INTEGRAL FORM OF THE EINSTEIN EQUATIONS AND A COVARIANT FORMULATION OF MACH'S PRINCIPLE

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Submitted to JETP editor April 12, 1966

J. Exptl. Theoret. Phys. (U.S.S.R.) 51, 1143-1150 (October, 1966)

A method is proposed for writing the Einstein equations in integral form by means of a covariant Green's function which is a two-point tensor. This approach includes a natural way of eliminating the difficulties associated with the nonlinearity of the Einstein equations. With a definite choice of the equation for the Green's function the integral form of the equations is a covariant way of writing Mach's principle, which is now equivalent to the requirement that the Einstein equations be valid not in differential form but in integral form. A number of examples are considered. In particular it is shown that the Friedmann model of a homogeneous and isotropic universe is incompatible with Mach's principle, because of the behavior of the metric near a singular point which is characteristic of this model.

1. INTRODUCTION

Mach's principle in its simplest form asserts that a statement about an acceleration has meaning only when we specify relative to what the acceleration occurs. A conclusion from this is that all inertial effects arise owing to accelerations relative to the system of remote galaxies. There have been repeated attempts to formulate in the framework of general relativity theory a principle according to which the local inertial properties of matter are determined by the distribution of matter in the Universe. The relevant bibliography, and also a survey of the difficulties that arise, can be found in a recent review volume.[1]

In the present paper, starting from the qualitative formulation of Mach's principle given by Einstein[2]: "The whole inertia, that is the entire field of the g_{ik}, is determined by the distribution of matter in the Universe," we propose writing the Einstein equations in integral form in such a way that by the use of a covariant bitensor retarded Green's function the value of the metric tensor g_{ik} at a given point is determined by the distribution of the matter energy-momentum tensor T_{ik} in all space. The Mach principle then simply means the requirement that the Einstein equations be valid not in differential form but in the integral form. This formulation is essentially a covariant method for imposing definite boundary conditions on the solutions of the Einstein equations. The large degree of arbitrariness in the choice of the equation for the Green's function is to a large extent removed by the condition that in the limiting case of small deviations h_{ik} from the Minkowski metric $g_{ik}^{(0)}$ the exact integral expression is to go over into the formula of Thirring and Einstein for the Mach principle[2,3]:

$$-\frac{1}{2}\left(h_{ik}-\frac{1}{2}g_{ik}^{(0)}h_m{}^m\right)=\frac{8\pi k}{c^4}\int\frac{[\delta T_{ik}]_{3\,\mathrm{ret}}}{r}d^3x, \quad (1)$$

which is written in the coordinate system defined by the relations

$$g^{(0)kn}\frac{\partial}{\partial x^n}\left(h_{ik}-\frac{1}{2}g_{ik}^{(0)}h_m{}^m\right)=0. \quad (2)$$

In the Appendix the gravitational field of a hollow cylinder rotating in empty space is found and it is shown that if the linear mass density μ of the cylinder satisfies the relation

$$2kc^{-2}\mu=1, \quad (3)$$

then in the internal region, free from Coriolis forces, the coordinate system always turns out to be that in which the cylinder is at rest. In Sec. 3 the condition (3) on the mass of the cylinder is obtained from the proposed covariant formulation of Mach's principle. In that section we also consider a number of cosmological models. The integral formulation of the Einstein equations at once excludes the "non-Mach" cases of empty and asymptotically flat space. Also in contradiction with it is the Friedmann model of a homogeneous and isotropic universe (open, closed, and flat), owing to the behavior of the metric near a singular point which is characteristic of this model. A model consistent with the Mach principle is the Einstein model of a static closed world, but this model is of only mathematical interest, since it holds if a nonphysical cosmological term is included in the Einstein equations.

2. INTEGRAL FORM OF THE EINSTEIN EQUATIONS

The following formula is a mathematical way of writing the qualitative formulation of the Mach principle which was given in the Introduction:

$$g_{ik}(x)=\frac{8\pi k}{c^4}\int G_{ik}^{\alpha\beta}(x,y)\,T_{\alpha\beta}(y)\sqrt{-g(y)}\,d^4y. \quad (4)$$

Here $G_{ik}^{\alpha\beta}(x, y)$ is a retarded tensor Green's function in the space defined by the metric g_{ik}.[4,5] It is a two-point tensor—i.e., the indices i, k refer to the point x and the indices α, β to the point y. In what follows it is understood that Latin letters refer to the point x and Greek letters to the point y. All indices run through the four values (0, 1, 2, 3) if there are no special stipulations.

We adopt the following general form of the covariant equation which the Green's function is required to obey:

$$A_{ik}^{mnpq}(x)\,G_{mn;\,pq}^{\alpha\beta}(x,y)+B_{ik}^{mn}(x)\,G_{mn}^{\alpha\beta}(x,y)$$
$$=\delta_i^{\alpha}\,\delta_k^{\beta}\,\delta(x-y)/\sqrt{-g}. \quad (5)$$

Here the semicolon denotes the operation of covariant differentiation; $\delta(x-y)$ is the four-dimensional Dirac delta function; δ_i^{α} is the Kronecker symbol, which is well defined, since in the right member of (5) $x = y$; and $A_{(..)}^{(....)}$ and $B_{(..)}^{(..)}$ are tensors which so far are arbitrary.

In order for the relation (4) to be an integral form of the Einstein equations[6] it is necessary that

$$B_{i\ k}^{mn}\,g_{mn}=R_{ik}-{}^1/_2 g_{ik}R. \quad (6)$$

This still does not determine the tensor B_{ik}^{mn} uniquely. We choose it in the following form

$$B_{i\ k}^{mn}=R_{i\ k}^{mn}-{}^1/_6 R\,(g_{i}g^{mn}-\delta_i^m\,\delta_k^n). \quad (7)$$

It follows from the result of Item D of Sec. 3 that the Riemann tensor $R_{i.k.}^{m\ n}$ must appear in the equation for the Green's function. We shall show that the differential term in the equation (5) for the Green's function is almost uniquely determined by the condition that Eq. (1) must be a consequence of (4) and (5). To do this we carry out a small variation of the energy-momentum tensor and the metric tensor, consistent with the Einstein equations and the condition (4):

$$T_{ik}\to T_{ik}+\delta T_{ik},\qquad g_{ik}\to g_{ik}+h_{ik}.$$

This sort of variation has been applied in[7,8] for integral relations containing scalar and vector Green's functions.

We get from Eqs. (4)—(7)

$$h_{ik}(x)=\int G_{ik}^{\alpha\beta}(x,y)\,\{A_{\alpha\beta}^{\gamma\delta\mu\nu}(y)\,h_{\gamma\delta;\,\mu\nu}(y)$$
$$+B_{\alpha\beta}^{\gamma\delta}(y)\,h_{\gamma\delta}(y)\}\sqrt{-g(y)}\,d^4y. \quad (8)$$

Here the Green's function, the tensors $A_{(..)}^{(....)}$ and $B_{(..)}^{(..)}$, and the covariant derivatives are defined in the space with the unperturbed metric g_{ik}.

The relation (8) follows from (4) provided that the unperturbed metric is consistent with (4). It is obvious that empty space with the Minkowski metric $g_{ik}^{(0)}$, which is the zeroth approximation in (1), is not compatible with (4). We find, however, that the tensor A_{ik}^{mnpq} in (5) is almost uniquely determined if we regard (8) as a relation independent of (4) and require that when we substitute in it $g_{ik} = g_{ik}^{(0)}$ in the coordinate system defined by the relations (2) it must go over into (1). This procedure is physically entirely justified, since we can assume that in (1) the unperturbed metric is the correct cosmological solution, compatible with (4), and the variations of the metric and of the energy-momentum tensor, for which (1) holds, are taken in a region of space small enough so that it can be regarded as approximately flat.

It is well known[6] that for the case considered the variation of the Einstein equations gives in the coordinate system (2)

$$-{}^1/_2 g^{(0)pq}(h_{ik}-{}^1/_2\,g_{ik}^{(0)}\,h_m^{\ m})_{;\,pq}=8\pi kc^{-4}\delta T_{ik}. \quad (9)$$

The expression (1), which is the integral form of this differential equation, can be rewritten in the following form:

$$-{}^1/_2(h_{ik}-{}^1/_2\,g_{ik}^{(0)}\,h_m^{\ m})=\frac{8\pi k}{c^4}\int S(x,y)\,\delta T_{ik}(y)\,d^4y, \quad (10)$$

where S(x, y) is the retarded scalar Green's function in flat space with the Minkowski metric and obeys the equation

$$g^{(0)pq}S_{;\,pq}(x,y)=\delta(x-y). \quad (11)$$

In order that Eq. (8), with $g_{ik} = g_{ik}^{(0)}$ in the coordinate system (2), be equivalent to Eq. (10), it is necessary that

$$-{}^1/_2\,A_{\alpha\beta}^{\gamma\delta\mu\nu}\,h_{\gamma\delta;\,\mu\nu}=\lambda^{-1}\cdot 8\pi kc^{-4}\delta T_{\alpha\beta}, \quad (12)$$

$$G_{ik}^{\alpha\beta}-{}^1/_2\,g_{ik}^{(0)}\,G_m^{m\alpha\beta}=\lambda\delta_i^{\alpha}\,\delta_k^{\beta}\,S(x,y). \quad (13)$$

Here λ is a constant whose value is determined in paragraph D of Sec. 3 from a consideration of the problem of the cylinder (λ = 2).

Comparison of (12) with (9), and also of (13) with (5), (11) enables us to find the tensor A_{ik}^{mnpq}, and when we also use (7) we can write the equation for the Green's function

$$\lambda^{-1}(\Box_x G_{ik}^{\alpha\beta} - {}^1/_2 g_{ik}\Box_x G_m^{m\alpha\beta})$$
$$+ [R_{i\cdot k\cdot}^{\cdot m\cdot n} - {}^1/_6 R(g_{ik}g^{mn} - \delta_i^m\delta_k^n)]G_{mn}^{\alpha\beta}$$
$$= \delta_i^\alpha\delta_k^\beta\delta(x-y)/\sqrt{-g}, \qquad (14)$$

(here $\Box_x$ is the covariant d'Alembertian operator acting at the point x). From this it is easy to get the boundary condition imposed on the Einstein equations by the integral form (4):

$$\int [g^{\mu\nu}(y)G_{ik\alpha;\mu}^{\alpha}(x,y)]_{;\nu}\sqrt{-g(y)}\,d^4y = 0, \qquad (15)$$

or, when we take into account the retarded character of the Green's function,

$$\lim \int G_{ik\alpha;\mu}^{\alpha}(x,y)g^{\mu 0}(y)\sqrt{-g(y)}\,d^3y = 0 \qquad (16)$$

for $y^0 \to -\infty$ (or at the initial time in a cosmological model).

3. EXAMPLES

To test the compatibility of some known solutions of the Einstein equations with the condition (4) there is no need to solve the equation (14) for the complete Green's function. We shall show how use of the symmetry properties of a concrete model decidedly simplifies the problem. If we introduce a notation for the right member of (4)

$$\frac{8\pi k}{c^4}\int G_{ik}^{\alpha\beta}(x,y)T_{\alpha\beta}(y)\sqrt{-g(y)}\,d^4y \equiv b_{ik}(x) \qquad (17)$$

and use (14), assuming that (4) is not satisfied, then we get a linear differential equation with a source, satisfied by the tensor b_{ik}:

$$\lambda^{-1}\Box(b_{ik} - {}^1/_2 g_{ik}b_m^m)$$
$$+ [R_{i\cdot k\cdot}^{\cdot m\cdot n} - {}^1/_6 R(g_{ik}g^{mn} - \delta_i^m\delta_k^n)]b_{mn}$$
$$\times = 8\pi k c^{-4}T_{ik}. \qquad (18)$$

The coefficients of this equation depend on the metric $g_{ik}(x)$. If the metric g_{ik} satisfies the Einstein equations, then $b_{ik} = g_{ik}$ is the solution of (18). The condition (4) means that this solution is the retarded inhomogeneous solution of the system of equations (18), and imposes definite boundary conditions on the metric. According to (17) the symmetry properties of a given metric make many properties of the tensor b_{ik} obvious from the start. This allows us to make considerable simplifications in the system of equations (18) and to write its inhomogeneous solution, using a simpler Green's function.

For example, in the case of a constant gravitational field it is obvious from (17) that the tensor b_{ik} does not depend on the time. The system (18) then becomes simpler, since all derivatives with respect to the time drop out, and the inhomogeneous solution $b_{ik} = g_{ik}$ can be written by means of the Green's function $D_{ik}^{\alpha\beta}(\mathbf{x}, \mathbf{y})$ in three-dimensional space. The corresponding boundary condition is of the form

$$\oint \frac{\partial D_{ik\alpha}^{\alpha}(\mathbf{x},\mathbf{y})}{\partial y^\mu}g^{\mu\nu}(\mathbf{y})\sqrt{-g(\mathbf{y})}\,df_{0\nu} = 0. \qquad (19)$$

Here the integral is taken over a surface inclosing the entire three-dimensional space.

A. The simplest case to consider is that in which the matter is located in the finite region of space. Then at large distances the metric is spherically symmetrical and goes over into the flat metric. The boundary condition (19) can be rewritten in the following form:

$$\lim_{r_y\to\infty}\left[\frac{\partial D_{ik\alpha}^{\alpha}(\mathbf{x},\mathbf{y})}{\partial r}g^{11}(\mathbf{y})\sqrt{-g(\mathbf{y})}\right] = 0, \qquad (20)$$

where $r_y \equiv y^1$ is the radial coordinate of the point **y**.

To verify (20) it is necessary to know the asymptotic behavior of the Green's function for large magnitudes of **y**. It is clear without calculation, however, that in the asymptotically flat region the components of the Green's function obey the ordinary Laplace equation, and therefore are proportional to $1/r_y$. Since in this region $g^{11} = 1$, $(-g)^{1/2} \sim r_y^2$, we see that, as was to be expected, the left member of (20) is equal to a constant, and is in general different from zero.

B. Let us consider the isotropic model of a static closed world[2] with the interval

$$-ds^2 = -c^2d\tau^2 + r^2(\sin^2\theta d\varphi^2 + d\theta^2)$$
$$+ dr^2/[1-(r/a)^2], \qquad (21)$$

where a is the constant radius of cuvature.

If we choose as the origin of coordinates the point of the three-dimensional hypersphere to which the surface integral (19) contracts, then in the "asymptotic region," i.e., for $r_y \to 0$ (sic), as in the preceding case, $g_{11} = 1$, $(-g)^{1/2} \sim r_y^2$. The boundary condition (19) is written the same as in (20), except that now the limit for $r_y \to 0$ is to be taken. It can be seen from this that (19) can be violated only if the Green's function has a singularity at $r_y = 0$. This is impossible, however, since the source in the equation for the Green's function is at $\mathbf{y} = \mathbf{x}$, i.e., at the opposite pole of the hypersphere. In fact, according to (21), the metric near the point $r_y = 0$ is such that the system of equations for $D_{ik}^{\alpha\beta}(\mathbf{x}, y)$ in this region contains terms with the ordinary Laplace operator and also terms associated with the curvature. The solution of the Laplace equation which is homogeneous at the origin is a constant. The presence in the equation of terms associated with the curvature does not change the good behavior of the Green's function at the point $r_y = 0$. Accordingly, the model in question is compatible with the boundary condition (19), i.e., we have shown that for the proposed formulation of Mach's principle Einstein's assertion that this model is compatible with Mach's ideas is correct.

C. From the point of view of the boundary condition (16) the closed, open, and flat models of a homogeneous and isotropic universe are equivalent, since the behavior of the metric near the singular point is the same for all of them. Therefore we can consider the simplest case of the flat model with the interval

$$-ds^2 = -c^2d\tau^2 + a^2(\tau)[(dx^1)^2 + (dx^2)^2 + (dx^3)^2]. \quad (22)$$

If near the singular point the equation of state is of the form

$$p = \alpha\varepsilon, \quad (23)$$

where α is a constant, then

$$a(\tau) = \text{const}\cdot\tau^{2/3(1+\alpha)}. \quad (24)$$

By the definition (17) of the tensor b_{ik} and from considerations of symmetry of the given problem we have

$$b_{ik} = 0 \text{ for } i \neq k; \quad \partial b_{ik} / \partial x^n = 0 \ (n = 1, 2, 3). \quad (25)$$

For the metric (22) we can separate out from the system (18) two equations for the functions

$$U_1(\tau) \equiv b_0{}^0 - {}^1\!/_2 b_m{}^m, \quad U_2(\tau) \equiv b_m{}^m. \quad (26)$$

Carrying out the calculation with the use of (25), we get

$$\ddot{U}_1 + \frac{3\dot{a}}{a}\dot{U}_1 - (\lambda + 8)\frac{\dot{a}^2}{a^2}U_1 + \frac{1}{2}(\lambda - 4) \times \frac{\dot{a}^2}{a^2}U_2 = -\lambda\frac{8\pi k}{c^2}T_0{}^0,$$
$$\ddot{U}_2 + \frac{3\dot{a}}{a}\dot{U}_2 - \lambda\left(\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2}\right)U_2 + 2\lambda\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right)U_1 = \lambda\frac{8\pi k}{c^2}T_m{}^m \quad (27)$$

The dot denotes differentiation with respect to the proper time τ. Since when the system (18) is written for the tensor b_i^k a particular solution is $b_i^k = \delta_i^k$, we see by (26) that a particular solution of (27) is $U_1 = -1$, $U_2 = 4$. In the present case the fundamental condition (4) means that this particular solution must be the retarded inhomogeneous solution. Writing it out by means of the retarded Green's function of the system (27), expressing the sources from the equations, and integrating by parts, we get in the usual way the boundary condition

$$\lim_{\tau\to 0}\left\{\frac{d}{d\tau}(V_{\xi}{}^1 - 4V_{\xi}{}^2) - \frac{3\dot{a}}{a}(V_{\xi}{}^1 - 4V_{\xi}{}^2)\right\} = 0$$
$$\text{for all } \xi = 1, 2, 3, 4. \quad (28)$$

Here $V_{\xi}^{1,2}$ are four linearly independent solutions of the homogeneous system of equations adjoint to (27), which determine the behavior of the Green's function with respect to its second argument. By substituting in these equations the $a(\tau)$ from (24) we can find these solutions in explicit form. They have a power-law dependence on τ, and a simple calculation shows that the boundary condition (28) is not satisfied for any value of the constant λ, nor for any value of α in the equation of state (23) of the matter. (Except for the case $\alpha \to \infty$, which means that there is an incompressible "core.")

D. It can be shown that the metric of the problem of the cylinder, analyzed in the Appendix [see (A.4)–(A.7)], is incompatible with the integral form (4) of the Einstein equations, i.e., with the boundary condition (19). In this problem, however, we have considered only a special version of the Mach principle, implying that in the coordinate system in which the interval is of the form (A.1) the g_{02} component of the metric tensor must be uniquely determined by specifying the angular velocity of the cylinder. According to the Appendix, a requirement for this is that the linear density of the cylinder satisfy the relation (3). We shall show that if the constant λ in (14) is equal to two, then this requirement is contained in the integral condition (8) for an infinitely small variation of the metric h_{02} caused by slow rotation of the cylinder.

Denoting the right member of (8) by $\tilde{b}_{ik}(x)$, we get for this tensor a system of differential equations analogous to the system (18) for the tensor b_{ik}, but with a different source. We write out the equation for the mixed component $\tilde{b}_2^0$ in the external region. The operations of raising and lowering indices and of covariant differentiation, and the Riemann tensor, are determined for the metric (A.4)–(A.6). We have

$$\frac{d^2\tilde{b}_2{}^0}{dr^2} - \frac{(1-2a)}{r}\frac{d\tilde{b}_2{}^0}{dr} + \left(\frac{a^2}{2} - a\right)\left(1 - \frac{\lambda}{2}\right)\frac{\tilde{b}_2{}^0}{r^2} = \lambda\left(\frac{r}{r_0}\right)^{-a+a^2/2}Q_2{}^0. \quad (29)$$

Here Q_2^0 is the mixed component of the expression in curly brackets in the integrand in (8). By definition $\tilde{b}_2^0 = h_2^0$ is a particular solution of (29). According to (A.2) or (A.7), for $r > r_0$ the component h_2^0 of the metric is constant.

The integral condition (8) implies that this particular solution must be an inhomogeneous solution of Eq. (29). The final form of the corresponding boundary condition cf. (28) is

$$\lim_{r\to\infty}(z_\xi + 1 - 2a) r^{\frac{z-1}{\xi}} = 0 \quad (\xi = 1, 2). \tag{30}$$

The index ξ numbers the linearly independent solutions of the homogeneous equation adjoint to (29). The solutions of this equation have a power-law dependence on r. The following expression is obtained for the power-law index z_ξ:

$$z_{1,2} = a \pm [(1-a)^2 + (1-\lambda/2)(a - a^2/2)]^{1/2}. \tag{31}$$

It is easy to see that the condition a = 1, i.e., the relation (3) for the linear mass density of the cylinder [cf. (A.3)], follows from (30) and (31) for $\lambda = 2$. We note that even when (3) is satisfied the relation (8) does not hold for the problem in question. In fact, for $\lambda = 2$ and a = 1 the two linearly independent solutions for which (30) holds become degenerate [cf. (31)]. There is then a new independent solution of the homogeneous equation adjoint to (29), which also determines the behavior of the Green's function of Eq. (29) with respect to its second argument, but for which the corresponding boundary condition is not satisfied.

The examples analyzed in this section (A, B, D) are mainly of an illustrative character. According to Sec. 3C, Mach's principle in the form (4) is not satisfied for the Friedmann model of a nonstationary isotropic universe, if a physically acceptable equation of state of the matter is prescribed near the singularity. It must be pointed out, however, that the compatibility of a given model with Mach's principle is determined by the behavior of the metric in the neighborhood of the singular point only [cf. (16)]. No restrictions beyond the Einstein equations are imposed on the behavior of the metric at later times. In particular, the Friedmann model is compatible with Mach's principle if near the singularity we assign a nonphysical equation of state corresponding to the presence of an incompressible "core." The question remains open as to whether, for physically admissible equations of state of the matter, there do exist any solutions of the Einstein equations compatible with (4) and (14).

I express my gratitute to L. L. Regel'son, who called my attention to this problem, and also to P. L. Vasilevskiĭ for many fruitful discussions of the question.

APPENDIX

For a cylindrical geometry with a stationary metric with the coordinates

$$x^0 = c\tau, \quad x^1 = r, \quad x^2 = \varphi, \quad x^3 = z$$

one can always choose a reference system so that the interval is of the form

$$-ds^2 = g_{00}(r)c^2 d\tau^2 + g_{11}(r)(dr^2 + dz^2) + g_{22}(r)d\varphi^2 + 2g_{02}(r)c\,d\varphi\, d\tau. \tag{A.1}$$

Let there be located at $r = r_0$ an infinitely thin heavy cylinder with finite linear mass density μ. The cylinder in general rotates with an infinitely small angular velocity ω relative to the system of coordinates defined by Eq. (A.2). The component of the metric tensor $g_{02} \equiv h_{02}$ is also small.

In empty regions the solution of the Einstein equations for the metric (A.1) can be found in general form. At $r = r_0$ one imposes the condition that the components of the metric tensor be continuous. On passing through the cylinder the first derivatives of g_{ik} with respect to r have discontinuities. The values of the discontinuities are determined from the Einstein equations, according to which the second derivatives of the components of the metric tensor at $r = r_0$ are proportional to the energy density ϵ—i.e., they become infinite. For consistency of the Einstein equations it is necessary to impose the condition that the pressure be finite at $r = r_0$.

We choose the rotation of the coordinate system in such a way that there is no Coriolis force in the external region, i.e.,[6]

$$-h_{02}/g_{00} = \text{const} \quad \text{for} \quad r > r_0. \tag{A.2}$$

Such a coordinate system is admissible for arbitrarily large r.

In the final result, which is presented below, the metric is expressed in terms of a dimensionless parameter a,

$$a \equiv 2kc^{-2}\mu, \tag{A.3}$$

and the angular velocity ω of the cylinder. For $r > r_0$ we have

$$g_{00} = -(r/r_0)^a, \tag{A.4}$$

$$g_{11} = g_{33} = (r/r_0)^{-a+a^2/2}, \tag{A.5}$$

$$g_{22} = r_0^2 (r/r_0)^{2-a}, \tag{A.6}$$

$$h_{02} = \Omega r_0^2 g_{00}/c. \tag{A.7}$$

For $r < r_0$ the diagonal components of g_{ik} are obtained by replacing $a \to -a$ in Eqs. (A.4), (A.5), and (A.6). In addition we have

$$h_{02} = -\Omega g_{22} / c, \quad r < r_0; \tag{A.8}$$

$$\Omega \equiv 2a\omega / (1 + a). \tag{A.9}$$

According to (A.8), in a new reference system defined by the transformation $\varphi' = \varphi - \Omega\tau$ the metric is static in the interior region, i.e.,

$$h_{02}' = 0, \quad r < r_0.$$

In this problem Mach's principle is satisfied for the rotational motion if in the new system of reference the cylinder is necessarily at rest, i.e., if $\Omega = \omega$; from this there follows the relation (3) of the main text [cf. (A.3) and (A.9)].

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Translated by W. H. Furry
138