DUAL INVARIANCE OF QUANTUM ELECTRODYNAMICS

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The interaction between a spinor particle having electric and magnetic charges and an electromagnetic field is considered. If it is assumed that the wave function of the particle is connected with the wave function of electron through dual rotation, then it follows that quantum electrodynamics is dually invariant. Hence the electric and magnetic charges of the particle are unobservable quantities. In experiment it is possible to measure only the sum of the squares of the electric and magnetic charges. If the charges are assumed scalar, the P- and T-parities of quantum electrodynamics also cannot be observed separately. Only C and PT have physical meaning. In the end of the article, the consequences of the assumption that the intermediate boson has definite electric and magnetic charges of equal values are discussed.

As is well known, free electromagnetic field is invariant with respect to rotations in dual space.

It will be shown in the present paper that quantum electrodynamics is dually invariant provided the appropriate definition of dual rotation of the electron wave function is accepted. In the end of the paper certain results are discussed which follow from the assumption that the weak interaction violates dual invariance of quantum electrodynamics.

In Mandelstam's paper[1] there was given a formulation of quantum electrodynamics without introducing potentials. The quantum theory of a particle having both electric and magnetic charges was constructed by Mandelstam's method in [2], where it was shown that the values of electric and magnetic charges may be arbitrary.

The equations of motion given in [1, 2] could be essentially simplified if one carried out the averaging by the method reported in [3].

Consider a spinor particle having both electric and magnetic charges. In this case the Maxwell equations take the form

$$\partial^\nu F_{\mu\nu} = eI_\mu, \tag{1}$$
$$\partial^\nu \tilde{F}_{\mu\nu} = gI_\mu, \tag{2}$$

where $F_{\mu\nu}$ is the tensor of electromagnetic field and the tilde denotes dual conjugation. Let us execute the dual rotation

$$h\mathscr{F}_{\mu\nu} = eF_{\mu\nu} + g\tilde{F}_{\mu\nu}, \tag{3}$$

$$h\tilde{\mathscr{F}}_{\mu\nu} = e\tilde{F}_{\mu\nu} - gF_{\mu\nu}, \tag{4}$$

where

$$e^2 + g^2 = h^2, \quad e = h\cos\theta, \quad g = h\sin\theta, \tag{5}$$

and θ is the angle of rotation in dual space. From (1)–(4) it follows that

$$\partial^\nu \mathscr{F}_{\mu\nu} = hI_\mu, \tag{6}$$

$$\partial^\nu \tilde{\mathscr{F}}_{\mu\nu} = 0. \tag{7}$$

Equation (7) means that $\mathscr{F}_{\mu\nu}$ can be expressed through one potential:

$$\mathscr{F}_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu. \tag{8}$$

In correspondence with [2, 3], we can put

$$C_\mu(x) = \frac{i}{4\pi^2}\int \mathscr{F}_{\mu\nu}(x+y)\frac{\partial}{\partial y_\nu}\left(\frac{1}{y^2}\right)d^4y, \tag{9}$$

and to the definition (9) it is necessary to add the rule for going round the poles $y^2 = 0$.

Let us introduce the notation

$$hC_\mu = eA_\mu + gB_\mu. \tag{10}$$

From (3), (4), (8), and (10) it follows directly that

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - (\widetilde{\partial_\mu B_\nu} - \widetilde{\partial_\nu B_\mu}),$$
$$\tilde{F}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + (\widetilde{\partial_\mu A_\nu} - \widetilde{\partial_\nu A_\mu}). \tag{11}$$

Substituting (11) into (1) and (2) we obtain

$$gA_\mu = eB_\mu. \tag{12}$$

The interaction between the electron and the electromagnetic field is introduced in minimal fashion by means of the substitution

$$\partial_\mu \to \partial_\mu - ihC_\mu = \partial_\mu - ieA_\mu - igB_\mu \tag{13}$$

in the Lagrange function. The Dirac equations take the form

$$i\gamma^\mu(\partial_\mu - ieA_\mu - igB_\mu)\psi - m\psi = 0,$$

$$i(\partial_\mu + ieA_\mu + igB_\mu)\bar{\psi}\gamma^\mu + m\bar{\psi} = 0. \tag{14}$$

The current in the right-hand sides of (1) and (2) is equal to

$$I_\mu = \bar{\psi}\gamma_\mu\psi. \tag{15}$$

The Maxwell equations (1) and (2), the Dirac equations (14), and Eqs. (15) and (11) describe a system consisting of a spinor particle having electric and magnetic charges and an electromagnetic field.

We quantize the free electromagnetic field according to the relation

$$[F_{\mu\nu}(x), F_{\rho\sigma}(x')] - i(g_{\mu\rho}\partial_\nu\partial_\sigma + g_{\nu\sigma}\partial_\mu\partial_\rho - g_{\mu\sigma}\partial_\nu\partial_\rho - g_{\nu\rho}\partial_\mu\partial_\sigma)D(x-x'). \tag{16}$$

It is easy to see that the quantities $\mathscr{F}_{\mu\nu}$ defined by the equations (3) and (4) satisfy the commutation relations (16), i.e.

$$[\mathscr{F}_{\mu\nu}(x), \mathscr{F}_{\rho\sigma}(x')] = [F_{\mu\nu}(x), F_{\rho\sigma}(x')]. \tag{17}$$

From (9) and (17) it follows[3]

$$[C_\mu(x), C_\nu(x')] = i\left(g_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right)D(x-x'), \tag{18}$$

and, therefore,

$$\langle T(C_\mu(x)C_\nu(x'))\rangle_0 = i\left(g_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right)D_c(x-x'). \tag{19}$$

We quantize the free spinor field in a usual way.

It is easy to construct the S-matrix for the above case, when the spinor particle has electric and magnetic charges. As follows from (14), the interaction term is determined by two potentials: A_μ and B_μ and, accordingly, by the charges e and g. But by means of the substitution (10) we introduce instead of the potentials A_μ and B_μ and the charges e and g the function C_μ and the constant h, and in constructing the Feynman diagram we make use of the relation (19). To each vertex of the Feynman diagram corresponds the constant h connected with the charges by Eq. (5).

For a free photon line we can also use C_μ, because the free electromagnetic field is dually invariant, and the potential C_μ is obtained from A_μ and B_μ by dual rotation. Thus the Feynman diagrams for the considered case differ from ordinary ones only in the sense that in our case the vertex is associated with the constant h.

We denote the wave function of the spinor particle having electric and magnetic charges, e and g respectively by ψ (e,g.). Our result is equivalent to the statement that the Feynman diagrams for the system ψ (e,g) and $F_{\mu\nu}$ completely coincide with the Feynman diagrams for the system ψ(h, 0) and $\mathscr{F}_{\mu\nu}$. We shall assume that the transition from ψ(e, g) to ψ(h, 0) means dual rotation through the angle θ. Therefore, quantum electrodynamics is invariant with respect to dual rotations by an arbitrary angle θ. We shall assume that the constants e and g are scalars. Then it follows from (1) and (2) that P- and T-parities are not conserved for sin θ different from zero. C and PT are conserved, but there is degeneracy with respect to the angle θ. In other words, in quantum electrodynamics P- and T-parities are unobservable quantities and it is meaningless to speak of their conservation or violation. The observable quantities are C and PT. Quite similarly, electric and magnetic charges of an electron are unobservable. The observable quantity is h ($h^2/4\pi = 1/137$).

As follows from (14), a spinor particle with electric and magnetic charges e and g has a dipole magnetic moment e/2m and a dipole electric moment g/2m. It is obvious that in electrodynamics the observable quantity is h/2m.

It is possible that there exists an interaction which violates dual invariance of quantum electrodynamics. To put it more accurately, we assume, first, that there exists an intermediate boson and, second, that the intermediate boson has electric and magnetic charges of equal values:

$$e = g = h/\sqrt{2} \tag{20}$$

and therefore it has equal electric and magnetic moments

$$\mu_E = \mu_M = e/2M. \tag{21}$$

The Salzmanns[4] have considered precisely the case when the intermediate boson has equal electric and magnetic moments, and have shown that the assumption (21) explains the $K_2^0 \to 2\pi$ decay, which T-parity is not conserved. They have also obtained for the ratio of the amplitudes of decays $K_2^0 \to 2\pi$ and $K_1^0 \to 2\pi$ a value which is in satisfactory agreement with experiment.[5]

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