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EXPERIMENTAL INVESTIGATION OF INSTABILITY IN AN INHOMOGENEOUS PLASMA

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We present the results of an investigation of an instability in a plasma produced by a hot-cathode arc discharge in magnetic fields up to 3000 Oe at pressures from 4×10^{-4} to 10^{-2} torr. It has been found that in the vicinity of the critical magnetic field this instability is characterized by the formation of tongues which protrude from the column and rotate in the ion direction under steady-state conditions. As H changes, the steady-state rotation disappears and when H > 2000 Oe the plasma becomes turbulent. Under these conditions we have estimated the transverse diffusion coefficient $D_\perp$ and the characteristic lifetime of the perturbations in the turbulent state γ'. It is shown that $D_\perp$ is of the same order of magnitude as the Bohm diffusion coefficient and that γ' is of the proper order of magnitude for the drift frequency. On the basis of a qualitative model we obtain an expression for the frequency of rotation of the tongue which is found to be in good agreement with the experimental observation.

1. INTRODUCTION

IN an earlier paper [1] we have described the results of experiments on an instability in an inhomogeneous plasma in an arc discharge contained in an equipotential volume at operating gas pressures ranging from 4×10^{-4} to 10^{-2} torr and magnetic fields up to 3000 Oe. It has been shown that this plasma is unstable. At magnetic fields greater than H_{cr}, the plasma exhibits strong oscillations accompanied by enhanced loss of particles across the magnetic field. The instability limits satisfy the condition $(\omega\tau)_i \sim 1$ (cf. Fig. 5 below), indicating the origin of the effect to be of a drift nature.[2] In the present work we present the results of experiments carried out to examine the change in the nature of the instability as a function of the discharge parameters; these experiments have also been designed to estimate the coefficient of anomalous diffusion. The experiments have been carried out on a device shown schematically in Fig. 1.[3]

2. CHANGE IN THE NATURE OF THE INSTABILITY

When the magnetic field is increased beyond some critical value the nature of the instability is modified. In the neighborhood of H_{cr} one always observes stably rotating plasma tongues which protrude from the arc column. The number of tongues varies from 1 to 3, depending on the discharge mode, but close to the instability boundary is always determined by the value of the azimuthal mode of the ion acoustic wave.[1] The stationary nature of the rotation of the tongues makes it possible, in certain cases, to study in detail the topological pattern of the density and potential in the unstable regime.[4] An analysis of the experimental results has shown that the development of the drift instability leads to the appearance of a narrow tongue; by drifting along this tongue the plasma escapes across the magnetic field. The density of charged particles in the tongue is approximately an order of magnitude greater than the density outside the tongue and the electron temperature reaches 1 eV. Outside the tongue the nature of the diffusion is classical. This is indicated by the low value of the charged particle density in these regions and also by the small value (~1) of the ratio of electron current to ion saturation current to a Langmuir probe.

In Fig. 2a we show a characteristic plasmogram and an oscillation spectrum at a probe as observed under these conditions. As the magnetic field is increased the number of tongues increases, a feature of which is clearly shown by the discontinuous change in the rotational frequency (Fig. 3). A similar jump is observed when the anode current I_a is

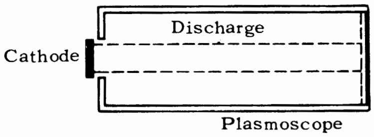


FIG. 1. Schematic diagram of the apparatus. Discharge length 40 cm, diameter of the outer cylinder 7.8 cm and cathode radius a = 0.5 cm.

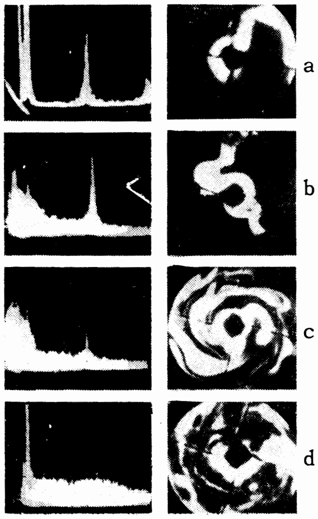


FIG. 2. Characteristic plasmograms in oscillation spectra at the probes. Operating gas is hydrogen. $p = 2.5 \times 10^{-3}$ torr. a) $H = 1800$ Oe, b) $H = 2000$ Oe, c) $H = 2300$ Oe, d) $H = 3000$ Oe. The sweep corresponds to frequencies from zero to 16 kHz and the pip at the origin is the zero marker.

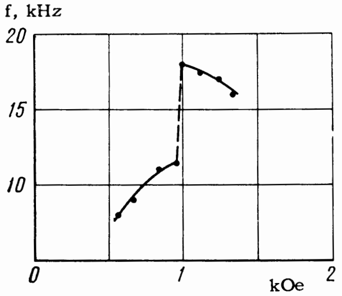


FIG. 3. The oscillation frequency at the probe as a function of magnetic field. Hydrogen, $p = 6 \times 10^{-4}$ torr, $I_a = 200$ mA.

increased (Fig. 4). The nature of the instability is independent of the anode voltage U_a.

Further increases in the magnetic field lead to a disturbance of the stationary rotation of the tongues and to the appearance of oscillations characterized by a continuous spectrum from 0 to 30—50 kHz. In this case the tongue structure of the instability is still maintained, as is indicated by the plasmograms observed under these conditions (Fig. 2b). The point at which the transition occurs is not accurately known; as a rule, it shifts toward lower magnetic fields with increasing anode current and reduced gas pressure (Fig. 5). The noise amplitude increases continuously with magnetic field while the amplitude of the oscillations at the rotational frequency is reduced (Fig. 2b, c) and at magnetic fields greater than H_T, it cannot be distinguished in the oscillation spectrum (Fig. 2d). The value of this magnetic field is essentially independent of the discharge parameters, being approximately 2000 Oe for hydrogen (Fig. 5). In this case, as is shown by an analysis of the plasmograms, one can no longer speak of a tongue structure of the instability.

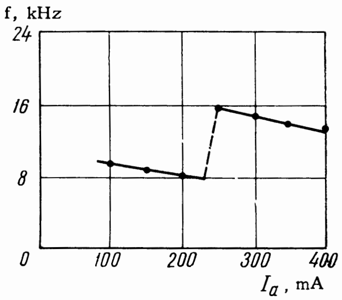


FIG. 4. Oscillation frequency at the probe as a function of anode current. Hydrogen, $p = 6 \times 10^{-4}$ torr, $H = 530$ Oe.

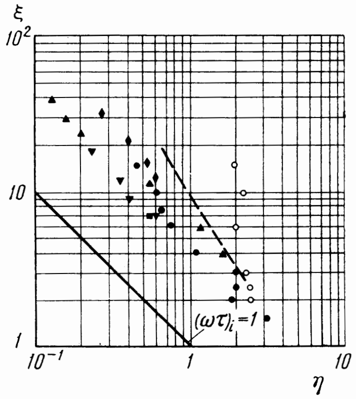


FIG. 5. Instability boundary in the coordinates $\xi = (\lambda_i/a)\sqrt{T_e/T_i} \sim 1/p$ and $\eta = a/r_i \sim H[^2]$ where a is the radius of the arc column, T_e and T_i are the electron and ion temperatures, λ_i is the ion mean free path, r_i is the ion Larmor radius, $U_a = 200$ V, $I_a = 200$ mA, ●) H_2, ▲) He, ▼) N_2, ♦) Ne, ■) Ar, the dashed curve is the limit for stationary rotation (H_2), ○ is the transition boundary of the plasma into the highly turbulent state (H_2).

Using two probes, one fixed and the other movable axially and radially, in all regimes of the instability we have been able to carry out a qualitative correlation analysis. The measurements have shown that the oscillations at the probes are always coherent for all points of the discharge at frequencies corresponding to the rotation of the tongue and that there are no phase shifts along the magnetic field over the entire length of the discharge. This result indicates that the tongue is a plasma formation which is extended along the axis of the device and rotates around the axis as a single unit. However, this correlation is not observed at fields greater than H_T. Thus, when $p = 10^{-3}$ torr and $H = 2500$ Oe, if one considers distances along the magnetic field that are greater than ~5 cm and along the radius that are greater than ~5 cm a fixed phase relation is no longer observed at both probes. The plasma makes a transition into a qualitatively new state which is characterized by strong turbulence.

3. INVESTIGATION OF DIFFUSION IN THE UNSTABLE STATE

Knowledge of the diffusion phenomena in the geometry being considered here can be obtained from the ratio of electron to ion saturation currents to a probe (i_e/i_i) which is located outside of the arc column, and from the nature of the density variation with radius. In the stable state the ratio (i_e/i_i) must be of order unity and the decay constant for the density at large distances from the column boundary is given by the familiar expression:[5]

$$q = (l/\pi)\sqrt{D_{i\perp}/D_{i0}}, \qquad (1)$$

where l is the length of the system while $D_{i\perp}$ and D_{i0} are the ion diffusion coefficients across and along the magnetic field. Measurements of q carried out in the subcritical regime[1] verify this expression and yield an estimate of $D_{i\perp}$.

In the unstable regime the ratio i_e/i_i increases by approximately an order of magnitude while the radial density distribution becomes much smoother (Fig. 6). Proceeding in a purely formal way in this case one can introduce a decay constant q and trace its behavior as a function of the discharge parameters. Then, using Eq. (1) one can estimate the effective transverse diffusion coefficient $D_{\perp\text{eff}}$. In Figs. 7 and 8 we show the dependence of q on magnetic field and pressure; the dependence of i_e/i_i on H is shown in Fig. 9. It is found that in high magnetic fields $q \sim 1/\sqrt{H}$ and $\sim\sqrt{p}$ while $i_e/i_i \sim 1/\sqrt{H}$. These results are in good agreement with the Bohm formula for the turbulent plasma diffusion coefficient

$$D_B = 10^7\, T_e / H. \qquad (2)$$

The value of $D_{\perp\text{eff}}$ determined from Eq. (1) for $p = 2.5 \times 10^{-3}$ torr and H = 2600 Oe is 1.4×10^4 cm^2/sec; this is also of the same order of magnitude as $D_B = 0.8 \times 10^4$ cm^2/sec for $T_e = 2$ eV.

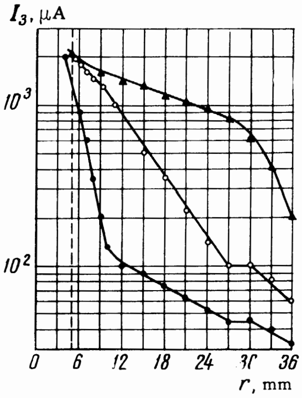


FIG. 6. Ion saturation current to the probe I_p as a function of radius. Hydrogen $p = 2.5 \times 10^{-3}$ torr, ●) H = 560 Oe, ○) H = 1900 Oe, ▲) H = 3000 Oe.

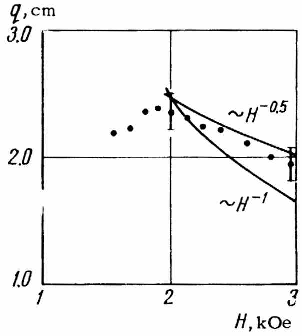


FIG. 7. The decay constant q as a function of magnetic field. Hydrogen, $p = 6 \times 10^{-4}$ torr.

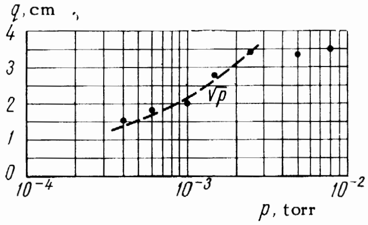


FIG. 8. The decay constant q as a function of pressure. Hydrogen, H = 2600 Oe.

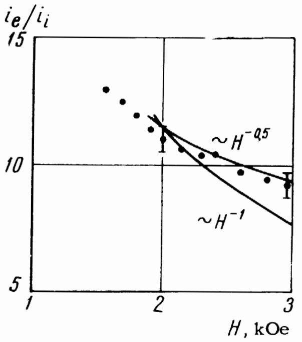


FIG. 9. The ratio i_e/i_i as a function of magnetic field. Hydrogen, $p = 6 \times 10^{-4}$ torr.

4. DENSITY OSCILLATIONS

We have indicated above that the transition of the plasma into the unstable state is characterized by strong oscillations. It is interesting to compare the change in the nature of the transverse diffusion (when the magnetic field is increased) with the corresponding changes in the density oscillations. The measurements have been carried out in hydrogen at a fixed pressure $p = 2.5 \times 10^{-3}$ torr for four values of the magnetic field. In this case the quantities H_{cr} and H_T were found to be 1740 and 2100 Oe respectively. An idea of the magnitude of the density oscillation can be obtained from the alternating component of the ion saturation current to a probe located at r = 1.6 cm.

Analysis of a large number of oscillograms for each value for the magnetic field makes it possible to plot a histogram (Fig. 10) which characterizes the probability W of observation of an oscillation amplitude in the density. On the basis of these data we determine the mean values of the density in a tongue $\bar{n}$ and the value of the mean deviation $\tilde{n}$.

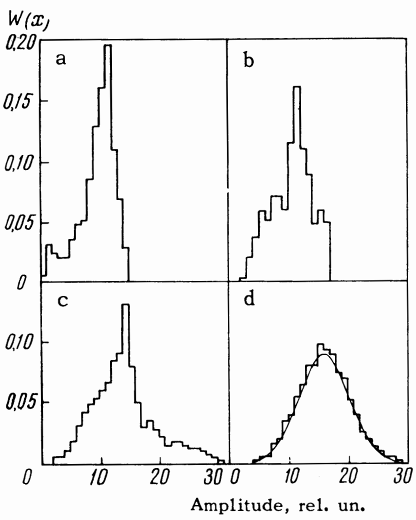


FIG. 10. The probability W(x) as a function of magnetic field, x – the value of the density in relative units. Hydrogen, $p = 2.5 \times 10^{-3}$ torr. a) H = 1800 Oe, b) H = 2000 Oe, c) H = 2300 Oe, d) H = 3000 Oe.

Simultaneously, using the averaged component of the probe characteristic, we determine the time-averaged value of the density $\bar{n}_t$ and the electron temperature T_e. In Fig. 11 we show the quantities $\bar{n}$ and $\bar{n}_t$ measured in this way as functions of the magnetic field; the value of T_e for H > 2000 Oe is essentially independent of magnetic field, being approximately 2 eV.

Near the critical magnetic field one observes a stably rotating tongue structure; the frequency of oscillation in the probe signal is a multiple of the rotational frequency of the tongue and the amplitude is essentially independent of time. In this case W is in the form of a δ-function. However, as the magnetic field increases one observes an increasing spread in the oscillation amplitude (Fig. 10, abc) and at magnetic fields of order of 2500—3000 Oe the quantity W becomes nearly Gaussian (Fig. 10d). This is a clear indication of the strong turbulence indicated above at magnetic fields greater than H_T. As the magnetic field is increased one also observes an increase in $\bar{n}$ and $\bar{n}_t$ and at large H (approximately 3000 Oe) these quantities are essentially the same (Fig. 11). The increase in density with increasing magnetic field does not contradict the reduction in q (Fig. 7) since the value of the density is, to a considerable degree, determined by the decay constant close to the boundary of the arc column and this constant increases with increasing field (Fig. 6).

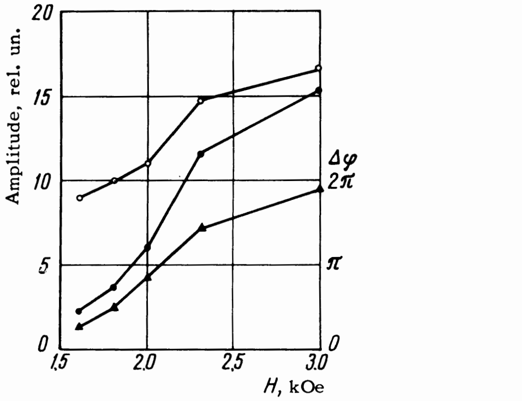
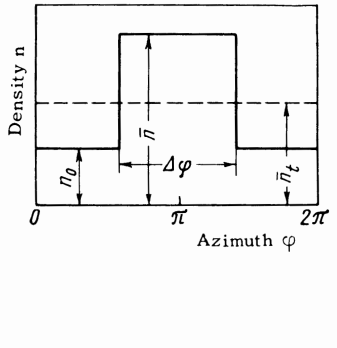


FIG. 11. The dependence of the quantities $\bar{n}$(○), $\bar{n}_t$(●) and $\Delta\varphi$(▲) on magnetic field. Hydrogen, $p = 2.5 \times 10^{-3}$ torr.

To a first approximation the entire cross section of the discharge can be divided into two regions. In the first (tongue) there is an anomalous loss of plasma across the magnetic field; in the second region the nature of the diffusion is essentially classical. Near H_{cr} this division is verified experimentally[4]; we will assume that this is also possible at high magnetic fields. Then each magnetic field corresponds to a definite ratio between these regions. In other words, at each value of H one can determine an effective tongue which has some required width $\Delta\varphi$ and charged particle density $\bar{n}$. Then, as is evident from Fig. 12, we must satisfy the relation

$$\Delta\varphi = 2\pi\frac{\bar{n}_t - n_0}{\bar{n} - n_0}, \tag{3}$$

FIG. 12.



where n_0 is the plasma density at a given radius in the unperturbed region. This quantity can be obtained by extrapolation of the corresponding values of the density in the subcritical regime to high magnetic fields. As a result of calculations which have been carried out following this procedure for each magnetic field we have obtained the values of $\Delta\varphi$ which are shown in Fig. 11. It is evident that as H increases the width of the effective tongue increases also and at fields ~3000 Oe, the quantity $\Delta\varphi$ is close to 2π. This means that in the transition to the highly turbulent state the anomalous loss across magnetic fields occurs over essentially the entire cross section of the discharge.

Knowing the values of $\tilde{n}$ and $\bar{n}$ one can estimate the averaged diffusion flux in the turbulent state Q. In this case $\bar{n}$ is the mean value of the density at a given radius while $\tilde{n}$ is the mean value of the oscillation amplitude. In accordance with[6]

$$Q = \gamma' q^2 \frac{\tilde{n}^2}{\bar{n}^2} \frac{dn}{dr},$$

whence

$$D_\perp \approx \gamma' q^2 \tilde{n}^2 / \bar{n}^2, \tag{4}$$

where γ' is the characteristic lifetime for a density perturbation. By equating the values of $D_\perp$ from Eqs. (1) and (4) we can estimate the quantity γ'. As a result, taking $p = 2.5 \times 10^{-3}$ torr and $H = 3000$ Oe we have $\tilde{n}/\bar{n} = 1/3$ (Fig. 10d) and $\gamma' \approx D_{i0} (\pi/l)^2 (\bar{n}/\tilde{n}) \approx 2 \times 10^4 \text{ sec}^{-1}$ which is of the order of magnitude of the drift frequency; in this case $\omega_{dr} = k_\varphi c T_e / Hq \approx 4.4 \times 10^4 \text{ sec}^{-1}$ if we take $\lambda_\varphi = 2\pi a$ and $T_e \approx 2$ eV.

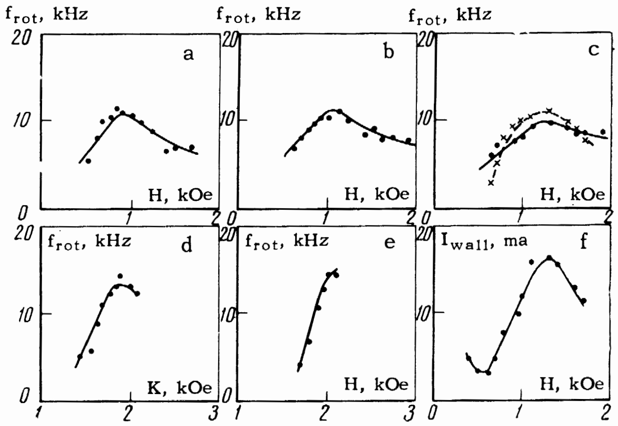


FIG. 13. The rotational frequency of the tongue $f_{rot} = \omega_{rot}/2\pi$ as a function of magnetic field and pressure. Hydrogen, a) $p = 6 \times 10^{-4}$, b) $p = 8 \times 10^{-4}$ torr, c) $p = 10^{-3}$ torr, d) $p = 2 \times 10^{-3}$ torr, e) $p = 3 \times 10^{-3}$ torr, f) the current to the side wall I_{wall} as a function of magnetic field $p = 10^{-3}$ torr.

5. ROTATION OF THE TONGUE

In Fig. 13 we show the frequency of rotation of the tongue as a function of magnetic field for various pressures; these data are obtained by analysis of the oscillation spectra observed at the probes. In the experiments described here we always observe rotation in the ion direction. As has been noted in[3] one can ascribe the rotation to two causes: ion drift and the presence of a current flowing in the tongue. However, in the majority of cases the azimuthal drift velocity of the ions in the tongue is smaller than the observed rotational velocity and need not be taken into account.

The current flowing in a tongue I_t produces a force $F = c^{-1} I_t \times H$ which causes rotation. The angular velocity of this rotation can be determined by equating the moment of the effective force to the change in the angular momentum of the ions as the result of their collisions with neutral particles:

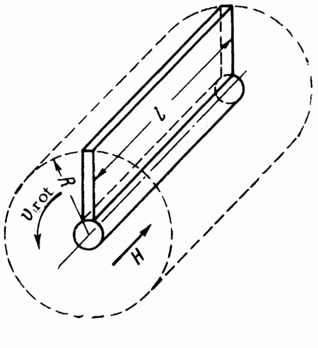
$$\Delta M = F \tau_i h_t / 2, \tag{5}$$

where τ_i is the time between collisions while h_t is the height of the tongue. The angular momentum of the tongue can be determined by regarding it as the solid body extended along the magnetic field over the entire length of the system l and having an azimuthal dimension δ_t and a radial dimension h_t (Fig. 14). Then the angular momentum associated with the rotation of the tongue around the axis of the system with frequency ω_{rot} is given by the expression

$$\Delta M = {}^1\!/_3 n_i m_i \delta_t l h_t{}^3 \omega_{rot}, \tag{6}$$

where n_i is the mean particle density in the tongue.

FIG. 14.



From Eqs. (5) and (6) we then have

$$\omega_{rot} = \frac{1}{c} I_t H \frac{3}{2 n_i m_i n_0 \sigma_i v_i h_t{}^2 \delta_t l} \tag{7}$$

where n_0 is the density of neutral particles, σ_i is the cross section for elastic ion collisions, v_i is the ion thermal velocity, and I_t is the total current flowing in the tongue. The quantity I_t can be determined from the expression $I_t \approx \alpha I_{wall}/n$ where α is some constant, n is the number of tongues and

I_{wall} is the current to the side wall of the chamber. In our case I_{wall} is always positive and is a function of the field and the discharge parameters.

In order to verify the validity of the model it is of greatest interest to determine the dependence of the rotational frequency on magnetic field; hence, we write Eq. (7) in the form

$$\omega_{rot} = kI_{wall}(H)\alpha H / n, \tag{8}$$

where k is some function of the other discharge parameters. To a first approximation it is assumed that n_i and δ_t are only weak functions of magnetic field. With $p = 10^{-3}$ torr, writing $\delta_t \approx a = 0.5$ cm, the radius of the arc column, $h_t \approx R = 4$ cm, the radius of the discharge chamber, $l = 40$ cm and $n_i \approx 5 \times 10^{10}$ particles/cm^3 [4] we find $k \approx 12$. Substituting the experimental value of I_{wall} (Fig. 13c) in Eq. (8) and taking account of the change of n with magnetic field we can then plot the function $\omega_{rot}(H)$. The results of the calculation are shown in Fig. 13c (dashed curve). It is clear that the curve is in good agreement with the experimental curve while the numerical values differ by approximately a factor of two ($\alpha \approx 2$).

6. CONCLUSION

The experimental results reported here indicate the following:

1. The nature of the instability changes with magnetic field. Near the instability boundary one always observes a stable rotation of the tongue; as the magnetic field is increased beyond this point the stationary rotation disappears and the plasma exhibits intense oscillations with a continuous frequency spectra ranging from zero to several tens of kHz. At magnetic fields greater than H_T one can no longer speak of a tongue structure for the instability. The oscillations then indicate a random nature as is indicated by the plasmograms, the oscillation spectra and the observed spread in oscillation amplitudes. Under these conditions the plasma evidently makes a transition to a qualitatively new state.

2. As the magnetic field is changed there is also a change in the nature of the anomalous loss of plasma across the magnetic field. Near H_{cr} this loss occurs in a relatively narrow region and is essentially more convective than diffusive in nature: the plasma ''flows'' from the arc column to the wall drifting in the azimuthal electric field of the tongue. As the magnetic field is increased one then observes a broadening of the noise spectrum and a growth of its amplitude, in which case the nature of the motion of the charged particles across the magnetic field evidently becomes more diffusive. This is verified by the form of the plasmograms, by the observed spread in oscillation amplitude, and by the absence of correlation in the oscillations over long distance.

3. Measurements of the mean values of the density and oscillation amplitude under conditions of strong turbulence allow us to estimate the characteristic lifetime perturbations in the turbulent state and these are found to be of the same order as the drift frequency.

4. The direction of frequency of the rotation of the tongue can be understood from a qualitative model in which the origin of the rotation is the interaction of the ion current flowing in the tongue with magnetic field.

In conclusion the author wishes to thank B. B. Kadomtsev, E. I. Dobrokhotov and A. V. Zharinov.

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