\times\exp\left\{(1-i)\frac{r+q}{3q}\frac{\xi_{20}}{\xi_{10}}v_0^{3q/2}\times\left[\left(1-\frac{2q}{r+q}\xi_{10}v_0^{-q}z\right)^{3/2}-1\right]\right\} \quad (3.21)$$

if $v(z) \gg 1$. For this case, S_{E_2} takes the form

$$S_{E_2}\sim\begin{cases}\exp\left\{-\dfrac{2}{3}(1-i)\dfrac{\xi_{20}}{\xi_{10}}v_0^{3q/2}\right\} & q>0\\ v_0^{1/4} & q<0.\end{cases} \quad (3.22)$$

If ω_1 satisfies relations III, then

$$E_2 = 2\zeta_2 E_2^0\left[1+\frac{q}{[(2r-q)(r-q)]^{1/2}}\frac{\omega_1}{c|\zeta_1|}z\right]^{\Omega''},$$

$$\Omega'' = \frac{1}{2}+\left[\frac{1}{4}-i\frac{[(2r-q)(r-q)]}{2q^2}\frac{\xi_{20}^2}{\omega_1^2|\zeta_1|^2}\right]^{1/2}. \quad (3.23)$$

The quantities ζ_2 and R_2 in the foregoing formulas can be determined with the aid of (129) and (1.30).

All the results can be readily used for a plasma.

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