PROPAGATION OF A LIGHT PULSE THROUGH A MEDIUM WITH POPULATION INVERSION

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It is noted that the normal amplification process changes sharply when condition (4) is no longer satisfied. In such a case, the intensity of output radiation tends to oscillate, and the light amplification or attenuation effects are themselves suppressed.

THE point of departure will be the quasi-classical equation system[1-4] describing the propagation of photons in a one-dimensional medium containing two-level atoms. The vector potential of the radiation will be sought in the form

$$\mathbf{A}_0(x, t) = \mathbf{A}(x, t) \exp(ikx - i\omega t) + \text{c.c.}$$

The amplitude **A**(x, t) is assumed to a variation slower than exponential as a function of x and t. The system of quasi-classical equations determining the field (with a definite linear polarization), the population excess $\Delta = n_2 - n_1$, and the "transition current" ρ has the following form:

$$\frac{\partial A}{\partial x} + \frac{1}{v}\frac{\partial A}{\partial t} + \frac{\beta}{2}A = \frac{2\pi}{\omega}\rho, \quad (1)$$

$$\frac{\partial \rho}{\partial t} + i\varepsilon\rho + \frac{\rho\Gamma}{2} = \frac{A\Delta|V|^2}{c\hbar}, \quad (2)$$

$$\frac{\partial\Delta}{\partial t} + \Delta\left(W_{13} + \frac{1}{\tau}\right) - n_0\left(W_{13} - \frac{1}{\tau}\right) = -\frac{2}{c\hbar}(A\rho^* + \rho A^*). \quad (3)$$

The following notation is used in (1)–(3): v is the velocity of light in the medium, β is the coefficient of absorption associated with nonresonant effects, $\epsilon = \omega_0 - \omega$ is the detuning at resonance, where $\epsilon/\omega \ll 1$, $n_0 = n_1 + n_2$ is the density of atoms at lower and upper working levels, Γ is the width of the absorption line (assumed to be Lorentzian), and $|V|^2$ is the squared modulus of the transition matrix element. The system (1)–(3) is solved for the following boundary and initial conditions. Let us assume that the injection of the pulse into the medium begins at t = 0. The intensity and phase of the incident radiation are given at the boundary of the medium (which occupies the half-space $x \geq 0$), i.e., when x = 0. At t = 0, the population excess is given as $\Delta = \Delta_0$; the "transition current" $\rho = 0$ and A(x, 0) = 0 for $x \geq 0$. The solution is sought for $x \geq 0$ and $t \geq 0$.

The second and third terms of (3) define effects due to pumping and spontaneous losses. Both terms, as well as the term for nonresonant losses in (1), can be neglected in the case considered below.

The equations as written (after omission of all the phenomenologically introduced terms), represent the quasi-classical limit of quantum electrodynamic equations discussed in [5,6]. To understand the nature of the solution obtained under these conditions, we shall consider the case of exact resonance, $\epsilon = 0$. This reduces the system (1)–(3) to a single equation for the real function A(x, t):

$$\frac{\partial A}{\partial x} + \frac{1}{v}\frac{\partial A}{\partial t} = \frac{\pi|V|\Delta_0}{\omega}\sin\left(\frac{2|V|}{c\hbar}\int_0^t A\,dt\right).$$

This equation can be solved numerically after conversion to an integral equation and application of initial and boundary conditions. The form of the equation readily shows that the boundedness of the derivative of A(x, t) imposes a linear limit upon the growth rate of A(x, t). Furthermore, we can expect an oscillating-type of solution whose frequency is determined by the value of the sine argument. Calculations based on the balance equations indicates a much more rapid (exponential) rise in intensity without any oscillations. This follows from considering the interaction of two-level atoms with the ambient medium.

Clearly, if the atomic relaxation is much faster than stimulated absorption or emission (condition (4)), so that the "transition current" has totally lost coherence, the solution will represent equations based on the neglect of any phase relationships (balance equations). Otherwise ρ will not lose coherence, and the amplification process will not have a sharply defined directivity and will be characterized by oscillations.

To verify this reasoning, let us consider a solution of (1)–(3) by perturbation theory. Let us assume that the amplitude of the input pulse changes weakly after passage through a medium with excess population. Then, substituting the initial values of intensities I_1 and A_1 into (2) and (3), we can determine $\rho(x, t)$ and $\Delta(x, t)$, and in turn, using (1), find the correction to the initial intensity. Let us consider the result of computing I, as an example. If the condition

$$16\sigma I_1 < \Gamma. \qquad (4)$$

is satisfied, the solution has the form quite similar to that of the corresponding balance equations

$$I = I_1 + I_1\sigma\Delta_0 x\left\{\exp\left[-2\sigma I_1\left(t - \frac{x}{v}\right)\right] - \exp\left[-\frac{\Gamma}{2}\left(t - \frac{x}{v}\right)\right]\right\}.$$

σ has in the equations the form $\sigma = 8\pi|V|^2/\omega c\hbar\Gamma$.

The corresponding solution in the case of the inverse inequality has the form

$$I = I_1 + \frac{\Delta_0 x}{2}\sqrt{\Gamma\sigma I_1}\sin\left\{\sqrt{\Gamma\sigma I_1}\left(t - \frac{x}{v}\right)\right\} \times\exp\left[-\frac{\Gamma}{4}\left(t - \frac{x}{v}\right)\right].$$

For the sake of simplicity, formulas are given for the case of $\epsilon = 0$ and for an initial input pulse of rectangular shape and length τ. The result given is valid within the range $\tau \geq t \geq x/v$ and when the second term is small. Within the range $t \leq x/v$ the intensity I is zero. A "tail" of a low intensity pulse (in comparison to the increment of I_1), with a width of the order of $1/\Gamma$, appears in the range $t \geq \tau$. A comparison of the equations reveals a differing nature of the amplification process.

When condition (4) is violated, the radiation intensity is oscillatory, and the light amplification or attenuation effects are suppressed.

For comparison with experiment, one should start with analogous formulas for $\epsilon \neq 0$ (too unwieldy to be given here) and one should try to decrease τ and Γ.

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