

## SLOWING DOWN OF DISLOCATIONS BY SCATTERING OF ELASTIC WAVES FROM IMPURITIES

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Submitted to JETP editor 30 April 1966

J. Exptl. Theoret. Phys. (U.S.S.R.) 51, 1207-1219 (April, 1966)

The deceleration of a rectilinearly moving dislocation in a crystal with impurity atoms is analyzed macroscopically. In the macroscopic approach the decelerating force of the moving dislocation is directly expressed in terms of the dispersion of the crystal elastic moduli, and therefore the dispersion of the elastic moduli due to the changes which the impurities introduce in the crystal-lattice dynamics is analyzed in detail. The dependence of the dislocation decelerating force  $f$  on the dislocation velocity  $V$  in a crystal with impurities is studied and it is pointed out that the curve  $f = f(V)$  may be strongly nonmonotonic if the impurities result in the appearance of quasilocal oscillations. The dislocation velocity corresponding to the peak on the  $f(V)$  curve is determined by the quasi-local oscillation frequency. Slowing down of dislocations by diffusion of impurity atoms in its elastic field is also considered. The relative contributions of each of the dissipative mechanisms to the dislocation decelerating force are estimated.

### INTRODUCTION

THE motion of a dislocation in a crystal is always accompanied by deceleration due to various physical processes whereby the dislocation energy is dissipated, and is determined by the real structure of the material. In this paper we analyze the deceleration of dislocations due to the presence of impurities in the crystal. A separate examination of this cause of dislocation deceleration is believed to be of interest in connection with the fact that its contribution depends on the impurity concentration, and can consequently be easily controlled.

The main mechanism of dislocation-energy dissipation in a crystal with impurities can be described macroscopically in terms of the elastic-modulus dispersion caused by the impurities and leading to the damping of elastic waves in such a crystal. A general calculation scheme for the corresponding resistance force, within the framework of elasticity theory, was indicated in an earlier paper by the authors<sup>[1]</sup>, where this force was directly expressed in terms of the dispersion of the elastic moduli. We start from the final formulas of that paper.

It must be borne in mind that a macroscopic consideration does not take into account the deceleration produced by impurities in the glide plane

of the dislocations as they interact directly with the nucleus of the dislocation<sup>[2]</sup>. This mechanism of action of impurities on the dislocation will not be discussed.

The dispersion of the elastic moduli of a crystal with impurities is caused by two circumstances. First, the impurity introduces appreciable changes in the dynamics of the crystal lattice; in particular, it noticeably deforms the spectrum of the lattice-vibration frequencies. Under certain conditions the rearrangement of the spectrum may affect the low frequency region of interest to us<sup>[3-5]</sup>, corresponding to long-wave vibrations of the crystal<sup>1)</sup>. In Sec. 1 of this article we obtain the dispersion of the long-wave oscillations of the crystal with interstitial impurities, and in Sec. 2 we discuss the dispersion-induced frequency dependence of the elastic moduli of the crystal. We analyze in detail the dependence of the elastic moduli on the frequency in the presence of the quasi-local oscillations that are connected with an interstitial impurity. The model used by us for the interstitial impurity allows us to carry out this investigation quite

<sup>1)</sup>The dynamics of crystal lattice with impurities has by now been extensively discussed, so that we refer only to papers whose results will be directly used in our article.

illustratively and simply. In Sec. 3 we discuss the dependence of the deceleration force on the dislocation velocity and call attention to the nonmonotonic character of this dependence. The decelerating force has a sharply pronounced maximum at a velocity  $V \sim a\omega_{\text{imp}}$ , where  $a$  is the lattice constant and  $\omega_{\text{imp}}$  is the frequency of the quasi-local oscillation.

We note that an attempt to calculate the decelerating force of a moving dislocation by impurities was made earlier by Takamura and Morimoto<sup>[6]</sup>, but the method investigated there and the results obtained there seem unsatisfactory to us.

The second cause of dispersion of elastic moduli and additional elastic-wave absorption is the impurity diffusion which arises in an inhomogeneous stress field. The resistance force due to this cause is analyzed in Sec. 4, where it is shown that at the dislocation velocities of interest to us it is proportional to the impurity diffusion coefficient. To avoid misunderstandings, we explain that the dislocation is assumed to move so rapidly, that there is no time for a quasistatic impurity cloud to be produced around it. In the other limiting case, considered by Cottrell et al.<sup>[7]</sup>, an inverse dependence of the decelerating force on the diffusion coefficient of the impurity will be obtained.

The decelerating force due to the impurity diffusion is linearly connected with the dislocation velocity  $V$  and at low temperatures it becomes appreciable only at very low velocities.

## 1. ELASTIC WAVES IN A CRYSTAL WITH INTERSTITIAL IMPURITIES

We consider a disordered solid solution of low concentration  $c$  ( $c \ll 1$ ), produced by interstitial impurities in a regular crystal lattice. Let  $m$  be the mass of the crystal atom and  $M$  the mass of the impurity atom. We denote by  $\chi(\mathbf{r})$  the amplitude of the displacement of the lattice atom with coordinate  $\mathbf{r}$ , where  $\mathbf{r}$  are integer vectors of the ideal-lattice points, and by  $\psi_j$  the displacement amplitude of an impurity atom occupying an interstice with coordinate  $\mathbf{R}_j$ . Then, in the harmonic approximation, the monochromatic oscillations (of frequency  $\omega$ ) of a lattice with impurities are described by the following system of equations:

$$\begin{aligned} \omega^2 \chi_i(\mathbf{r}) - \sum_{\mathbf{r}'} L_{ih}(\mathbf{r} - \mathbf{r}') \chi_h(\mathbf{r}') \\ = \sum_{j, \mathbf{r}'} U_{ih}(\mathbf{r} - \mathbf{R}_j, \mathbf{r}' - \mathbf{R}_j) \chi_h(\mathbf{r}') + \sum_j A_{ih}(\mathbf{r} - \mathbf{R}_j) \psi_k^{(j)}, \end{aligned} \quad (1)$$

$$\omega^2 \psi_i^{(j)} - \omega_{ik}^2 \psi_k^{(j)} = \frac{m}{M} \sum_{\mathbf{r}'} A_{ih}(\mathbf{r}' - \mathbf{R}_j) \chi_h(\mathbf{r}'), \quad (2)$$

where  $L_{ik}$  is the matrix of elastic constants of the ideal lattice,  $\omega_{ik}^2$  the matrix of the elastic coupling of the impurity atom with the lattice, and the matrices  $U_{ik}$  and  $A_{ik}$  describe the corresponding perturbations of the elastic constants<sup>2)</sup>. Repeated indices imply summation throughout.

The matrices  $U_{ik}$ ,  $\omega_{ik}^2$ , and  $A_{ik}$  satisfy the obvious relations

$$\begin{aligned} \sum_{\mathbf{r}'} U_{ih}(\mathbf{r} - \mathbf{R}_j, \mathbf{r}' - \mathbf{R}_j) + A_{ih}(\mathbf{r} - \mathbf{R}_j) = 0, \\ \omega_{ih}^2 + \frac{m}{M} \sum_{\mathbf{r}} A_{ih}(\mathbf{r} - \mathbf{R}_j) = 0. \end{aligned} \quad (3)$$

With the aid of (2) we can eliminate from (1) the amplitudes of the impurity atoms  $\psi$  and reduce (1) to the form<sup>[3]</sup>

$$\begin{aligned} \omega^2 \chi_i(\mathbf{r}) - \sum_{\mathbf{r}'} L_{ih}(\mathbf{r} - \mathbf{r}') \chi_h(\mathbf{r}') \\ = \sum_{j, \mathbf{r}'} \Lambda_{ih}(\omega^2; \mathbf{r} - \mathbf{R}_j, \mathbf{r}' - \mathbf{R}_j) \chi_h(\mathbf{r}'), \end{aligned} \quad (4)$$

where

$$\begin{aligned} \Lambda_{ih}(\omega^2; \xi, \xi') = U_{ih}(\xi, \xi') \\ + \frac{m}{M} A_{in}(\xi) A_{lk}(\xi') \Delta_{ln}(\omega^2) \left/ \prod_{\alpha=1}^3 (\omega^2 - \omega_{\alpha}^2) \right., \end{aligned} \quad (5)$$

$\Delta_{in}(\omega^2)$  are the cofactors of the elements of the determinant  $\text{Det} |\omega^2 \delta_{in} - \omega_{in}^2|$ , and  $\omega_{\alpha}^2$  are the roots of the equation  $\text{Det} |\omega^2 \delta_{in} - \omega_{in}^2| = 0$  and have the meaning of "bare" natural frequencies of the impurity atom in the lattice.

Equation (4) together with relations (3) now determines only the amplitudes of the oscillations of the lattice atoms, and the perturbations of the lattice oscillations by the impurities are completely described by the matrix  $\Lambda_{ik}$ . Further analysis of (4) calls for knowledge of the explicit coordinate dependence of the matrix  $\Lambda_{ik}$ . Being interested in the qualitative aspect of the problem, we shall attempt to cast this matrix in a relatively simple form. As already noted in the introduction, we are interested in the singularities introduced by the impurities in the long-wave deformations of the crystal, relative to which the crystal can be regarded as a continuous medium, and the impurities and the perturbations caused by them can be regarded as pointlike.

<sup>2)</sup>Equations (1) and (2) are an obvious generalization of the equations considered in [4]. However, the mass ratio  $M/m$  was erroneously omitted from the right side of (2) in [4] so that the results of [4] pertain only to the case  $M = m$ .

In addition, wishing to obtain the results in closed form, we must recognize that the scattering of elastic waves and the deceleration of the dislocations are described by relatively simple formulas only in the isotropic approximation. Therefore it is reasonable to confine ourselves henceforth to the simplest crystal model, namely a continuous isotropic medium. This makes it possible to simplify to the utmost all the equations, retaining at the same time the essential features of the phenomena under consideration.

In this approximation, the coordinates  $\mathbf{r}$  of the lattice atoms can be assumed to run through a continuous set of values, and the matrix  $U_{ik}$  is naturally chosen in the form

$$U_{ik}(\xi, \xi') = U_0 v_0^2 \delta(\xi) \delta(\xi') \delta_{ik}, \quad (6)$$

where  $\delta(\xi)$  is a  $\delta$ -function,  $\delta_{ik}$  is the Kronecker symbol, and  $v_0$  is the volume per unit cell of the crystal lattice and has the meaning of a normalization constant. Then the relations between the force constants go over into

$$\omega_0^2 = U_0 m / M, \quad (7)$$

where  $\omega_0$  is the "bare" natural frequency of the impurity ( $\omega_{ik}^2 = \omega_0^2 \delta_{ik}$ ), which is unique in the isotropic model, and for the matrix  $\Lambda_{ik}$  we obtain as a result of the simplifications the following expression:

$$\Lambda_{ik}(\omega^2; \xi, \xi') = \Lambda_0(\omega^2) \delta(\xi) \delta(\xi') \delta_{ik},$$

$$\Lambda_0(\omega^2) = \omega^2 U_0 / (\omega^2 - \omega_0^2). \quad (8)$$

A general method for the analysis of equations with a perturbation potential of the type (6) was developed by I. M. Lifshitz<sup>[3]</sup>. Following this method, we put in (4)  $\mathbf{r} = \mathbf{R}_j$  and, using the Green's tensor  $G_{in}^\omega(\mathbf{r})$  for the equation of the stationary oscillations of the ideal lattice, we rewrite (4) in the form

$$\{\delta_{in} + \Lambda_0(\omega^2) G_{in}^\omega(0)\} \chi_n(\mathbf{R}_j)$$

$$= -\Lambda_0(\omega^2) \sum_{j' \neq j} G_{in}^\omega(\mathbf{R}_j - \mathbf{R}_{j'}) \chi_n(\mathbf{R}_{j'}). \quad (9)$$

For elastic oscillations whose wavelengths greatly exceed the average distance between impurities, we can speak of waves that are plane "in the mean," putting  $\chi(\mathbf{R}) = \chi_0 \exp(i\mathbf{k} \cdot \mathbf{R})$ , where  $\chi_0$  is the wave amplitude averaged over the impurities<sup>[3]</sup>. Averaging of (9) over the impurities yields in the first approximation in the concentration  $c$

$$[\delta_{in} + \Lambda_0(\omega^2) G_{in}^\omega(0) + c \Lambda_0(\omega^2) g_{in}(\omega^2, \mathbf{k})] \chi_n^{(0)} = 0, \quad (10)$$

where  $g_{in}(\omega^2, \mathbf{k})$  denotes the Fourier transform of the Green's function  $G_{in}^\omega(\mathbf{r})$ :

$$G_{in}^\omega(\mathbf{r}) = \frac{v_0}{(2\pi)^3} \int d\mathbf{k} g_{in}(\omega^2 + i0, \mathbf{k}) e^{i\mathbf{k}\mathbf{r}},$$

$$g_{in}(\omega^2, \mathbf{k}) = \sum_{\mathbf{r}} G_{in}^\omega(\mathbf{r}) e^{-i\mathbf{k}\mathbf{r}}.$$

From (10) we get the following dispersion equation for the vibrations of a crystal with interstitial impurities:

$$\text{Det} |\delta_{in} + \Lambda_0(\omega^2) G_{in}^\omega(0) + c \Lambda_0(\omega^2) g_{in}(\omega^2, \mathbf{k})| = 0. \quad (11)$$

As is well known, the tensors  $g_{in}(\omega^2, \mathbf{k})$  and  $G_{in}^\omega(0)$  in the general anisotropic case can be represented in the form

$$g_{in}(\omega^2, \mathbf{k}) = \sum_{\alpha=1}^3 \frac{e_i^{(\alpha)}(\mathbf{k}) e_n^{(\alpha)}(\mathbf{k})}{\omega_{\alpha}^2(\mathbf{k}) - \omega^2}; \quad (12)$$

$$G_{in}^\omega(0) = \sum_{\alpha=1}^3 \left[ \oint \frac{n_{in}^{(\alpha)}(z)}{z^2 - \omega^2} dz + i \frac{\pi}{2} \frac{n_{in}^{(\alpha)}(\omega)}{\omega} \right],$$

$$n_{in}^{(\alpha)}(z) = \frac{v_0}{(2\pi)^3} \oint_{\omega_{\alpha}(\mathbf{k})=z} dS_{\mathbf{k}} \frac{e_i^{(\alpha)}(\mathbf{k}) e_n^{(\alpha)}(\mathbf{k})}{|\nabla_{\mathbf{k}} \omega_{\alpha}(\mathbf{k})|} \quad (13)$$

here  $e^{(\alpha)}(\mathbf{k})$  is the unit polarization vector,  $\omega_{\alpha}(\mathbf{k})$  the dispersion law ( $\alpha$  is the number of the branch of the acoustic spectrum of the ideal crystal), and the integral with respect to  $z$  is taken in the sense of principal value.

On going over to an isotropic medium, the second-rank tensor  $n_{in}(\omega)$  should go over into a scalar

$$n_{in}(\omega) = 1/3 \nu(\omega) \delta_{in}, \quad \nu(\omega) = \frac{v_0}{(2\pi)^3} \oint_{\omega(\mathbf{k})=\omega} dS_{\mathbf{k}} \frac{1}{|\nabla_{\mathbf{k}} \omega(\mathbf{k})|}$$

where  $\nu(\omega)$  is the frequency density of the corresponding branch of the spectrum (at low frequencies  $\nu(\omega) = v_0 \omega^2 / 2\pi^2 s^3$ , where  $s$  is the speed of sound). This quantity determines the imaginary part of the tensor  $G_{in}^\omega(0)$  in the isotropic case. To calculate the real part of the tensor  $G_{in}^\omega(0)$  in the first nonvanishing approximation in the small parameter  $\omega/\omega_D$  ( $\omega_D$  is the Debye frequency) we can put  $\omega = 0$  in the denominator of the integrand of expression (13) for  $G_{in}^\omega(0)$ . This greatly simplifies (13):

$$G_{in}^\omega(0) = \frac{3}{\Omega_1^2} \left( 1 + i \frac{\omega}{\Omega_2} \right) \delta_{in},$$

$$\frac{1}{\Omega_1^2} = \frac{1}{18} \sum_{\alpha=1}^3 \int \frac{\nu_{\alpha}(z)}{z^2} dz, \quad \Omega_2 = \frac{v_0}{9\pi^3} \left( \frac{1}{s_l^3} + \frac{2}{s_t^3} \right) \Omega_1^2, \quad (14)$$

where  $s_l$  and  $s_t$  are respectively the velocities of the longitudinal and transverse sound waves in an ideal crystal, and obviously  $\Omega_1 \sim \Omega_2 \sim \omega_D$ .

The tensor  $g_{in}(\omega^2, \mathbf{k})$  has in the same approximation the form

$$g_{in}(\omega^2, \mathbf{k}) = \frac{\kappa_i \kappa_n}{s_i^2 k^2 - \omega^2} + \frac{\delta_{in} - \kappa_i \kappa_n}{s_i^2 k^2 - \omega^2}, \quad \kappa = \frac{\mathbf{k}}{k}. \quad (15)$$

When (14) and (15) are substituted in (11), the latter reduces to two equations of the type

$$s^2 k^2 = \omega^2 - \frac{c \Lambda_0(\omega^2)}{1 + 3\Omega_1^{-2} \Lambda_0(\omega^2) (1 + i\omega/\Omega_2)}, \quad (16)$$

one of which determines the dispersion law for longitudinal waves ( $s = s_l$ ) and the other for transverse waves ( $s = s_t$ ) in a crystal with interstitial impurities. The fact that the presence of impurities does not lead to "entanglement" of the longitudinal and transverse waves in the crystal is the consequence of the isotropy of the perturbation-potential model (6) chosen by us.

For further analysis it is convenient to introduce in place of the parameters  $c$ ,  $\omega_0$ ,  $m/M$ ,  $\Omega_1$ , and  $\Omega_2$  the new parameters

$$\varepsilon = c \frac{M}{m}, \quad \omega_{\text{imp}} = \omega_0 \left(1 + 3 \frac{M}{m} \frac{\omega_0^2}{\Omega_1^2}\right)^{-1/2},$$

$$\beta = \frac{M}{m} \frac{\omega_{\text{imp}}^3}{\Omega_1^2 \Omega_2}, \quad (17)$$

and, using (7), rewrite (16) in the form

$$s^2 k^2 = \omega^2 \left[ \varphi_1 \left( \frac{\omega}{\omega_{\text{imp}}} \right) + i \varphi_2 \left( \frac{\omega}{\omega_{\text{imp}}} \right) \right],$$

$$\varphi_1(x) = 1 + \frac{\varepsilon(1-x^2)}{(1-x^2)^2 + \beta^2 x^6}, \quad \varphi_2(x) = \frac{\varepsilon \beta x^3}{(1-x^2)^2 + \beta^2 x^6}. \quad (18)$$

The frequency  $\omega_{\text{imp}}$  has the meaning of a renormalized natural frequency of the impurity atom.

Let us stop to discuss the limitations under which (18) is valid. As already noted above, the length of the elastic wave  $\lambda$  should be much larger than the mean distance between impurities  $a c^{-1/3}$  ( $a =$  lattice parameter). In the presence of dispersion in the isotropic medium, the wavelength  $\lambda(\omega)$  is given by the formula  $\lambda(\omega) = 2\pi/\text{Re}\{k(\omega)\}$ , where  $k(\omega)$  is the wave-vector value obtained from (18). Therefore this condition is expressed by the following inequality:

$$a(2\pi)^{-1} \text{Re}\{k(\omega)\} \ll c^{1/3}. \quad (19)$$

In the absence of dispersion, (19) is replaced by  $\omega/\omega_D \ll c^{1/3}$ .

We note that for substitutional impurities, a dispersion equation similar to (16) and (18) was obtained by a somewhat different method by Slutskin and Sergeeva<sup>[5]</sup> with a more general perturbation-potential form than used in our case.

## 2. DISPERSION OF ELASTIC MODULI

In final analysis we are interested in the deceleration of a linear dislocation moving in a crys-

tal with constant of velocity  $V$  ( $V \ll s$ ). In the isotropic approximation, this force per unit dislocation length is equal to<sup>[1]</sup>

$$f(V) = \frac{2b_1^2}{\pi V} \int_0^{k_0 V} \text{Im} \left\{ \mu(\omega) \left[ 1 - \frac{\mu(\omega)}{\sigma(\omega)} \right] \right\} d\omega$$

$$+ \frac{b_2^2}{\pi V} \int_0^{k_0 V} \text{Im} \{ \mu(\omega) \} d\omega. \quad (20)$$

Here  $\mu(\omega)$  is the complex shear modulus and  $\sigma(\omega)$  is a combination of elastic moduli, which enters into the definition of the longitudinal speed of sound in the absence of dispersion (if  $\lambda$  and  $\mu$  are Lamé coefficients, then  $\sigma = \lambda + 2\mu$ )<sup>3)</sup>;  $b_1$  and  $b_2$  denote respectively the edge and screw components of the Burgers dislocation vector;  $k_0$  is a certain "cut-off" parameter for the integration limits, with magnitude of the order of the reciprocal lattice constant ( $k_0 a \sim k_0 b \sim 1$ ). The appearance of the parameter  $k_0$  is connected with the need of eliminating from consideration deformations in the nucleus of the dislocation, since these cannot be described within the framework of the continual theory<sup>[1]</sup>.

Thus, the value of the decelerating force is determined completely by the dispersion of the elastic moduli. The explicit form obtained by us for the dependence of the wave vector on the frequency (18) enables us to investigate in detail the modulus dispersion due to the impurities.

Using the definition of  $\sigma(\omega)$  and  $\mu(\omega)$  in (18), we obtain

$$\sigma(\omega) = \rho s_l^2 \frac{\varphi_1(\omega/\omega_{\text{imp}}) - i\varphi_2(\omega/\omega_{\text{imp}})}{\varphi_1^2(\omega/\omega_{\text{imp}}) + \varphi_2^2(\omega/\omega_{\text{imp}})}$$

$$\mu(\omega) = \rho s_t^2 \frac{\varphi_1(\omega/\omega_{\text{imp}}) - i\varphi_2(\omega/\omega_{\text{imp}})}{\varphi_1^2(\omega/\omega_{\text{imp}}) + \varphi_2^2(\omega/\omega_{\text{imp}})} \quad (21)$$

where  $\rho$  is the density of the medium, and the remaining notation is the same as in Sec. 1.

Inasmuch as the upper limit of integration in (20) is finite, the relation  $f = f(V)$  will be determined by the behavior of the corresponding integrands in the frequency interval  $(0, k_0 V)$ . Consequently, we must ascertain under what conditions do the singularities of the imaginary parts of the elastic moduli appear in this interval. In addition, it must be remembered that the dispersion law (18) was derived under certain limitations, so that we must know the frequency interval in which the formulas in (21) become meaningless.

<sup>3)</sup>We introduce the symbol  $\sigma(\omega)$  in order to avoid confusion, since in this text  $\lambda(\omega)$  denotes throughout the length of the elastic wave corresponding to the frequency  $\omega$ .

We shall show first that the frequency interval in which (21) may be incorrect is very small. We shall also verify later that this interval was outside the region of frequencies in which the most essential change in the integrands of (20) occurs.

If the dispersion is small, then  $\lambda \sim s/\omega$  and the inequality (19) reduces to the simple condition

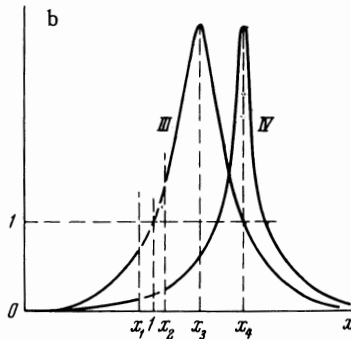
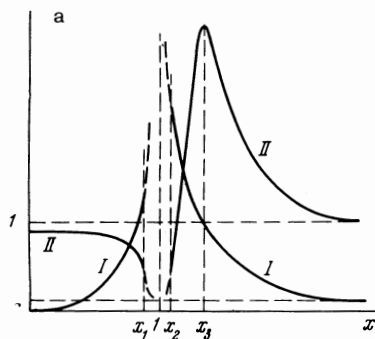
$$\omega / \omega_D \ll c^{1/3}, \quad c \ll 1. \quad (22)$$

However, in the presence of quasi-local oscillations in the crystal, the dispersion of the elastic waves has a resonant character<sup>[4,5]</sup>, and then an interpretation of inequalities (19) calls for a detailed analysis of the dispersion equation. In the case of substitution impurities, a detailed analysis of this type was carried out in<sup>[5]</sup>. All the qualitative singularities of the dispersion of elastic waves, noted in that paper, appear also in the case of interstitial impurities, so that we confine ourselves to a simple listing of these singularities, and indicate their connection with the parameters of the impurity model assumed by us.

The singularities in the propagation of elastic waves in a crystal with impurities appear in the vicinity of the quasi-local frequency<sup>[4,5]</sup>. For interstitial impurities, the role of the quasi-local frequency is played by the renormalized natural frequency  $\omega_{\text{imp}}$  of the impurity atoms, if this frequency lies in the low-frequency end of the spectrum. We put  $\omega_{\text{imp}} \ll \omega_D$  and assume that (22) is satisfied for all the frequencies in question, so that the inequality

$$\omega_{\text{imp}} / \omega_D \ll c^{1/3} \quad (23)$$

is certainly satisfied. The wavelength dispersion  $\lambda(\omega) = 2\pi/\text{Re}\{k(\omega)\}$  and of the wave damping coefficient  $\gamma(\omega) = \text{Im}\{k(\omega)\}$  are shown for these conditions in Fig. (a) (all the plots in our figure are in an arbitrary scale and are schematic only). Curve I shows the frequency dependence of the damping coefficient:



Dependence on the dimensionless frequency  $x = \omega/\omega_{\text{imp}}$  of the following quantities: I—the dimensionless damping coefficient  $s\gamma(\omega)/\omega_{\text{imp}}$ ; II—the dimensionless wavelength  $\omega\lambda(\omega)/2\pi s$ ; III— $\gamma(\omega)\lambda(\omega)/2\pi$ ; IV—the dimensionless imaginary part of the elastic moduli  $F(\omega/\omega_{\text{imp}}) = -\rho^{-1}s_t^{-2}\text{Im}\mu(\omega) = -\rho^{-1}s_l^{-1}\text{Im}\sigma(\omega)$ . The quantities  $x_1, x_2, x_3,$  and  $x_4$  are equal to:

$$\begin{aligned} x_1 &= 1 - c^{1/3}Mm^{-1}(\omega_{\text{imp}}/\omega_D)^2, \\ x_2 &= 1 + c^{1/3}Mm^{-1}(\omega_{\text{imp}}/\omega_D)^{3/2}, \\ x_3 &= (1 + 1/2cM/m)^{1/2}, \quad x_4 = (1 + cM/m)^{1/2} \end{aligned}$$

$$\gamma(\omega) = \frac{\omega}{s} \left[ \frac{\sqrt{\varphi_1^2 + \varphi_2^2} - \varphi_1}{2} \right]^{1/2},$$

and curve II the frequency dependence of the wavelength:

$$\lambda(\omega) = 2\pi \frac{s}{\omega} \left[ \frac{2}{\sqrt{\varphi_1^2 + \varphi_2^2} + \varphi_1} \right]^{1/2}.$$

The plots show the dimensionless quantities  $s\gamma(\omega)/\omega_{\text{imp}}$  and  $\omega\lambda(\omega)/2\pi s$ .

We see that at very low frequencies ( $\omega \ll \omega_{\text{imp}}$ ) usual Rayleigh scattering of the elastic wave by the impurities takes place, leading to small dispersion:

$$\gamma(\omega) = \frac{1}{2s\omega_{\text{imp}}^3} \frac{\varepsilon\beta}{(1+\varepsilon)^{1/2}} \omega^4, \quad \lambda(\omega) = \frac{2\pi s}{\omega} \frac{1}{(1+\varepsilon)^{1/2}}.$$

However, in a narrow region of frequencies  $\delta\omega \sim \omega_{\text{imp}}c^{1/3}(M/m)(\omega_{\text{imp}}/\omega_D)^2$  near the quasi-local frequency  $\omega_{\text{imp}}$ , the damping coefficient increases in resonant fashion ( $s\gamma(\omega)/\omega_{\text{imp}} \gg 1$ ), and the wavelength becomes very small ( $\omega\lambda(\omega)/2\pi s \ll 1$ ). In the figure, the points  $x = x_1$  and  $x = x_2$  correspond to those frequencies at which  $\lambda(\omega) = ac^{-1/3}$ , and therefore the requirement (19) is certainly satisfied inside the interval  $(x_1, x_2)$ . The latter circumstance makes Eq. (18) and on all the formulas that follow from it doubtful. Accordingly, the corresponding sections of the curve are shown dashed. Bearing in mind condition (23), we can conclude that  $\delta\omega \ll \omega_{\text{imp}}cM/m$ . Inasmuch as it is natural to assume that at low impurity concentrations the total mass of the impurity atoms is smaller than the mass of the initial crystal ( $cM/m < 1$ ), we have  $\delta\omega \ll \omega_{\text{imp}}$ , that is, the discussed frequency interval is much smaller than the magnitude of the quasi-local frequency.

Directly adjacent to the region of resonant damping of the elastic waves is a frequency interval of width  $\Delta\omega = \omega_{\text{imp}}(x_4 - 1) \sim \omega_{\text{imp}}cM/m$ , in which the characteristic attenuation length of the elastic wave becomes smaller than its wavelength:

$\lambda(\omega)\gamma(\omega) > 2\pi$  (the point  $x = x_4$  is determined by the condition  $\lambda(\omega)\gamma(\omega) = 2\pi$ ). A plot of the function  $\lambda(\omega)\gamma(\omega)/2\pi$  is shown in Fig. (b) (curve III). We note first that all our formulas remain in force in the frequency interval  $\Delta\omega$ . Indeed, the condition  $\lambda(\omega)\gamma(\omega) > 2\pi$  is due essentially to the anomalous increase in the wavelength (at the point  $\omega = \omega_{\text{imp}}x_3$  the function  $\omega\lambda(\omega)/2\pi s$  is maximal and its order of magnitude is  $\epsilon/\beta \sim c(\omega_D/\omega_{\text{imp}})^3 \gg 1$ ), so that the inequality (19) is certainly satisfied in this region.

The damping coefficient  $\gamma(\omega)$  in the interval  $\Delta\omega$  is not very large, but when  $\lambda(\omega)\gamma(\omega) > 2\pi$ , the crystal becomes "opaque" to the elastic waves. In this frequency interval, the reflection coefficient of the elastic waves from the crystal with impurities is practically equal to unity<sup>4</sup>. The occurrence of a crystal "opacity" band, due to the impurities with quasi-local oscillations, was predicted in<sup>[4]</sup>. However, the estimate given there for the width of the interval  $\Delta\omega$  pertains to the case  $M = m$ . The dependence of  $\Delta\omega$  on  $M/m$  was discussed in the paper by Slutskin and Sergeeva<sup>[5]</sup>.

It is very important that the "opacity" interval  $\Delta\omega$  is much broader than the region of resonant

damping of the elastic waves  $\delta\omega$ :  $\Delta\omega \gg \delta\omega$ . This can be easily verified by using the aforementioned estimates of  $\delta\omega$  and  $\Delta\omega$ :

$$\frac{\delta\omega}{\Delta\omega} \sim c^{-2/3} \left( \frac{\omega_{\text{imp}}}{\omega_D} \right)^2 \ll 1. \quad (24)$$

It is important to note that under the condition (23) the inequality (24) does not depend on the mass ratio  $M/m$ .

At frequencies lying to the right of the "opacity" region ( $\omega \gg \omega_{\text{imp}}$ ), the dispersion again becomes small, and condition (19) reduces to inequality (22). We note an obvious and natural analogy between the dispersion of the elastic waves in a crystal with impurities that possess a quasi-local frequency, on the one hand, and the anomalous dispersion of the electromagnetic waves near the natural atomic or molecular frequencies, on the other.

Bearing in mind all the foregoing, we proceed to analyze the imaginary parts of the elastic moduli  $\mu(\omega)$  and  $\sigma(\omega)$ . The imaginary parts of these quantities, in accord with (21), are proportional to the function  $F(\omega/\omega_{\text{imp}})$ , which is determined by the expression

$$F(x) = \frac{\varphi_2(x)}{\varphi_1^2(x) + \varphi_2^2(x)} = \frac{\epsilon\beta[(1-x^2)^2 + \beta^2x^6]x^3}{(1-x^2)(1+\epsilon-x^2)[(1-x^2)(1+\epsilon-x^2) + 2\beta^2x^6] + \epsilon^2\beta^2x^6 + \beta^4x^{12}}. \quad (25)$$

The general form of this function, in the presence of a quasi-local frequency in the low-frequency region and when condition (22) is satisfied for all the frequencies in question, is shown in Fig. (b) (curve IV). The function  $F(x)$  has a clearly pronounced maximum at the point  $x = x_4 = (1 + \epsilon)^{1/2}$  at which

$$F(x_4) \sim \epsilon/\beta \sim c(\omega_D/\omega_{\text{imp}})^3 \gg 1.$$

At low frequencies ( $x = \omega/\omega_{\text{imp}} \ll 1$ ) the function  $F(x)$  is described by the expression

$$F(x) = \frac{\epsilon\beta}{(1+\epsilon)^2} x^3. \quad (26)$$

<sup>4</sup>For the coefficient of reflection of elastic waves incident from an ideal crystal on a crystal with impurities, we have the following formula:

$$R(\omega) = \left\{ \left[ 1 - \left( 1 + c \frac{M}{m} \right) \frac{\omega}{2\pi s} \lambda(\omega) \right]^2 + \left[ \frac{1}{2\pi} \lambda(\omega) \gamma(\omega) \right]^2 \right\} \times \left\{ \left[ 1 + \left( 1 + c \frac{M}{m} \right) \frac{\omega}{2\pi s} \lambda(\omega) \right]^2 + \left[ \frac{1}{2\pi} \lambda(\omega) \gamma(\omega) \right]^2 \right\}^{-1}.$$

It is easy to see that, for all frequencies lying inside the interval  $\Delta\omega$ , the coefficient  $R$  is equal to unity, accurate to terms of the order of magnitude of the small parameters  $(\beta/\epsilon)^{2/3} \ll 1$  or  $(\beta/\epsilon)^{1/2} \ll 1$ .

At high frequencies ( $x \gg 1$ ) this function decreases quite rapidly:

$$F(x) \cong \varphi_2(x) \cong \frac{\epsilon}{\beta} \frac{1}{x^3}. \quad (27)$$

The half-width of the peak of the curve  $F(x)$  has an order of magnitude

$$\epsilon\beta \sim c(M/m)^2(\omega_{\text{imp}}/\omega_D)^3.$$

We deem it essential to point to the following circumstances, which results from the anomalous character of the dispersion near the quasi-local frequency. First, the maximum of the function  $F(x)$ , i.e., in fact, the maximum of the imaginary parts of the elastic moduli, does not coincide with the maximum of the elastic-wave damping coefficient. The relative displacement of the peaks of the two curves is  $\Delta x = x_4 - 1 \sim cM/m$ . It is small when  $cM/m \ll 1$ , but increases linearly with increasing mass ratio  $M/m$ . Second, the sharp increase in the function  $F(x)$  takes place in a frequency interval corresponding to the "opacity" of the crystal (region of "total internal reflection"). The maximum value for this function (at  $x = x_4$ ) lies at the end of the interval  $\Delta\omega$ , where  $\lambda(\omega)\gamma(\omega) \gg 1$ . The fact that the main rise in the dislocation-energy loss takes place precisely in

the interval  $\Delta\omega$  seems perfectly natural to us.

The entire region where an appreciable change takes place in  $F(x)$  lies outside the interval  $\delta\omega$  and is much broader than this interval. If we use expression (25) to describe formally the function  $F(x)$  in the interval  $\delta\omega$ , then it turns out that  $F(x)$  is a smooth function without any singularities in this interval. Therefore, in integrating over the frequencies, the narrow interval  $\delta\omega$  near the quasi-local frequency cannot make an appreciable contribution to the deceleration force  $f(V)$ . Consequently, the function  $F(x)$  can be regarded as specified by formula (25) for all values of its argument, and there is no need at all for separating the interval  $\delta\omega$  of resonant absorption of the elastic waves.

### 3. FORCE OF SLOWING DOWN OF A DISLOCATION BY IMPURITIES

To analyze the deceleration force of a dislocation in a medium with interstitial impurities, we rewrite (20) and insert the function  $F(x)$  into the expression for  $f(V)$ :

$$f(V) = -\rho s_t^2 \frac{\omega_{\text{imp}}}{\pi V} \left[ 2b_1^2 \left( 1 - \frac{s_t^2}{s^2} \right) + b_2^2 \right] \int_0^{k_0 V / \omega_{\text{imp}}} F(x) dx. \quad (28)$$

We shall use the behavior of the function  $F(x)$  described in Sec. 2, and establish the character of the dependence of the deceleration force on the dislocation velocity.

If the dislocation velocity is sufficiently low,

$$V \ll (\omega_{\text{imp}} / k_0) (1 + \varepsilon)^{1/2} \sim a\omega_{\text{imp}},$$

then we can calculate the deceleration force by using expression (26) for the function  $F(x)$  at low values of its argument. It then turns out that the deceleration force increases with velocity like

$$f(V) = -\alpha^2 V^3, \quad \alpha^2 = \frac{\rho s_t^2 k_0^4}{4\pi\omega_{\text{imp}}^3} \left[ 2b_1^2 \left( 1 - \frac{s_t^2}{s^2} \right) + b_2^2 \right] \frac{\varepsilon\beta}{(1 + \varepsilon)^2}. \quad (29)$$

The order of magnitude of the coefficient  $\alpha^2$  is (see (17)):

$$\alpha^2 \sim \rho\varepsilon\beta \frac{s^2 k_0^2}{\omega_{\text{imp}}^3} \sim \varepsilon \frac{M}{m} \frac{\rho s^2 k_0^2}{\omega_D^3} \sim c \left( \frac{M}{m} \right)^2 \frac{b\mu}{s^3}, \quad (30)$$

where  $\mu$  is the shear modulus of the ideal crystal.

At sufficiently large dislocation velocities

$$V \gg (\omega_{\text{imp}} / k_0) (1 + \varepsilon)^{1/2} \sim a\omega_{\text{imp}},$$

it is possible to put in (28) an infinite upper limit of integration, using the rapid decrease of the function  $F(x)$  at large values of its argument ( $x \gg 1$ ). As a result we find that at such velocities the slowing-

down force is inversely proportional to the velocity:

$$f(V) = -b\mu V_0 / V, \quad V_0 = \frac{\omega_{\text{imp}}}{\pi b} \left[ 2b_1^2 \left( 1 - \frac{s_t^2}{s^2} \right) + b_2^2 \right] \int_0^\infty F(x) dx. \quad (31)$$

Let us estimate the value of  $V_0$ . An estimate of the integral entering into the definition of  $V_0$  can be obtained by multiplying the half-width of the peak of the  $F(x)$  curve by its height:

$$\int_0^\infty F(x) dx \sim \varepsilon\beta \frac{\varepsilon}{\beta} = \left( c \frac{M}{m} \right)^2.$$

Consequently, in order of magnitude,

$$V_0 \sim (cM/m)^2 a\omega_{\text{imp}}. \quad (32)$$

Comparing the behavior of the decelerating force  $f(V)$  at large and small velocities, we can conclude that it has a clearly pronounced maximum. The velocity ( $V_{\text{max}}$ ) at which  $f(V)$  reaches a maximum is determined by the root of the equation

$$\int_0^z F(x) dx = zF(z), \quad z = \frac{k_0 V}{\omega_{\text{imp}}}. \quad (33)$$

Since  $F(x)$  has a very sharp peak in a small vicinity of the point  $x = x_4$ , we have in order of magnitude

$$V_{\text{max}} \sim x_4 \omega_{\text{imp}} / k_0 \sim a\omega_{\text{imp}}. \quad (34)$$

Thus, the deceleration force of the dislocation, as a function of the dislocation velocity, has a sharply nonmonotonic character if the crystal contains impurities with quasi-local frequencies. If the impurity does not produce any quasi-local oscillations, then the dependence of the deceleration force on the dislocation velocity is described by formula (29) for all velocities that admit of a macroscopic analysis. It must be borne in mind, however, that the coefficient entering into this formula is defined very roughly, since it contains a factor  $k_0^4$ , that is, a parameter whose order of magnitude can only be estimated. As regards formula (31), it is accurate in this sense, since it contains only the macroscopic characteristics of the medium in the region of its applicability.

### 4. DISLOCATION DECELERATION DUE TO DIFFUSION OF IMPURITIES

We consider, finally, the deceleration due to diffusion of impurities in the elastic field of the dislocation. In the analysis of this deceleration we confine ourselves, as already noted in the introduction, to rather high dislocation velocities, at which the deviation of the impurity concentration  $c(\mathbf{r}, t)$  from its equilibrium value  $c_0$  at each point of the crystal can be regarded as small:

$$c(\mathbf{r}, t) = c_0 + c_1(\mathbf{r}, t), \quad c_1(\mathbf{r}, t) \ll c_0.$$

We assume, as before, that the medium is isotropic. We denote by  $\mathbf{u}(\mathbf{r}, t)$  the displacement vector of the elements of the medium, and by  $D$  the impurity diffusion coefficient. Then the elastic motion of the medium and the impurity diffusion associated with this motion are described in an approximation that is linear in the strains and in the concentration gradient by the following system of equations:

$$\begin{aligned} \frac{\partial^2}{\partial t^2} \mathbf{u}(\mathbf{r}, t) &= (s_l^2 - s_t^2) \text{grad div } \mathbf{u}(\mathbf{r}, t) + s_t^2 \Delta \mathbf{u}(\mathbf{r}, t) \\ &\quad - \frac{\eta K}{\rho} \text{grad } c_1(\mathbf{r}, t), \\ \frac{\partial}{\partial t} c_1(\mathbf{r}, t) &= D \Delta \left[ c_1(\mathbf{r}, t) - c_0 \frac{\eta a^3 K}{kT} \text{div } \mathbf{u}(\mathbf{r}, t) \right]. \end{aligned} \quad (35)$$

Here  $\eta$  is a coefficient characterizing the equilibrium relative change in volume when a single impurity is introduced<sup>[8]</sup> (usually  $\eta \sim 1$ ),  $K$  is the modulus of the volume compression,  $a^3$  is the atomic volume, and  $kT$  is the temperature in energy units.

Since we want to obtain the elastic-modulus frequency dependence brought about by the impurity diffusion, let us find the dispersion of the oscillations, described by (35), of the vector  $\mathbf{u}$  and of the concentration  $c_1$ . It is easy to imagine that the dispersion of interest to us will be determined by a parameter  $\tau\omega$ , where

$$\tau = c\eta^2 a^3 \frac{D}{kT} \frac{K^2}{\rho s_l^4} \sim c\rho a^3 \frac{D}{kT} \sim cm \frac{D}{kT}. \quad (36)$$

We can verify that  $\tau\omega \ll 1$  for all reasonable values of the temperature and for all frequencies. Therefore we can confine ourselves in the calculations to the first nonvanishing power of  $\tau\omega$ . We then obtain for longitudinal waves the dispersion equation

$$s_l^2 k^2 = \omega^2 (1 + i\tau\omega). \quad (37)$$

In our approximation, of course, there is no dispersion of the transverse waves ( $s_t^2 k^2 = \omega^2$ ).

Using (37), we obtain expressions for the complex moduli of elasticity of the medium:

$$\mu(\omega) = \rho s_l^2, \quad \sigma(\omega) = \rho s_l^2 / (1 + i\tau\omega). \quad (38)$$

Substituting (38) in formula (20) for the dislocation deceleration force, we obtain the following dependence of this force on the dislocation velocity<sup>5)</sup>:

$$f(V) = -\alpha_1^2 V, \quad \alpha_1^2 = \frac{\rho s_l^4 b_l^2 k_0^2}{\pi s_l^2} \tau. \quad (39)$$

It is easy to estimate the order of magnitude of the coefficient  $\alpha_1^2$  in formula (39):

$$\alpha_1^2 \sim \rho s_l^2 \tau \sim \mu \tau. \quad (40)$$

It is of interest to compare (39) with the magnitude of the deceleration force (29) due to the elastic scattering of the sound waves by impurities. The ratio of the force (39) to the force (29) is of the order

$$\begin{aligned} \left(\frac{m}{M}\right)^2 \frac{\omega_D \tau}{c} \left(\frac{s}{V}\right)^2 &\sim \left(\frac{m}{M}\right)^2 \left(\frac{s}{V}\right)^2 \frac{\omega_D D}{kT} \rho a^3 \\ &\sim \left(\frac{m}{M}\right)^2 \left(\frac{s}{V}\right)^2 \frac{\omega_D D}{kT/m}. \end{aligned} \quad (41)$$

At low temperatures, the diffusion velocity ( $\sqrt{\omega_D D}$ ) is vanishingly small compared with the average thermal velocity ( $\sqrt{kT/m}$ ), and therefore the ratio (41) is small even when  $V \sim a\omega_{\text{imp}}$ , i.e., in the region, discussed in Sec. 3, of nonmonotonic variation of the deceleration force. Only at very low velocities does this ratio become of the order of unity.

However, at sufficiently high temperatures, the ratio (41) increases rapidly with the increasing temperature, and the moving-dislocation energy dissipation caused by the impurity diffusion in its elastic field may turn out to be the main mechanism for the deceleration of the dislocation by the impurities.

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<sup>5)</sup>Naturally, in an isotropic medium a deceleration of this type is experienced only by the edge component of the dislocation.