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PARTICLE RELAXATION IN A MAXWELL GAS

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The effect of particle interaction on their distribution and relaxation is considered. It is shown that the quantum energy indeterminancy due to the interaction leads to the appearance of power-law tails in the momentum distribution. A time-dependent distribution function characterizing the relaxation of a particle in a Maxwell gas has been obtained by taking into account quantum corrections. The particle is assumed to possess a definite momentum at the initial time. A kinetic equation which takes the quantum energy indeterminacy into account is derived.

THE energy spectrum of a system can, in a large number of cases, be described in terms of quasiparticles. The quasiparticle states are not, strictly speaking, stationary states, but decay with time as the result of interaction processes. The presence of damping means that the energy of the quasiparticles is not real, and contains an imaginary correction, proportional to the reciprocal of the relaxation time. With account of this quantum correction to the energy, the momentum distribution of the quasiparticles, and consequently also of the particles, cannot be described by the usual Bose or Fermi distributions. It is shown in the present research that the particle momentum distribution is determined by an integral over the energy of the ordinary distribution function, multiplied by the dispersion function width, equal to the quantum energy indeterminacy. Such a dependence leads to the appearance of power-law "tails" in the momentum distribution of the particles. In considering specific physical effects, one must keep in mind that the particles described by such tails are virtual, so that their energy and momentum are not connected by the usual relation.

The noted quantum-energy indeterminacy of quasiparticles also leads to the appearance of specific quantum corrections to the kinetic equations. Actually, in their calculation, the collision terms will no longer have their usual form with a delta function describing the energy conservation law. Using the terminology of the theory of spectral line widths, one can say that because of the relaxation process, there is a "natural width" of this delta function. We obtain here a solution of the problem of the relaxation of the particle distribution in a Boltzmann gas with account of the effects mentioned. It is shown that this process does not take place monotonically, and the distribution function contains oscillatory terms. A kinetic equation is found for such a function in the simplest case of a gas of infinitely heavy particles.

1. We consider a system of interacting particles in thermal equilibrium. We assume that the interaction in the system is weak in the sense that there exists the concept of quasiparticles characterized by a momentum $\mathbf{p}$. For simplicity in the subsequent calculations, we also assume the conditions of applicability of perturbation theory to be satisfied (the generalization to the case of the gas approximation does not present any difficulty). The energy of the quasiparticles is $\epsilon(\mathbf{p})$ and, since dispersion will not be important in what follows, we set

$$\varepsilon(\mathbf{p}) = {}^1\!/_2 p^2 \tag{1}$$

(the system of units is chosen in which $\hbar = m = 1$). For definiteness, let the particles obey Fermi statistics. In the case of zero temperature, the mean number of particles in a state with given momentum $\mathbf{p}$, as is well known, has the form[1]

$$n(\mathbf{p}) = \frac{1}{\pi}\int_{-\infty}^{\mu} d\omega \,\mathrm{Im}\, G(\mathbf{p},\omega), \tag{2}$$

μ is the chemical potential of the system. Under our assumptions, the Green's function G is equal to

$$G = (\omega - \varepsilon(\mathbf{p}) - i\gamma)^{-1}, \tag{3}$$

where γ is the damping decrement of the quasiparticle states. Consequently,

$$n(\mathbf{p}) = \frac{1}{\pi} \int_{-\infty}^{\mu} d\omega \frac{\gamma}{[\omega - \varepsilon(\mathbf{p})]^2 + \gamma^2}. \tag{4}$$

In the absence of interaction ($\gamma \rightarrow 0$), this expression transforms to the usual Fermi distribution function. For γ not equal to zero, tails appear in the distribution function, which fall off with momentum according to a power law

$$n(\mathbf{p}) = \frac{1}{\pi \varepsilon^2(\mathbf{p})} \int_{-\infty}^{\mu} d\omega\, \gamma(\mathbf{p}, \omega), \quad p \gg p_f. \tag{5}$$

It is natural to expect a similar situation to arise for a temperature different from zero. Therefore, we shall not consider Eq. (5) in further detail, but shall turn at once to the case of an arbitrary temperature. It is evident beforehand that, for sufficiently large p, a mixture of states of each quasiparticle, falling off according to a power law, can exceed the exponential contribution of the fundamental part. The mean number of particles is described in the following fashion:

$$\begin{aligned} n(\mathbf{p}) &= \frac{1}{\beta} \sum_{\varepsilon_n} e^{+i\cdot 0} G(\mathbf{p}, \varepsilon_n) \\ &= \int_{-\infty}^{\infty} \frac{d\omega}{e^{\beta(\omega-\mu)} + 1} \frac{1}{\pi} \frac{\gamma(\mathbf{p}, \omega)}{[\omega - \varepsilon(\mathbf{p})]^2 + \gamma^2(\mathbf{p}, \omega)}. \end{aligned} \tag{6}$$

Here $G(p, \epsilon_n)$ is the single-particle temperature Green's function of a system of interacting particles,[2] $\beta = T^{-1}$. The function γ (for $s = 1/2$) has the form[3]

$$\begin{aligned} \gamma(\mathbf{p}, \omega) = \pi \int d\mathbf{q}\, d\mathbf{p}_1 V_{\mathbf{q}} (2V_{\mathbf{q}} - V_{\mathbf{q}-\mathbf{p}-\mathbf{p}_1}) \\ \times [n_{\mathbf{p}-\mathbf{q}} (n_{\mathbf{p}_1-\mathbf{q}} - n_{\mathbf{p}_1}) + n_{\mathbf{p}_1} (1 - n_{\mathbf{p}_1-\mathbf{q}})] \\ \times \delta(\omega - \varepsilon_{\mathbf{p}-\mathbf{q}} + \varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}_1-\mathbf{q}}), \end{aligned} \tag{7}$$

V_q is the Fourier component of the interaction potential, $n_p = \{\exp[\beta(\epsilon_p - \mu)] + 1\}^{-1}$, $dq = d^3q/(2\pi)^3$.

We denote by p_f the characteristic momentum in the system (for example, this is the Fermi momentum at low temperatures and the mean thermal momentum at high temperatures). For $p \gg p_f$, the maxima of the functions entering into the integral (6) are widely separated and, consequently, there are two regions which give the largest contribution. It is not difficult to see that the region of the maximum of the dispersion equation gives the usual result, and is determined by the inequality $\omega \gg \omega_f$, where $\omega_f \cong \min[\mu, \beta^{-1}]$. On the other hand, $\{\exp[\beta(\omega - \mu)] + 1\}^{-1}$ is of the order of unity in the interval $-\infty < \omega \lesssim \omega_f$ and falls off exponentially for $\omega > \omega_f$. Therefore, for large momenta, Eq. (6) can be represented as the sum of two components:

$$n(\mathbf{p}) = n_{\mathbf{p}} + \frac{1}{\pi} \int_{-\infty}^{\omega_f} d\omega \frac{\gamma(\mathbf{p}, \omega)}{(\omega - \varepsilon_{\mathbf{p}})^2} \equiv n_{\mathbf{p}} + \nu_{\mathbf{p}}, \quad p \gg p_f. \tag{8}$$

Substituting the value γ from (7) in ν_p, and integrating with the delta function, we get

$$\begin{aligned} \nu_{\mathbf{p}} = \int d\mathbf{q}\, d\mathbf{p}_1 V_{\mathbf{q}} (2V_{\mathbf{q}} - V_{\mathbf{q}-\mathbf{p}-\mathbf{p}_1}) \\ \times \frac{n_{\mathbf{p}-\mathbf{q}} (n_{\mathbf{p}_1-\mathbf{q}} - n_{\mathbf{p}_1}) + n_{\mathbf{p}_1} (1 - n_{\mathbf{p}_1-\mathbf{q}})}{(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}-\mathbf{q}} + \varepsilon_{\mathbf{p}_1} - \varepsilon_{\mathbf{p}_1-\mathbf{q}})^2}, \end{aligned} \tag{9}$$

with the condition

$$-\infty < \varepsilon_{\mathbf{p}-\mathbf{q}} - \varepsilon_{\mathbf{p}_1} + \varepsilon_{\mathbf{p}_1-\mathbf{q}} \lesssim \omega_f. \tag{10}$$

It is easy to conclude that the region of nonexponential contribution is limited by the order equalities

$$|\mathbf{p} - \mathbf{q}| \sim |\mathbf{p}_1 - \mathbf{q}| \sim p_f, \quad q \sim p_1 \sim p \gg p_f.$$

Neglecting unimportant terms in (9), we finally obtain

$$\nu_{\mathbf{p}} = \frac{|V_{\mathbf{p}}|^2}{(2\varepsilon_{\mathbf{p}})^2} \int d\mathbf{q}\, d\mathbf{p}_1 n_{\mathbf{p}-\mathbf{q}} n_{\mathbf{p}_1-\mathbf{q}} = \frac{n^2 |V_{\mathbf{p}}|^2}{16 \varepsilon_{\mathbf{p}}}, \quad p \gg p_f, \tag{11}$$

where n is the particle number density.

In the case of short-range interaction $|V_p|^2 \cong \pi\sigma$ ($p_f \ll p \ll 1/a$, a is the region of action of the potential)

$$\nu_p = \frac{\pi}{4} \frac{n^2 \sigma}{p^4}, \tag{12}$$

σ is the integral scattering cross section. For Coulomb interaction, $V_p = 4\pi e^2/p^2$ and

$$\nu_{\mathbf{p}} = 4\pi^2 n^2 e^4 / p^8. \tag{13}$$

The value of ν_p (13) is comparable with the Maxwell term when

$$\frac{\varepsilon(p)}{T} - 4 \ln \frac{\varepsilon(p)}{T} = \ln \frac{m^{1/2} T^{5/2}}{\hbar n e^4}, \tag{14}$$

which, for hydrogen gas under normal conditions, gives $\epsilon(p)/T = 10$—20.

We note that the numerical corrections have a truly quantum character and vanish as ħ approaches zero.

2. Let us consider further the problem of diffusion in momentum space of a particle with initial momentum p_0 moving in an ideal gas of particles of mass M. The total Hamiltonian has the form

$$\begin{aligned} H = H_0 + H_c + H_{int} = \sum \varepsilon_{\mathbf{p}} a_{\mathbf{p}}^{+} a_{\mathbf{p}} + \sum \varepsilon_{\mathbf{p}}' b_{\mathbf{p}}^{+} b_{\mathbf{p}} \\ + \sum V_{\mathbf{q}} b_{\mathbf{p}}^{+} \overset{+}{a}_{\mathbf{p}_1-\mathbf{q}} a_{\mathbf{p}_1} b_{\mathbf{p}-\mathbf{q}} \end{aligned} \tag{15}$$

(we make use of a system of units in which the mass of the particle $m = \hbar = 1$), where a_p^+ and a_p are the creation and annihilation operators of the particle; b_p^+ and b_p are the corresponding operators referring to the particles of the gas; $\epsilon_p = p^2/2$, $\epsilon_p' = p^2/2M$ and V_q is the Fourier component of the interaction potential.

We denote the eigenfunctions of H_0 and H_c by $|p\rangle = a_p^+|0\rangle$ and $|s\rangle$ ($|ps\rangle \equiv |p\rangle|s\rangle$). Then the formal solution of this equation with the complete Hamiltonian (15) is

$$\Phi(t) = e^{-iHt}a_{\mathbf{p}_0}{}^+|0s\rangle. \tag{16}$$

The particle distribution function at the instant of time $t > 0$ with initial condition $\mathbf{p} = \mathbf{p}_0$ at $t = 0$ is obviously determined by the equality

$$n_{\mathbf{p}_0}(\mathbf{p}, t) = (\Phi(t),\ a_{\mathbf{p}}{}^+ a_{\mathbf{p}}\Phi(t))_s. \tag{17}$$

The index s on the matrix element denotes the average over all states of the system with matrix density

$$\rho = \exp\{\beta(\Omega + \mu N - H_c)\}.$$

Substituting the value of $\Phi(t)$ (16) in (17) and introducing the Heisenberg operators

$$\tilde{a}_{\mathbf{p}}(t) = e^{iHt}a_{\mathbf{p}}e^{-iHt},$$

we get by means of simple transformations

$$n_{\mathbf{p}_0}(\mathbf{p}, t) = (\langle s|G^*(\mathbf{pp}_0, t)G(\mathbf{pp}_0, t)|s\rangle)_s. \tag{18}$$

It must be kept in mind that the single-particle Green's functions

$$G(\mathbf{pp}_0, t - t') = -i\langle 0|T\{\tilde{a}_{\mathbf{p}}(t)\tilde{a}_{\mathbf{p}_0}{}^+(t')\}|0\rangle \tag{19}$$

are operators over a variable system. It is interesting that for $t < 0$, the mean $\langle T\{\tilde{a}_p(t)\tilde{a}_{p_0}{}^+(0)\}\rangle = 0$ and, consequently, the distribution function (18) has meaning only for positive times, as should be the case.

Furthermore, it is convenient to introduce the two-particle Green's function according to the formula

$$K(\mathbf{pp}_0, tt_0;\ \mathbf{pp}_0, t't_0') = (\langle s|G^*(\mathbf{pp}_0, t - t_0)G(\mathbf{pp}_0, t' - t_0')|s\rangle)_s. \tag{20}$$

The distribution function (18) is expressed in its terms in the following simple fashion

$$n_{\mathbf{p}_0}(\mathbf{p}, t) = K(\mathbf{pp}_0, t0;\ \mathbf{pp}_0, t0). \tag{21}$$

Because of the time homogeneity, K depends only on three times, for example, $t - t'$, $t_0 - t_0'$ and $\frac{1}{2}(t + t') - \frac{1}{2}(t_0 + t_0')$. Taking this circumstance into account and also (21), we find

$$n_{\mathbf{p}_0}(\mathbf{p}, t) = \int \frac{d\omega\, d\omega_0\, d\lambda}{(2\pi)^3} e^{-i\lambda t}K(\mathbf{p}_0\mathbf{p},\ \omega_0\omega;\ \lambda). \tag{22}$$

The rules of the diagram technique for K are obtained by means of the usual procedure—transformation to the interaction representation with subsequent expansion of the S matrix in powers in H_{int} (the only singularity is the simultaneous use of the temperature and zero-temperature techniques of the Green's function). In view of the simplicity of the operation set forth, there is no need for going into the procedure in detail. Therefore, we shall at once write down the final results. Here, for the sake of simplicity of exposition, we again limit ourselves to consideration of the region of applicability of the Born approximation. Furthermore, we shall not be interested in the "shift" of the energy of the particle and, consequently, in the calculation of the self-energy part Σ, we shall keep only its imaginary part which corresponds to the "width."

The lines of interaction with momentum $q = (\mathbf{q}, \nu)$ (denoted on the graph by a wavy line), corresponds to the function $D(q)$

$$D(q) = 2\pi|V_{\mathbf{q}}|^2\int d\mathbf{p}\, n_{\mathbf{p}}\delta(\nu + \varepsilon_{\mathbf{p}}' - \varepsilon_{\mathbf{p}-\mathbf{q}}')$$

$$= 2\pi|V_{\mathbf{q}}|^2 n\left(\frac{\beta}{4\pi\varepsilon_{\mathbf{q}}'}\right)^{1/2}\exp\left[-\frac{\beta}{4\pi\varepsilon_{\mathbf{q}}'}(\nu - \varepsilon_{\mathbf{q}}')^2\right]. \tag{23}$$

The latter enters with negative sign if the line belongs to a definite single particle Green's function. Straight lines in the graph correspond to the Green's function (19) averaged over the states of the system; G and G* have opposite directions.

The zero Green's function is equal to

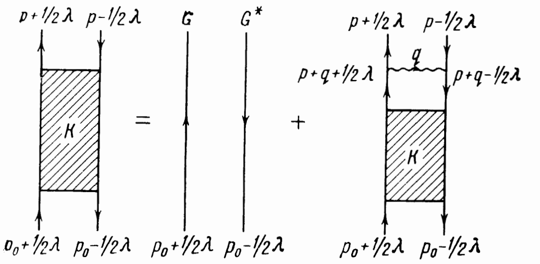
$$G^{(0)}(p) = (\omega - \varepsilon_{\mathbf{p}} + i\delta)^{-1}. \tag{24}$$

The exact G function is described in the well-known fashion[2]

$$G(p) = (\omega - \varepsilon_p - \Sigma(p))^{-1}, \tag{25}$$

where Σ is in turn determined, generally speaking, by the integral equation

$$\Sigma(p) = \int dqD(q)G(p - q),\quad dq = d\mathbf{q}\frac{d\nu}{2\pi}. \tag{26}$$



The equation which K satisfies is represented graphically in the drawing (all straight lines correspond to exact G). In analytic form,

$$K(p_0, p;\lambda) = G^*(p - {}^1\!/_2\lambda)G(p + {}^1\!/_2\lambda)\delta(p - p_0)$$

$$+ G^*(p - {}^1\!/_2\lambda)G(p + {}^1\!/_2\lambda)\int dqD(q)K(p_0, p + q;\lambda),$$

$$p \pm {}^1\!/_2\lambda = (\mathbf{p}, \omega \pm {}^1\!/_2\lambda),\ p_0 \pm {}^1\!/_2\lambda = (\mathbf{p}_0, \omega_0 \pm {}^1\!/_2\lambda),$$

$$\delta(p - p_0) = (2\pi)^4\delta(\mathbf{p} - \mathbf{p}_0)\delta(\omega - \omega_0). \tag{27}$$

For solution of the given equation, we shall express it in another equivalent form

$$K(p_0, p;\lambda) = G^*(p_0 - {}^1\!/_2\lambda)G(p_0 + {}^1\!/_2\lambda)\Big\{\delta(p - p_0)$$

$$+ \int dqD(q)K(p_0 - q, p;\lambda)\Big\}. \tag{27'}$$

Integrating this equation over p, we see that the function

$$\varphi(p_0, \lambda) = \int dp\, K(p_0, p, \lambda) \tag{28}$$

obeys the relation

$$\varphi(p_0, \lambda) = G^*(p_0 - {}^1\!/_2\lambda)G(p_0 + {}^1\!/_2\lambda)\{1$$

$$+ \int dqD(q)\varphi(p_0 - q, \lambda)\}. \tag{29}$$

Forming the difference $G^{-1}(p_0 + {}^1\!/_2\lambda) - G^{*-1}(p_0 - {}^1\!/_2\lambda)$ by using Eqs. (25) and (26) for G and Σ, it is not difficult to show that the solution of Eq. (29) is

$$\varphi(p_0, \lambda) = \lambda^{-1}[G^*(p_0 - {}^1\!/_2\lambda) - G(p_0 + {}^1\!/_2\lambda)], \tag{30}$$

where λ^{-1} must be taken in the sense $(\lambda + i\delta)^{-1}$ [this is evident from the definition of $G^{(0)}$ (24)].

The satisfaction of the law of conservation of number of particles follows immediately from these relations. Actually, noting that the poles of G and G* in λ lie in different half planes, and that $G(p_0, -0)$, we have

$$\int d\mathbf{p}n_{\mathbf{p}_0}(\mathbf{p}, t) = \int \frac{d\omega_0 d\lambda}{(2\pi)^2} e^{-i\lambda t}\varphi(p_0, \lambda)$$

$$= \int \frac{d\omega_0}{2\pi} e^{-i\cdot 0} iG(p_0) = iG(\mathbf{p}_0, +0) = 1.$$

We shall now seek K in the form

$$K(p_0, p;\lambda) = G^*(p - {}^1\!/_2\lambda)G(p + {}^1\!/_2\lambda)\{\delta(p - p_0)$$

$$+ \varphi(p_0, \lambda)f(p_0 p, \lambda)\}. \tag{31}$$

A simple substitution of this quantity in (27) yields

$$f(p_0 p, \lambda) = D(p_0 - p) + \int dq[D(p - q) - D(p_0 - p)]$$

$$\times G^*(q - {}^1\!/_2\lambda)G(q + {}^1\!/_2\lambda)f(p_0 q, \lambda). \tag{32}$$

The equation for f can be solved by successive approximations. Limiting ourselves to the zero approximation, in which f does not depend on λ and is equal to $D(p_0 - p)$, we obtain the following function as a solution of the integral equation (27):

$$K(p_0, p;\lambda) = G^*(p - {}^1\!/_2\lambda)G(p + {}^1\!/_2\lambda)\{\delta(p - p_0)$$

$$+ \varphi(p_0, \lambda)D(p_0 - p)\}. \tag{33}$$

The desired distribution is found by substituting Eq. (33) in Eq. (22), and integration over the variables indicated. The calculations are not difficult if one notes that it suffices to take the values of the eigenenergy parts at the poles of the corresponding Green's function. We write down the final result:

$$n_{\mathbf{p}_0}(\mathbf{p}, t) = (2\pi)^3\delta(\mathbf{p} - \mathbf{p}_0)e^{-2\Sigma_0 t} + \int d\mathbf{p}_1 n_{\mathbf{p}_1}\Gamma_{\mathbf{p}_0}(\mathbf{p}_1;\mathbf{p}, t), \tag{34}$$

where

$$\Gamma_{\mathbf{p}_0}(\mathbf{p}_1;\mathbf{p}, t)$$

$$= \frac{|V_{\mathbf{p}_0 - \mathbf{p}}|^2}{\Sigma}\frac{\Sigma_+}{E^2 + \Sigma_+^2}\Big\{1 + \frac{\Sigma_-}{E^2 + \Sigma_-^2}e^{-\Sigma_+ t}$$

$$\times\Big[e^{-\Sigma_- t}\frac{E^2 + \Sigma_+^2}{\Sigma_+} - 2\Sigma\cos Et\Big]$$

$$- \frac{2\Sigma}{\Sigma_+}\frac{E^2}{E^2 + \Sigma_-^2}e^{-\Sigma_+ t}\Big[\cos Et + \frac{2\Sigma_0}{E}\sin Et\Big]\Big\},$$

$$\Sigma_0 = \operatorname{Im}\Sigma(\mathbf{p}_0), \qquad \Sigma = \operatorname{Im}\Sigma(\mathbf{p}), \Sigma_\pm = \Sigma \pm \Sigma_0,$$

$$E = \varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}_0} + \varepsilon'_{\mathbf{p}_1 + \mathbf{p}_0 - \mathbf{p}} - \varepsilon_{\mathbf{p}_1}'.$$

The imaginary part of Σ appearing here is found by perturbation theory, i.e., from Eq. (26), in which the exact G is replaced by $G^{(0)}$. For example, in the case of short range effects,

$$\operatorname{Im}\Sigma(\mathbf{p}) = \frac{n\sigma}{2p}\mu^2\Big(p^2 + \frac{1}{2}p_f^2\Big), \tag{35}$$

the reduced mass $\mu = M/(1 + M)$, the mean thermal momentum $p_f = (2M\beta^{-1})^{1/2}$.

As was also to be expected, for large values of the time the distribution function is described by an integral of the type (6). The classical limit as ħ → 0 is obvious. The most evident specific features of the distribution (34) are seen in the simple example of an infinitely heavy mass for the particles of the medium. It is interesting that in this case the solution (33) is an exact solution of Eq. (27) with $D(q) = 2\pi|V_q|^2 n\delta(\nu)$.

In (34) the limit is taken directly by letting M go to infinity, we obtain (short range)

$$n_{\mathbf{p}_0}(\mathbf{p}, t) = (2\pi)^3 \delta(\mathbf{p} - \mathbf{p}_0) e^{-\gamma t} + \frac{2\pi}{p_0} \frac{\gamma}{(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}_0})^2 + \gamma^2}$$
$$\times \left\{ 1 - e^{-\gamma t} \left[\cos(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}_0}) t + \frac{\gamma}{\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}_0}} \sin(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}_0}) t \right] \right\}; \tag{36}$$

where $\gamma = n\sigma p_0$. Thus the establishment of equilibrium of the distribution function is accompanied by unique quantum oscillations.

We now find the kinetic equation that describes the relaxation of the given particle in a gas of infinitely heavy particles. The Fourier component $n_{p_0}(\mathbf{p}, t)$ (36) is equal to

$$n_{\mathbf{p}_0}(\mathbf{p}, \lambda) = (2\pi)^3 \delta(\mathbf{p} - \mathbf{p}_0) \frac{i}{\lambda + i\gamma}$$
$$+ \frac{2\pi i}{\lambda p_0} \frac{\gamma}{(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}_0})^2 - (\lambda + i\gamma)^2}. \tag{37}$$

It is convenient to introduce a function integrated over the angles of the vector $\mathbf{p}$,

$$\varphi_\lambda(\varepsilon - \varepsilon_0) = \int \frac{do}{(2\pi)^3} n_{\mathbf{p}_0}(\mathbf{p}, \lambda). \tag{38}$$

The kinetic equation for such a function should have the form

$$-i\lambda\varphi_\lambda(\varepsilon - \varepsilon_0) = \frac{1}{p_0} \delta(\varepsilon - \varepsilon_0) - \gamma\varphi_\lambda(\varepsilon - \varepsilon_0)$$
$$+ \frac{p_0}{2\pi^2} \int d\varepsilon' S_\lambda(\varepsilon - \varepsilon') \varphi_\lambda(\varepsilon' - \varepsilon_0). \tag{39}$$

It is then not difficult to find a Fourier-type kernel of the equation $S_\lambda(\tau)$ expressed in terms of the Fourier component $\varphi_\lambda(\tau)$. By determining the latter by means of (37) and (38) and by inverting $S_\lambda(\tau)$ we get

$$S_\lambda(\varepsilon) = \pi\gamma \frac{\gamma - i\lambda}{p_0} \int_{-\infty}^{\infty} d\tau \frac{e^{-i\varepsilon\tau}}{\gamma - i\lambda \exp[|\tau| (\gamma - i\lambda)]}$$
$$= \frac{2\pi^2\gamma}{p_0} \frac{1}{\pi} \frac{\gamma - i\lambda}{\varepsilon^2 + (\gamma - i\lambda)^2} \frac{1}{2} [(1 - \xi) F(1, 1, 2 + \xi; \delta)$$
$$+ (1 + \xi) F(1, 1, 2 - \xi; \delta)], \tag{40}$$

where $\xi = i\epsilon/(\gamma - i\lambda)$ and $\delta = \gamma/(\gamma - i\lambda)$. The expression in square brackets is a smooth function of the energy; therefore, with sufficient accuracy,

$$S_\lambda(\varepsilon) \cong \frac{2\pi^2\gamma}{p_0} \frac{1}{\pi} \frac{\gamma - i\lambda}{\varepsilon^2 + (\gamma - i\lambda)^2}. \tag{40$'$}$$

The characteristic dispersion shape of this function, with $\gamma - i\lambda$ as the "damping decrement," is a reflection of the fact, pointed out at the beginning, of the smearing out of the delta function of the law of energy conservation.

In conclusion, we shall show how the transition to the classical kinetic equation takes place in the quantum-mechanical equations. For this purpose, we introduce the Planck constant in explicit fashion and formally allow it to approach zero. We shall mark the delta function arising here by the index ħ. First of all, we have

$$G^* \left(p - \frac{1}{2}\lambda\right) G\left(p + \frac{1}{2}\lambda\right) \to \frac{\pi\delta_\hbar(\omega - \varepsilon_{\mathbf{p}})}{\hbar \left(\Sigma - \frac{i}{2}\lambda\right)},$$
$$\Sigma(p) \to \frac{\pi}{\hbar} \int dq\, D(q) \delta_\hbar(\nu - \varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{p}-\mathbf{q}}). \tag{41}$$

We substitute these relations in the equation for K (27), multiply it by $\hbar(\Sigma - \tfrac{1}{2}i\lambda)\exp(-i\lambda t)$ and, deriving the initial condition from it, we integrate over ω, ω_0 and λ. As a result, we find

$$\frac{\partial n(\mathbf{p}, t)}{\partial t} = -\frac{2\pi}{\hbar} n(\mathbf{p}, t) \int dq\, D(q) \delta_\hbar(\nu - \varepsilon_{\mathbf{p}} + \varepsilon_{\mathbf{p}-\mathbf{q}})$$
$$+ \frac{2\pi}{\hbar} \int dq\, D(q) \delta_\hbar(\nu + \varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}+\mathbf{q}}) n(\mathbf{p} + \mathbf{q}, t). \tag{42}$$

Recalling the definition (23) of D(q), we can write down the resultant equation in the form

$$\frac{\partial n(\mathbf{p}, t)}{\partial t} = \int d\mathbf{q}\, d\mathbf{p}_1 [n(\mathbf{p} - \mathbf{q}, t) n_{\mathbf{p}_1} - n_{\mathbf{p}_1 - \mathbf{q}} n(\mathbf{p}, t)]$$
$$\times \frac{2\pi}{\hbar} |V_{\mathbf{q}}|^2 \delta_\hbar(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}-\mathbf{q}} + \varepsilon'_{\mathbf{p}_1 - \mathbf{q}} - \varepsilon'_{\mathbf{p}_1}),$$

which corresponds accurately to the ordinary classical kinetic equation.

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