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SPLITTING OF IONIC LINES OWING TO REVOLUTION OF THE IONS IN A MAGNETIC FIELD

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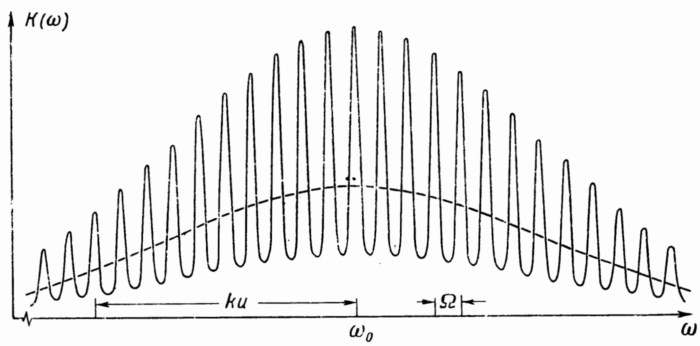
It is shown that if the cyclotron frequency Ω of the revolutions of an ion in a magnetic field exceeds the natural width γ of a spectral line, then in the radiation directed transverse to the magnetic field the Doppler contour of each Zeeman component is split into a number of peaks. The distance between adjacent peaks is Ω, and the width of each peak is determined by the natural line width γ.

It is well known that for a gas at not too high pressure the width of an atomic or ionic line is due to the Doppler effect, and as a rule considerably exceeds the natural line width γ.

In the present paper it is shown that if a plasma is placed in a sufficiently strong magnetic field, then in the radiation in a direction transverse to the field the spectral lines of the ions are decidedly altered in shape. Namely, when the condition $\Omega \gtrsim \gamma$ is satisfied, where Ω is the Larmor frequency of the ion, the Doppler contour of an ionic line is split into a series of peaks with widths equal to the natural line width γ and separated from each other by the amount Ω. Accordingly, when observed transverse to the magnetic field, the contour of such a line takes the form shown in the figure. This sort of splitting occurs for each of the Zeeman components of the line (the distance between Zeeman components is larger than the splitting considered here by a factor $\sim M/m$, where M is the mass of the ion and m that of the electron).

The explanation of this effect is extremely simple. In a sufficiently strong magnetic field an ion makes several revolutions in a Larmor orbit during the time of emission of radiation. For $\Omega > \gamma$ the revolving radiator emits in the plane of revolution a discrete spectrum of frequencies $\omega_0 + n\Omega$, where ω_0 is the characteristic frequency of the stationary radiator, Ω is the angular frequency of revolution, and $n = 0, \pm 1, \pm 2, \ldots$. Since the frequency of revolution Ω of the ion does not depend on its velocity and is determined only by the magnetic field, in the direction perpendicular to the magnetic field all of the ions emit the same spectrum of frequencies. The only thing dependent on the velocity of the ion and the phase of its revolution is the distribution of the radiated energy over



Splitting of an individual Zeeman component of an ionic line observed transverse to the magnetic field [Eq. (8)]; $\pi\gamma/\Omega = 1$, $ku/\Omega = 10$. The dashed curve is the Doppler contour as observed for $\gamma \gg \Omega$.

this spectrum; this will of course be different for different ions. Averaging over the ions leads to the spectrum shown in the figure.

These qualitative arguments are confirmed by the following simple calculation. Let the magnetic field $\mathbf{H}$ be directed along the z axis, and let the spontaneous emission from the ions be observed in the direction of the x axis. We denote the transition frequency for a chosen Zeeman component of an ionic line by ω_0, and the natural line width by γ. In a semiclassical treatment the field $\mathbf{E}(t)$ produced by one ion at the point of observation x_1 is of the form

$$\mathbf{E}(t) = \mathbf{E}A(t),$$

$$A(t) = \exp[(i\omega_0 - \gamma/2)(t - t_0) - ik(x_1 - x(t)) + i\psi]. \tag{1}$$

Here t_0 is the time at which the atom became excited, $k = \omega_0/c$, $x(t)$ is the coordinate of the ion along the x axis at the time t, and ψ is the initial phase. The amplitude $\mathbf{E}$ of the electric field is proportional to the dipole-moment matrix element for the transition. The total intensity of the Zeeman component of the line and the polarization of the radiation are determined according to the usual rules and are not considered further here.

To obtain the spectrum of the radiation we must find the correlation function

$$K(t-t') = \gamma\langle A(t)A^*(t')\rangle, \quad (2)$$

where the angle brackets denote averaging over the ensemble (i.e., averaging over the times of excitation t_0), and calculate the Fourier transform of the quantity $K(\tau)$. The coordinate $x(t)$ of the ion, which appears in (1), is

$$x(t) = \Omega^{-1}\{v_x \sin\Omega(t-t_0) - v_y[1-\cos\Omega(t-t_0)]\} + x_0, \quad (3)$$

where x_0, v_x, v_y are the coordinate and the projection of the velocity of the ion at $t = t_0$; $\Omega = eH/Mc$ is the Larmor frequency of the ion, e being the charge of the ion and M its mass. Using (1) and (3), we get

$$\gamma A(t)A^*(t') = \gamma\exp\{i\omega_0(t-t') - {}^1/_2\gamma(t+t'-2t_0) + ikv_x\Omega^{-1}[\sin\Omega(t-t_0) - \sin\Omega(t'-t_0)] + ikv_y\Omega^{-1}[\cos\Omega(t-t_0) - \cos\Omega(t'-t_0)]\}. \quad (4)$$

We shall assume that the velocity distribution of the ions is Maxwellian. Then the probability that the velocity of the ion is in the limits $\mathbf{v}$ to $\mathbf{v} + d\mathbf{v}$ is $F(\mathbf{v})d^3v$, with

$$F(\mathbf{v}) = (u\sqrt{\pi})^{-3}\exp(-v^2/u^2)$$

[$u = (2T/M)^{1/2}$ is the most probable speed of the ion, T being the temperature of the ions in energy units]. Multiplying the expression (4) by the quantity $F(\mathbf{v})d^3v$, integrating over the time of excitation t_0 from $-\infty$ to the smaller of the times t and t', and integrating over velocities, we find ($\tau = t - t'$)

$$K(\tau) = \exp\left(-\frac{\gamma}{2}|\tau| + i\omega_0\tau - \frac{k^2u^2}{\Omega^2}\sin^2\frac{\Omega\tau}{2}\right). \quad (5)$$

As normalized to unity, the spectral density of the radiation for an individual Zeeman component is

$$K(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} K(\tau)e^{-i\omega\tau}\,d\tau.$$

Let us break the range of integration up into segments of length $2\pi/\Omega$ and introduce the new variable of integration $z = \Omega\tau$. We then get

$$K(\omega) = \frac{1}{2\pi\Omega}\sum_{n=-\infty}^{\infty}\int_{(2n-1)\pi}^{(2n+1)\pi}\exp\left(-\frac{\gamma}{2\Omega}|z| + i\frac{\omega_0-\omega}{\Omega}z - \frac{k^2u^2}{\Omega^2}\sin^2\frac{z}{2}\right)dz.$$

By means of obvious transformations we bring this expression into the form

$$K(\omega) = 2J_1\left\{\mathrm{Re}\sum_{n=0}^{\infty}\exp\left[\left(-\frac{\gamma}{2\Omega} + i\frac{\omega_0-\omega}{\Omega}\right)2\pi n\right] - 1\right\} + J_2, \quad (6)$$

where

$$J_1 = \frac{1}{2\pi\Omega}\int_{-\pi}^{\pi}\exp\left(-\frac{\gamma}{2\Omega}z + i\frac{\omega_0-\omega}{\Omega}z - \frac{k^2u^2}{\Omega^2}\sin^2\frac{z}{2}\right)dz,$$

$$J_2 = \frac{1}{2\pi\Omega}\int_{-\pi}^{\pi}\exp\left(-\frac{\gamma}{2\Omega}|z| + i\frac{\omega_0-\omega}{\Omega}z - \frac{k^2u^2}{\Omega^2}\sin^2\frac{z}{2}\right)dz.$$

We shall assume that the Doppler width is much larger than the natural line width and the cyclotron frequency of the ion, i.e., $ku \gg \gamma, \Omega$. Then the important values of z in the integrals J_1 and J_2 are $z \lesssim \Omega/ku \ll 1$. Therefore we can write

$$J_1 = J_2 = \frac{1}{2\pi\Omega}\int_{-\infty}^{\infty}\exp\left(i\frac{\omega_0-\omega}{\Omega}z - \frac{k^2u^2}{4\Omega^2}z^2\right)dz = \frac{1}{ku\sqrt{\pi}}\exp\left[-\left(\frac{\omega-\omega_0}{ku}\right)^2\right]. \quad (7)$$

Summing the progression in (6) and using the expression (7), we find the following final expression for the spectral density of the radiation within the range of a single Zeeman component of the ionic line:

$$K(\omega) = \frac{1}{ku\sqrt{\pi}}\exp\left[-\left(\frac{\omega-\omega_0}{ku}\right)^2\right] \times \frac{\sinh\pi\gamma/\Omega}{\cosh\pi\gamma/\Omega - \cos 2\pi(\omega-\omega_0)/\Omega}. \quad (8)$$

Accordingly, the shape of the line is that of the Doppler contour modulated by a periodic function of the frequency ω with the period Ω. The integral

$$\int_{-\infty}^{\infty} K(\omega)\,d\omega = 1$$

is $\sim\exp(-k^2u^2/\Omega^2)$, apart from small correction terms.

If $\gamma \gg \Omega$, then

$$K(\omega) = \frac{1}{ku\sqrt{\pi}}\exp\left[-\left(\frac{\omega-\omega_0}{ku}\right)^2\right] \times\left[1 + 2\exp\left(-\frac{\pi\gamma}{\Omega}\right)\cos 2\pi\frac{\omega-\omega_0}{\Omega}\right],$$

i.e., the shape of the line differs from the Doppler shape by an exponentially small oscillating correction. If, on the other hand, $\gamma \ll \Omega$, then the Dopper contour is distinctly split up into a series of peaks separated by the interval Ω and having the Doppler shape with half-value width equal to γ. The heights of the peaks are proportional to the magnetic field. The figure shows the contour of a single Zeeman component for the case in which $\pi\gamma/\Omega = 1$, which means that during the lifetime in the excited state the ion makes half a revolution in its Larmor orbit.

If the observation is not made at right angles to the magnetic field, then all of the peaks will be broadened owing to the free thermal motion of the ions along the field. The width of each peak will then (if $k_z u > \gamma$) be determined by the quantity $k_z u$, where k_z is the component of the wave vector in the direction of the magnetic field. Therefore for the effect to appear the deviation of the angle of observation from a right angle must not be larger than Ω/ku (in radians). If the gas pressure is high the peaks can also be broadened owing to collisions.

We point out the possibility of obtaining beats between different peaks at frequencies that are multiples of Ω, in an experiment such as the well known one done by Forrester and others.[1]

The splitting of ionic lines can also in principle appear in the absorption of light propagated transverse to a strong magnetic field. Within the extent of each Zeeman component the absorption coefficient is then of the form (8).

From the point of view of quantum mechanics the effect described here is due to the fact that the emission (or absorption) of a photon is accompanied by transitions between the Landau levels of the ion.

Equation (8) is valid for $ku \gg \gamma, \Omega$. For arbitrary ratios of these quantities one can calculate the Fourier transform of the expression (5) and easily obtain

$$K(\omega) = \frac{1}{2\pi} \exp\left(-\frac{k^2u^2}{2\Omega^2}\right) \times \sum_{n=-\infty}^{\infty} I_n\left(\frac{k^2u^2}{2\Omega^2}\right) \frac{\gamma}{(\gamma/2)^2 + (\omega - \omega_0 + n\Omega)^2}, \tag{9}$$

where I_n is the modified Bessel function with index n. For $ku \gg \gamma, \Omega$ one can use the asymptotic form for the functions I_n and get (8) from the expression (9). If, on the other hand, the magnetic field is so strong that $ku \ll \Omega$, then in (9) we must keep only the term with $n = 0$, and then

$$K(\omega) = (2\pi)^{-1}\gamma[(\gamma/2)^2 + (\omega - \omega_0)^2]^{-1},$$

i.e., the line is of the Lorentz shape with width γ. This narrowing of the line is due to the fact that for $ku \ll \Omega$ the mean Larmor radius of the ion is much smaller than the wavelength, and is analogous to the narrowing of lines studied by Dicke.[2]

If the natural line width is not too large, then for light ions the Larmor frequency Ω becomes comparable with the quantity γ for magnetic fields of the order of some tens of kOe. If the line has fine structure or hyperfine structure this of course leads to a smearing out of the effect if the distance between the components of the structure is small in comparison with the Doppler width.

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[1] Forrester, Gudmundsen, and Johnson, Phys. Rev. **99**, 1691 (1955).

[2] R. Dicke, Phys. Rev. **89**, 472 (1953).

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