# *VELOCITY OF FOURTH SOUND IN* $He^3$–$He^4$ *SOLUTIONS*

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The expression obtained earlier[4] for the velocity of fourth sound for weak solutions of $He^3$–$He^4$ has been extended to the case of high concentrations. The relative oscillations of pressure, temperature, and concentration in the fourth-sound wave are calculated. All the calculations are valid for the region of temperatures and concentrations where $He^3$ takes part only in the normal motion. An experimental investigation has been carried out of the temperature and concentration dependence of the velocity of fourth sound in helium isotope solutions with $He^3$ content amounting to 6.30, 11.05, 15.56 and 19.53 mole %. The experimental results show an excellent agreement with theory, from which it follows that $He^3$ takes part only in the normal motion in solutions of the concentrations studied. It is shown that in narrow channels, the density of the superfluid component of the solution is identical with the corresponding values for the macroscopic volume.

## 1. INTRODUCTION

IN superfluid $He^4$, as is well known, the propagation of two independent waves is possible under ordinary condition, the so-called first and second sound. Both the superfluid (with density $\rho_s$) and the normal fluid (with density $\rho_n$) components take part in these two sound processes. But in the first-sound wave if we neglect the coefficient of thermal expansion because of its smallness, the fluids oscillate in phase (the velocity of the normal component $v_n$ is equal to the velocity of the superfluid component $v_s$, $v_s = v_n$), whereas in the second sound they are strictly in phase opposition such that $\rho_n v_n + \rho_s v_s = 0$, that is, there is no transport of matter.

However, under certain conditions, namely, when the motion of the normal component is completely hindered, as has been shown theoretically[1,2] and verified experimentally,[3] a new type of oscillation is possible—"fourth sound."[1)] Here the fourth sound is propagated only in the superfluid component of the helium. The velocity of fourth sound $u_4$ is expressed in symmetric fashion by the velocity of first sound $u_1$ and second sound $u_2$:

$$u_4^2 = \frac{\rho_s}{\rho} u_1^2 + \frac{\rho_n}{\rho} u_2^2 \qquad (1)$$

($\rho = \rho_n + \rho_s$ is the total density of the liquid helium).

In superfluid solutions of $He^3$–$He^4$, according to Sanikidze and Chernikova,[4] the propagation of fourth sound is also possible, the velocity of which, for weak solutions, is equal to

$$u_4^2 = \frac{\rho_s}{\rho}\left(1 + 2\frac{c}{\rho}\frac{\partial \rho}{\partial c}\right) u_1^2 + \frac{\rho_n}{\rho} u_2^2, \qquad (2)$$

where c is the weight concentration of $He^3$ in the solution, and the quantities $\rho_s$, $\rho_s$, $\rho$, $u_1$, $u_2$ refer to the solutions.

The expression for $u_4$ can also be obtained for solutions of arbitrary concentration. By using the system of hydrodynamic equations which describe the propagation of fourth sound in a solution,[4] and neglecting the coefficient of thermal expansion, we have

$$u_4^2 = \frac{\rho_s}{\rho} u_1^2 \left[1 + \frac{c}{\rho}\frac{\partial \rho}{\partial c}\right]^2 \Big/ \left[1 + \frac{\rho_s}{\rho_n}\left(\frac{c}{\rho}\frac{\partial \rho}{\partial c}\right)^2\right] + \frac{\rho_n}{\rho} u_2^2 \left[1 + \frac{\rho_s}{\rho_n}\left(\frac{c}{\rho}\frac{\partial \rho}{\partial c}\right)^2\right], \qquad (3)$$

where $u_1$ and $u_2$ refer to solutions of arbitrary concentration and, in accord with [5],

$$u_1^2 = \left(\frac{\partial P}{\partial \rho}\right)_{T,c}\left[1 + \frac{\rho_s}{\rho_n}\left(\frac{c}{\rho}\frac{\partial \rho}{\partial c}\right)^2\right], \qquad (4)$$

($\mu_1$ and $\mu_2$ are chemical potentials of $He^3$ and $He^4$ in the solution; $\sigma$ is the entropy per unit mass of

[1)] For completeness, we remark that "third sound" corresponds to the oscillations of the thickness of a superfluid film of helium under conditions when the motion of the normal component is hindered.

$$u_2^2 = \frac{\rho_s}{\rho_n}\left[\left(\sigma - c\frac{\partial\sigma}{\partial c}\right)^2\left(\frac{\partial T}{\partial\sigma}\right)_{P,c} + c^2\frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)\right]\left[1+\frac{\rho_s}{\rho_n}\left(\frac{c}{\rho}\frac{\partial\rho}{\partial c}\right)\right]^{-1}, \quad (5)$$

$$Z = \rho(\mu_3 - \mu_4) \quad (6)$$

the solution). It is easy to see that for small c, Eq. (3) transforms to Eq. (2).

For elucidation of the features of fourth sound, we compute the amplitude of oscillation of the pressure, temperature, and concentration, connected with its propagation, both in pure $He^4$ and in $He^3$–$He^4$ solutions. By considering a plane sound wave and writing down the variable parts of the pressure P′, temperature T′, and concentration c′ in the sound wave in the form

$$P' = av_s, \quad T' = bv_s, \quad c' = dv_s, \quad (7)$$

it is possible, with the help of a method used by E. M. Lifshitz,[6] to obtain for the case of $He^4$,

$$a = \frac{\rho_s u_1^2}{u_4}, \quad b = -\frac{\rho_n}{\rho\sigma}\frac{u_2^2}{u_4}, \quad (8)$$

and for $He^3$-$He^4$ solution with concentration C

$$a_c = \frac{\rho_s}{u_4}\left(\frac{\partial P}{\partial\rho}\right)_{T,c}\left[1+\frac{c}{\rho}\left(\frac{\partial\rho}{\partial c}\right)_{T,P}\right],$$

$$b_c = -\frac{\rho_s}{\rho u_4}\left(\frac{\partial T}{\partial\sigma}\right)_{P,c}\left[\sigma - c\left(\frac{\partial\sigma}{\partial c}\right)_{P,T}\right],$$

$$d_c = -\frac{\rho_s}{\rho}\frac{c}{u_4}. \quad (9)$$

In the relations (8) and (9), terms are omitted which contain the coefficient of thermal expansion, in view of its smallness. In such an approximation, the waves of fourth sound are oscillations of both pressure and temperature (and also of the concentration in solutions), as is seen from Eqs. (8) and (9), while in this same approximation, first sound is only a pressure wave in $He^4$, and second sound, only a temperature wave.[2)]

However, the relative amplitude of oscillation of the temperature in the fourth sound wave is much less than the oscillation of the pressure. Let $P_0$ and $T_0$ be the equilibrium values of the pressure and temperature, and

$$T'/T_0 = kP'/P_0. \quad (10)$$

Then, for the case of $He^4$,

$$k = -\frac{\sigma}{C}\frac{P_0}{\rho u_1^2} \quad (11)$$

(C is the heat capacity of the helium), which, for $T = 1.5°K$ amounts to $k \approx 1\times10^{-4}$.

For a $He^3$–$He^4$ solution with weight concentration c, we have

$$k_c = -P_0\left[\sigma - c\left(\frac{\partial\sigma}{\partial c}\right)_{P,T}\right] \times C\rho\left(\frac{\partial P}{\partial\rho}\right)_{T,c}\left[1+\frac{c}{\rho}\left(\frac{\partial\rho}{\partial c}\right)_{P,T}\right]; \quad (12)$$

for $T_0 = 1.5°K$ and $c = 0.16$, the value of $k_c \approx -2\times10^{-4}$.

In solutions, the relative amplitude of the concentration oscillations is also small. Actually, if we write, similarly to (10),

$$c'/c_0 = l_c P'/P_0, \quad (13)$$

then

$$l_c = -P_0\Big/\rho\left(\frac{\partial P}{\partial\rho}\right)_{T,c}\left[1+\frac{c}{\rho}\left(\frac{\partial\rho}{\partial c}\right)_{P,T}\right]. \quad (14)$$

For a solution with concentration $c = 0.16$ and $T = 1.5°K$, the value of $l_c \approx -6\times10^{-4}$.

Therefore, the observation of fourth sound in $He^4$ and in $He^3$–$He^4$ solutions is most easily carried out by detecting the pressure oscillations. However, it is possible in principle to excite both pressure and temperature oscillations.

In a previous research[7] it was shown that fourth sound can be propagated in a solution with molar concentration 6.3% $He^3$; the temperature dependence of its velocity agrees well with theoretical predictions. In this paper, a method is described and experimental results are given on the determination of the velocity of fourth sound in solutions with molar concentrations up to 20%.

## 2. EXPERIMENTAL METHOD

Fourth sound is achieved, as has been pointed out earlier, under the condition of complete retardation of the normal component ($v_n = 0$). This can be accomplished if we impregnate with the helium a "filter" with characteristic channel dimension d much smaller than the penetration depth $\delta$ of the viscous wave, i.e., $d \ll \delta = (2\eta_n/\omega\rho)^{1/2}$, where $\eta_n$ is the viscosity of the normal component and $\omega$ the angular frequency of the oscillations. It is then seen that the admissible dimension of the channels of the filter depends on the temperature and the concentration of the solution, and on the frequency of the oscillations. For solutions with $He^3$ content up to 20 mol. %, the estimate of the

[2)] In $He^3$-$He^4$ solutions, oscillations of temperature and concentration also take place, in addition to those of pressure (this will be published).

least value of $\delta$ in the temperature range 1.5°K–$T_\lambda$ for a frequency of oscillation of $\sim 10^5$ Hz gives $\sim 0.5\mu$. Starting from this point, a filter was chosen with dimensions less than $0.5\mu$.

For such a filter, we used finely divided rouge ($Fe_2O_3$), which is compressed by a pressure of 40 kg/cm$^2$. The mean dimension of the rouge particles, as a study by means of an electron microscope has shown, was less than $0.5\mu$, and their shape was arbitrary. A copper cylindrical resonator, with inner diameter 20 mm and length 10 mm, was filled with the filter (Fig. 1). Identical capacitors were placed at each end, one of which served as the sound radiator, the other as the detector. Each such capacitor consisted of a massive electrode as one plate and an aluminum disk of thickness $4\mu$ as the other. This latter plate was mounted on a lavsan polyester film of thickness $10\mu$, which served as the dielectric layer. The entire capacitor was attached to the resonator by screws.

Rectangular pulses of $2\mu$ sec duration were fed to the transmitter from the blocking oscillator. The repetition frequency was 200 Hz, with a rise time of 0.1 $\mu$sec and amplitude of 400 V. In addition, the radiator was polarized by a 200 V dc.

The received pulses, after amplification, were fed to an I2-9A short-time-interval detector used as an indicator (Fig. 2). The indicator sweep was synchronized with the trigger of the blocking generator by means of paired-pulse generators contained in the I2-9A meter. The received pulses had the form shown in Fig. 3, with a pulse width $10\mu$sec at the level $1/\sqrt{2}$ of the maximum amplitude. The principal part of the spectrum of such a pulse contains low frequencies up to $10^5$ Hz, which does not disturb the conditions for production of fourth sound.

In the determination of the velocity of fourth sound, it is essential to take into account the effect of multiple scattering of the sound wave by the particles of the filter. This effect leads to the result that the measured velocity of fourth sound $u_4'$ is less than the true velocity $u_4$, because of the increase in the actual path traversed by the wave. Ordinarily, this is taken into account by the introduction of a correction $n = u_4/u_4'$, which has the meaning of an index of refraction.

In the present research we determined n by calibration measurements of the velocity of fourth sound in pure $He^4$. The correction was chosen in such a way that for some temperature, for example, 1.6° K, the sound velocity was identical with the value given by Eq. (1), which, according to the data of Shapiro and Rudnick[3] is in excellent

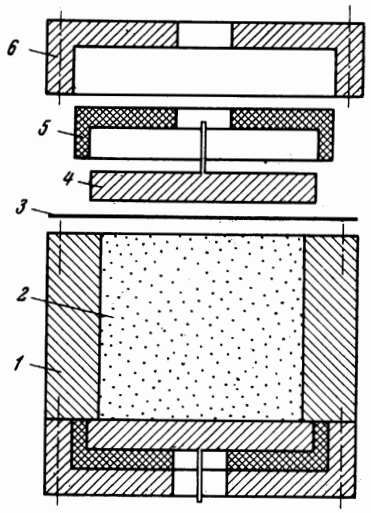


FIG. 1. Resonator (upper part is shown schematically): 1–housing; 2–rouge filter; 3–film; 4–massive electrode; 5–insulation packing; 6–cover.

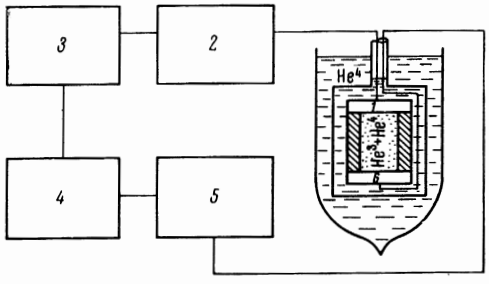


FIG. 2. Measuring apparatus: 1–radiator; 2–blocking generator; 3–paired pulse generatore; 4–display unit; 5–amplifier; 6–receiver.

agreement with experiments for $He^4$. If the correction is chosen correctly, then such an agreement should be observed throughout the entire temperature range. For the filter used in this research, n was found to be equal to 1.21.[3)] With account of this correction, the temperature dependence of the velocity, obtained experimentally, is in excellent agreement with that computed from Eq. (1), and also with the data of Rudnick and Shapiro.[3]

The application of the correction for multiple scattering of the sound wave makes it difficult to obtain accurate absolute values of the fourth sound velocity. The relative accuracy is determined by the error in the measurement of the time intervals. The I2-9A apparatus used for this purpose guarantees the measurement of the time intervals

[3)]The value n = 1.21 differs somewhat from that given previously[7] in connection with the use in that research of values of $\rho_n$ for pure $He^4$ taken from Dash and Taylor.[8]

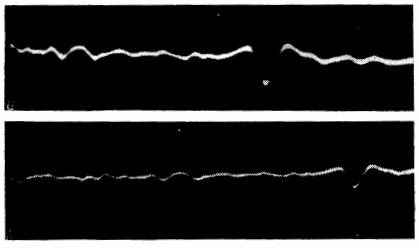

FIG. 3. Pulses of fourth sound in a solution with 11.05% $He^3$: a–temperature 1.60°K; b–temperature 1.85°K. Only the first pulses are shown. The left edge coincides with the start of the sweep.

with an error not exceeding ± 0.01% of the measured value. However, the actual error was larger in our case, because the quality of the front of the received pulse deteriorated as the λ point was approached, and also with increase in the concentration. Therefore, the error in the time measurements averaged about ±0.8% of the measured value.

A number of features of the experimental method were due to the research in solutions of $He^3$–$He^4$. The solution of the investigated concentration was condensed by means of a unit cooled by liquid nitrogen in a copper container, isolated from the outer container, in which the resonator was placed. The necessary temperature was obtained by the flow of gas over the liquid $He^4$ in the outer vessel; here the pressure was maintained and measured with an accuracy of ± 0.1 mm Hg. After a certain amount of solution was condensed, the condensing system was turned off, so that further measurements were carried out with a constant amount of solution. Inasmuch as the volume of the liquid filling the channel is approximately equal to the remaining volume, one could determine the $He^3$ concentration in the channels by the vapor pressure of the solution in the container. Measurements showed that the vapor pressure of the solution corresponds to the equilibrium value for a given concentration. This means that the solution outside the filter was not enriched by $He^3$.

In the present research, solutions were studied with molar concentration of $He^3$ amounting to 6.30, 11.05, 15.56 and 19.53%. The solutions were obtained by mixing the pure isotopes. The error in the determination of the concentration did not exceed ±0.05% $He^3$ for all concentrations.

## 3. RESULTS AND DISCUSSION

The obtained values of the fourth-sound velocity for $He^3$–$He^4$ solutions of different concentrations as a function of the temperature are given in the

Smoothed values of the fourth sound velocity (m/sec) for different temperatures and solution concentration

| T, °K | mol. % $He^3$ | | | |
|---|---|---|---|---|
| | 6.30 | 11.05 | 15.56 | 19.53 |
| 1.60 | 190.2 | 168.5 | 145.5 | 128.5 |
| 1.65 | 184.3 | 163.0 | 139.6 | 120.8 |
| 1.70 | 177.5 | 156.4 | 132.1 | 111.7 |
| 1.75 | 170.4 | 149.1 | 122.7 | 99.5 |
| 1.80 | 162.8 | 139.6 | 110.8 | 83.0 |
| 1.85 | 153.1 | 128.0 | 95.0 | — |
| 1.90 | 141.7 | 112.7 | — | — |
| 1.95 | 127.7 | 90.1 | — | — |
| 2.00 | 110.4 | — | — | — |
| 2.05 | 80.5 | — | — | — |

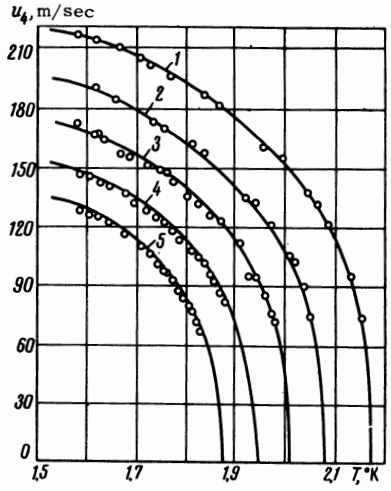


FIG. 4. Temperature dependence of the velocity of fourth sound: curve 1–pure $He^4$; 2–6.30% $He^3$; 3–11.05% $He^3$; 4–15.56% $He^3$ (molar concentrations are shown). The solid lines correspond to values computed from Eq. (3).

table. Graphically, they are shown in Fig. 4. The theoretical curves, computed by Eq. (3), are plotted for a comparison with experimental data. The values of $\rho$, $\rho_n$, $u_1$, and $u_2$ for the solutions studied were taken from [8-12].

The maximum departure of $u_4$ from the mean experimental value for solutions of isotopes and for $He^4$ does not exceed 1.0%, and the mean-square deviation does not exceed 0.7%. For a solution with a concentration of 11.05% $He^3$, these deviations are respectively equal to 1.5% and 1.2%, which is related to the poorer conditions of measurement.

The theoretical curves for solutions with concentrations of $He^3$ up to 11.05% agree with the experimental data in the limits of accuracy of the experiment. For the remaining concentrations, the deviation amounts to 2%, which is obviously connected with the insufficient accuracy of the experimental values used in the calculation. For the solutions studied, the principal contribution to the

velocity of fourth sound is the first term of Eq. (3), and the most important quantities in it are proportional to $c\rho^{-1}\partial\rho/\partial c$. The contribution from them increases with increase in concentration, and for solutions with a $He^3$ content equal to 20%, they change the value of the entire first term by almost 25%. The fact that the velocity of fourth sound in solutions depends on $c\rho^{-1}\partial\rho/\partial c$ is due to the oscillations of the concentration in the sound wave, together with the oscillations of pressure and temperature. The contribution of the second term of Eq. (3), which contains $u_2$, does not exceed 3%.

Figure 5 shows the isotherms of the second sound velocity as a function of the concentration of the light isotope. At temperatures close to the λ point of the solutions studied, the velocity falls off rapidly with increase in the concentration. For very low temperatures, the dependence on the concentration is approximately linear.

The theoretical consideration of fourth sound in $He^3-He^4$ solutions, completed by Sanikidze and Chernikova,[4] were made under the assumption that $He^3$ makes a contribution only to the normal component of the solution. Usually, this is assumed for weak solutions of the helium isotopes. The excellent agreement of the experimental results with the theory in the present research shows that even for concentrated solutions (up to 20% $He^3$), $He^3$ takes part only in the normal motion. Here the density of the normal component of the solution does not differ from the corresponding values for the macroscopic liquid solution of the same concentration.

Preliminary measurements of the absorption coefficient of fourth sound in solutions of helium isotopes show that it depends appreciably on both the temperature and the concentration of the solution.

The error increases with increase in the temperature, and becomes very large near the λ point, which makes it difficult to obtain reliable values of the velocity. The absorption also increases with increase in the concentration of $He^3$ in the solution. Whereas 8–10 distinct echo pips are produced on the display by the 6.3% solution, only 3–4 pips are seen for the 19.53% solution.

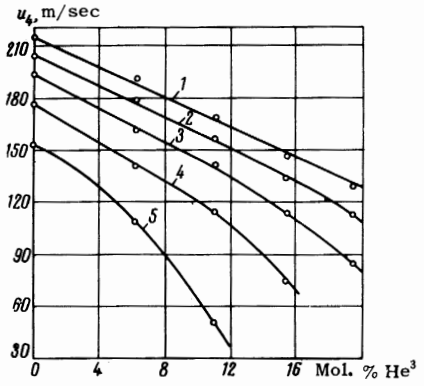


FIG. 5. Isotherms of fourth sound velocity: curve 1–T = 1.60°K; 2–T = 1.70°K; 3–T = 1.80°K; 4–T = 1.90°K; 5–T = 2.00°K.

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