DENSITY OF THE NORMAL COMPONENT IN CONCENTRATED HELIUM ISOTOPE SOLUTIONS

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The density of the normal component of concentrated helium-isotope solutions is measured by the oscillating disk-pile technique at temperatures ranging from the λ point of the solution down to $1.45°$ K. For solutions containing up to 15.5% of He^3, and for temperatures not exceeding $1.6°$ K, a linear dependence of the normal component density on the weight concentration has been found. For solutions containing 21.7% of He^3, a departure from linearity is observed in the direction of smaller values of the density of the normal component. The value obtained for the effective mass of He^3 (2.35 atomic masses of He^3) agrees with the data of other authors.

THE study of the temperature dependence of the normal-component density of solutions of helium isotopes enables us to obtain a number of important facts about the properties of the He^3–He^4 system. Direct measurements of the normal component density[1-3] and the velocity of second sound[4,5] in weak helium isotope solutions have yielded experimental data on the energy spectrum of the impurity (He^3) dissolved in He II, which had been considered theoretically by Pomeranchuk.[6]

However, the presently available data on the normal-density component pertains to solutions with a He^3 molar concentration up to 11%,[7] and such data are completely lacking for higher concentrations, although they would be of significant interest. In this connection, the present work is devoted to the experimental study of the temperature dependence of the normal density component ρ_n for solutions of higher concentrations.

The measurements were made by means of a vibrating column of disks, used earlier in experiments with He^4[8] and with weak He^3–He^4 solutions. Disks of 3.00 cm diameter were prepared from aluminum foil of 1.3×10^{-3} cm thickness and arranged in a pile. The distance between the disks (1.5×10^{-2} cm) was so chosen that complete entrainment of the normal component was assured under the given experimental conditions. For protection against mechanical damage, the pile, consisting of 60 disks, was placed in an aluminum container of 3.10 cm diameter, 1.00 cm height, and wall thickness of 10^{-2} cm, rigidly connected to the disks. This system was suspended on an elastic wire from a straight glass rod and could execute torsional vibrations about a vertical axis, which made it possible to determine the moment of iner-

tia of the pile together with the liquid entrained by it. The apparatus differed in construction little from the apparatus used in other researches for similar purposes.[2,3]

In the experiments, the period θ and the damping γ of the oscillations of the described system were measured directly. The period in vacuum amounted to 14.137 ± 0.002 sec. The relative normal component density ρ_n/ρ_λ was computed from these same data (here ρ_λ is the density of the solution at the λ point) according to the formulas obtained in the solution of the corresponding hydrodynamic problem.[2] 1) An additional account of the correction for the entrainment of the liquid in the region between the outer disks and the inner surface of the container showed that it was smaller in magnitude than the error of measurement.

The normal-component density was measured in solutions with molar concentrations of He^3 equal to 11.05, 15.56, 19.53, and 27.00%, obtained by mixing pure He^3 and He^4. The depletion of the He^3 solutions due to the vapor phase did not exceed 0.05%. The total error in the determination of the concentration of the solutions amounted to no more than 0.1%. The measurements were made starting with temperatures just above the λ point of the solutions down to $1.45°$ K.

The results of the experiments are shown in Table I and Fig. 1. These give the temperature dependence of ρ_n/ρ_λ for the solutions of given concentration. Curve 1 represents the data for pure He^4, obtained as the result of a series of

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1)Numerical calculations were made on the M-20 electronic computer. We take this opportunity to thank M. A. Belaeva and I. D. Borisov for help in carrying out these calculations.

Table I. Temperature dependence of ρ_n/ρ_λ for solutions of various molar concentrations of He^3.

$c = 0$		$c = 11.05\%$		$c = 15.56\%$		$c = 19.53\%$		$c = 27.00\%$	
T, °K	ρ_n/ρ_λ, %	T, °K	ρ_n/ρ_λ, %	T, °K	ρ_n/ρ_λ, %	T, °K	ρ_n/ρ_λ, %	T, °K	ρ_n/ρ_λ, %
2,200	100.0	2,080	100.0	1.962	100.0	1.897	100.0	1.766	100.0
2,150	87,5	2,018	100.0	1.942	100.0	1.877	100.0	1.753	100.0
2,100	75,0	2,000	94.4	1.933	97.1	1.872	97.5	1.737	94.7
2,000	55,0	1.996	90.7	1.924	93.7	1.861	95.9	1.727	89.8
1.825	35,0	1.967	81.8	1.923	92.5	1.853	87.9	1.718	91.1
1.744	26,6	1.948	76.5	1.897	84,0	1.825	84.5	1.694	82.6
1.575	15.5	1.930	75.3	1.876	76.3	1.787	76.7	1.680	81.9
1.534	14,0	1.912	71,4	1.846	72.5	1.745	71.3	1.663	80.2
1.510	12.2	1.883	65.8	1.794	66.7	1.713	66.2	1.636	75,1
1,473	11.1	1.848	60,6	1.767	60.5	1.681	64.3	1.599	70,9
		1.833	59,5	1.747	59.3	1.651	58.8	1.554	65,3
		1.800	54.6	1.694	54.2	1.601	55.6	1.498	63.4
		1.756	50,3	1.642	51,5	1.555	53.6	1.475	60,7
		1.683	44.2	1.575	46.9	1.487	48.9	1.436	57.9
		1.595	37.9	1.560	44.9	1.446	48.2	1.433	58.0
		1.547	35,2	1.529	43,2				
		1.535	34.7	1.500	41,0				
		1.515	32.7	1.455	38,9				
				1.452	38,2				

control experiments carried out both at the beginning of the experiments with the solutions and in the middle of them. It is clearly evident that these data agree well with the results obtained in other researches,[2, 3, 7-10] which are shown on the same curve. Sharp breaks were observed on all the curves, corresponding to the λ temperatures of the solutions, in which the results obtained for the transition temperature also agree excellently with the data of other researches.[11] The relative error in the determination of ρ_n/ρ_λ for the solutions did not exceed 5%.

Isotherms were constructed from the given temperature dependence of ρ_n/ρ_λ for solutions with different amounts of He^3; these are shown in Fig. 2. Here the dependence of ρ_n/ρ_λ on the weight concentration of the solutions is shown. For the isotherms 1–4, a linear dependence of ρ_n/ρ_λ on x is clearly seen (x is the weight concentration of He^3) for x < 15.5 wt.%. This fact demonstrates the possibility of describing the impurity correc-

tion to the normal-component density ρ_n^{imp} by the formula

$$\rho_n^{imp} = \rho_p(\mu / m_3)x, \qquad (1)$$

where ρ_p is the density of the solution, μ the effective mass of the impurity excitations (He^3) in the solution, m_3 the mass of the He^3 atom. Thus, the linear dependence of the normal component density on the concentration of $He^3 - He^4$ solution holds not only for weak solutions, but also for concentrated ones.

The effective mass of the impurity excitations (He^3) computed by Eq. (1) amounted to $2.3_5 m_3$. This value of the effective mass is in excellent agreement with similar data obtained in a number of other researches for solutions of lower concentrations.[7, 13-15] However, as is seen from Table II, the scatter of the values of μ obtained by the different authors exceeds the measurement error.
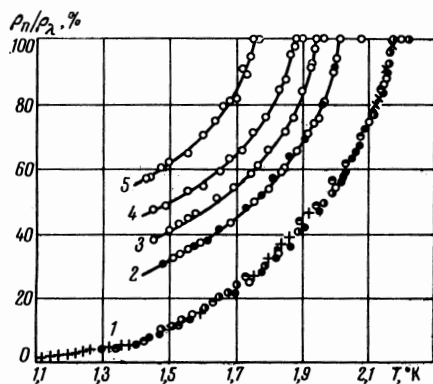

FIG. 1. Temperature dependence of ρ_n/ρ_λ. Curve 1–pure He⁴; ◑–from [8]; ◐–from [3]; ●–from [7]; +–from [9]; ×–from [10]; O–present work; curve 2–11.05 mol. % He³; curve 3–15.56 mol. % He³; curve 4–19.53 mol. % He³; curve 5–27.00 mol. % He³.
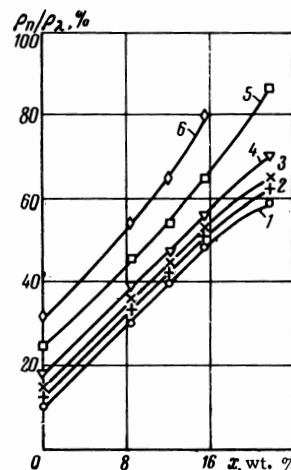
FIG. 2. Isotherms of ρ_n/ρ_λ as a function of the weight concentrations: 1–T = 1.45°K; 2–1.50°K; 3–T = 1.55°K; 4–T = 1.60°K; 5–T = 1.70°K; 6–T = 1.80°K.

Table II. Values of the effective mass of the impurity (He^3) in
He II, obtained in various researches.

μ/m_3	T, °K	c, mol.%	Ref.	μ/m_3	T, °K	c, mol.%	Ref
2,7	0,9—1.8	4.4	[1]	2.7—2.9	0,2—0,5	0,35	[12]
3.0	1.4—1.8	0—8,3	[3]	2.5	0,1—1.0	0,037—0,15	[13]
2.8	1,26—1.8	0—0,8	[4]	2.4	0,01—0,1	5	[14]
2.0—3.6	0,2—1,8	0.32—4,3	[5]	2.5	0,35—1.3	2—5	[15]
2,5	1,2—1.8	0—11	[7]	2.35	1.45—1.8	0—15	Present work

We now turn our attention to the fact that there is a deviation of the values of the normal component density from the linear dependence for solutions with content of He^3 of more than 15 wt.% for temperatures 1.60–1.45°K, in the direction of smaller values; this deviation clearly exceeds the error of measurement.

The deviation in the direction of large values of ρ_n/ρ_λ for sufficiently high temperatures was also observed in other researches[3,7] and is connected with the nearness to the λ point of the solutions. These deviations are seen especially clearly in Fig. 3, in which the dependence of ρ_n on x is shown for the temperatures 1.8–1.45°K. Curve 7 shown in this drawing corresponds to the values of the total density of the isotope solutions of helium for the temperatures 1.80–1.45°K.[16]

Because of the lack of a theory of concentrated solutions of helium isotopes, it is not possible to give a unique explanation of the experimentally observed features of the dependence of $\rho_n(x)$ for low temperatures; one can only mark some features of the research paths of such an explanation, within the framework of the Landau theory.
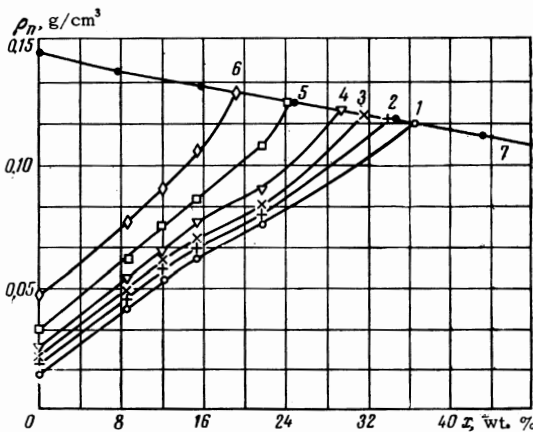

FIG. 3. Dependence of the normal-component density and the total density on the weight concentration of the solution. Curves 1-6–the same as in Fig. 2; curve 7–dependence of the total density of the solution on the concentration. [16]

It is well known that the calculation of the normal component density of helium isotope solutions has been carried out by Pomeranchuk[6] under the following assumptions:

1. The total density of the normal component of the solution is determined by the expression

$$\rho_n = \rho_{n4} + \rho_n^{imp}, \qquad (2)$$

ρ_{n4} is the normal-component density contributed by the He^4, and ρ_n^{imp} that contributed by the impurity.

2. The dependence of energy ϵ of the impurity excitations on the pressure p has the form

$$\varepsilon = \varepsilon_0 + p^2/2\mu. \qquad (3)$$

3. The impurity excitations are described by Boltzmann statistics.

It is easy to see that the direct integration of the Landau formula[17]

$$\rho_n = \frac{4\pi}{3} \int_0^\infty p^4 \frac{\partial n(\varepsilon)}{\partial \varepsilon} \, dp$$

for particles with the dispersion law (3) leads, for any distribution law $n(\epsilon)$, to the expression

$$\rho_n^{imp} = N\mu, \qquad N = \int_0^\infty n \, d^3p$$

(where N is the number of excitations), which is equivalent to Eq. (1), and consequently the third assumption is not necessary. In particular, this means that the observed change in the dependence of ρ_n on x is not associated with the approach to temperature degeneracy of the Fermi gas of the impurities as the concentration of the solution increases.

The simplest explanation of the deviation from a linear dependence of $\rho_n(x)$ for higher concentrations is connected with the assumption of the dependence of the effective mass of the impurity excitations on the concentration of He^3. However, for sufficiently high concentrations, it is naturally impossible to exclude the possibility of a more

radical change in the dispersion law (3) or gener-
ally a violation of the conditions of applicability of
the description of He II in terms of a gas of quasi-
particles.

It must also be noted that even within the frame-
work of the Landau theory, in the use of Eq. (2)
for the computation of the impurity density of the
normal component according to the experimental
data, the value of ρ_{n4} is shown to be somewhat in-
determinate. In the Pomeranchuk theory for weak
solutions, it is assumed that $\rho_{n4} = \rho_{n4}^0$ (ρ_{n4}^0 is the
normal component density of pure He^4). However,
for concentrated solutions, one can expect the
presence of He^3 to lead to an appreciable change
in the parameters of the phonon and roton spec-
tra, which in turn produces a change in the contri-
bution of He^4 to the density of the normal compo-
nent of the solution. Qualitatively, the presence of
such an effect can evidently be verified by meas-
uring the normal-component density of the same
solutions at much lower temperatures, where
$\rho_n^{imp} \gg \rho_{n4}$, and by comparing the resultant values
of the effective mass here with the corresponding
values found at higher temperatures.

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