NON-LINEAR INSTABILITY OF OPTICAL FREQUENCIES IN PARTIALLY IONIZED PLASMA

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Nonlinear instability mechanisms that can lead to an increase of optical frequency radiation intensity, as well as to emission of frequencies that differ from the transition frequency by the plasma frequency, are considered for plasma containing atoms with inverse level population.

1. We are interested in the nonlinear interaction of plasma waves with optical ones, whose frequencies are close to the frequency $\omega_{12} = (\epsilon_2 - \epsilon_1)/\hbar$ of the transition between levels having an inverse population. We shall show that at sufficiently low plasma-wave intensity the nonlinear frequency buildup can greatly exceed the linear one.[1)]

When estimating the nonlinear instability, we shall assume for the sake of simplicity that the dielectric constant of the plasma can be described, for frequencies close to ω_{12}, by the formula

$$\varepsilon^t \approx 1 - \omega_{0*}^2(\nu_1 - \nu_2)/(\omega^2 - \omega_{12}^2), \tag{1}$$

where $\omega_{0*}^2 = 4\pi n_* e^2 |f_{12}|/m_e$; n_*—concentration of atoms having a transition frequency ω_{12}; $\nu_1 = n_1/n_*$ and $\nu_2 = n_2/n_*$—relative populations of levels n_1 and n_2; $|f_{12}|$—oscillator strength; n_1, n_2—concentrations of the atoms at levels ϵ_1 and ϵ_2; $\omega - \omega_{12} \gg \gamma$, where γ characterizes the width of the line under consideration.

Assuming that $\omega_{12} \gg \omega_{0e} = (4\pi ne^2/m_e)^{1/2}$ (n—plasma electron concentration), and also that $\omega_{12} \gg \omega_{0*}$, we can readily see that the contribution of the non-ionized atoms in the region of frequencies determining plasma oscillations is negligibly small. Nonlinear interaction of frequencies close to ω_{12} with the plasma frequencies leads to a buildup of the frequencies ω_{12} in the case when the energy of the transverse waves is negative:[2]

$$\frac{\partial}{\partial\omega}\omega^2\varepsilon^t \approx 2\omega\left(1 + \frac{\omega^2(\nu_1 - \nu_2)\omega_{0*}^2}{(\omega^2 - \omega_{12}^2)^2}\right) < 0,$$

which is possible when $\nu_2 > \nu_1$; $\Delta\omega/\omega_{12} = (\omega - \omega_{12})/\omega_{12} \ll 1$, and

$$\Delta\omega < {}^1/_2\omega_{0*}\sqrt{\nu_2 - \nu_1}. \tag{2}$$

The latter condition, by virtue of $\Delta\omega \gg \gamma$ is satisfied for a sufficiently large concentration of the non-ionized atoms; $\omega_{0*} \gg \gamma$.

Let us consider the nonlinear interaction between plasma waves and waves satisfying condition (2). Also participating in this interaction are transverse waves whose frequencies differ from those satisfying Eq. (2) by $\pm\omega_{0e}$. The most effective nonlinear instability occurs when the indicated waves have positive energy, while the frequency of the negative-energy waves (2) is the largest. This is possible for those frequencies which satisfy, besides (2), the condition $\Delta\omega > {}^1/_2\,\omega_{0*}\sqrt{\nu_2 - \nu_1} - \omega_{0e}$. These conditions determine the width of a packet of negative-energy waves, for which the nonlinear instability is effective.

In the statistical description of the wave interaction we can use equations having, in the simplest one-dimensional case and when $\omega_{12} \gg \omega_{0e}$, the approximate form

$$\frac{\partial N_1^t}{\partial t} \approx \frac{\partial N_2^t}{\partial t} = \beta N^l(N_1^t + N_2^t), \tag{3}$$

$$\frac{\partial N^l}{\partial t} \approx \beta\left(\frac{\omega_{12}}{\omega_{0e}}\right)^3 N^l(N_1^t + N_2^t). \tag{4}$$

[1)]The results apply equally well to a heavy-gas plasma, in which part of the ions is in an excited state, and to a partially ionized plasma, in which it is the neutral atoms which are excited. Both linear and nonlinear instabilities can occur in the presence of population inversion (see, for example, [1]), and the development of the instability leads to an equalization of the populations. We consider here the initial stage of development of a nonlinear instability, assuming that the level populations remain unchanged.

where

$$N^l = 2\pi^2 \int dk_\perp |E_k{}^l|^2 \frac{\partial \varepsilon^l}{\partial \omega}\Big|_{\omega=\omega^l(k)},$$

$$N_{1,2}^t = 2\pi^2 \int dk_\perp |E_k{}^t|^2 \frac{1}{\omega^2}\Big| \frac{\partial}{\partial \omega} \omega^2 \varepsilon^t \Big|_{\omega=\omega^t(k)}$$

are the one-dimensional distribution functions of the plasma-wave quanta and of the transverse waves of respectively negative (index 1) and positive (index 2) energies. Equations (3) and (4) can be obtained both by directly averaging over the phases the nonlinear equations describing the interaction between the transverse and longitudinal waves in the plasma, and from simple probability considerations (with account taken of the processes of simultaneous creation and annihilation of three waves). An approximate value for the coefficient β can be obtained when the following inequalities are satisfied

$$\Delta k = k - \frac{\omega_{12}}{c} \ll \frac{\omega_{12}}{c};$$

$$\omega_{0*} \sqrt{\nu_2 - \nu_1} \ll \Delta k \ll \frac{\omega_{0*}{}^2}{\gamma}(\nu_2 - \nu_1),$$

$$\beta = \frac{e^2 \omega_{0e}{}^3 \omega_{0*}{}^2 (\nu_2 - \nu_1)}{32\pi m_e{}^2 \omega_{12}{}^2 (\Delta k)^2}. \tag{5}$$

It follows from (3) and (4) that a nonlinear instability develops more rapidly than exponentially, since the characteristic increment of the instability is a growing function of the time. Solutions (3) and (4) have a structure of the type $N \to \text{const}/(t_0 - t)$ as $t \to t_0$. Under conditions when the initial energy densities of the transverse waves W_{10}^t and W_{20}^t and of the longitudinal waves W_0^l satisfy the inequality $W_{10}^t + W_{20}^t \ll \omega_{12} W_0^l/\omega_{0e}$, the characteristic time t_0 for the development of the instability is estimated at

$$t_0 \approx \frac{4\Delta\omega (\Delta k)^2 c^2}{\omega_{0e}\omega_{12}\omega_{0*}{}^2(\nu_2 - \nu_1)} \frac{n m_e c^2}{W_0{}^l} \ln \frac{W_0{}^l \omega_{12}}{\omega_{0e}(W_{10}{}^t + W_{20}{}^t)}, \tag{6}$$

where $\Delta\omega$ is the width of the spectrum for which effective generation is possible. If Δk is of the order of ω_{0*}/c, then the maximum estimate is

$$t_0 \approx \frac{\Delta\omega}{\omega_{0e}\omega_{12}} \frac{n m_e c^2}{W_0{}^l} \ln \frac{W_0{}^l \omega_{12}}{\omega_{0e}(W_{10}{}^t + W_{20}{}^t)}. \tag{7}$$

At appreciable values of W_0^l and especially small $\Delta\omega$, the times t_0 can be much shorter than the times of the linear instability $t \sim 1/\gamma$. We note that the energy of the generated transverse waves should not exceed some maximum value of the order of

$$W_{max}^t \approx \frac{\omega_{12}}{\omega_{0e}} W_{max}^l$$

$$\approx n m_e c^2 \frac{W_0{}^l}{W_{10}{}^t + W_{20}{}^t} \left(\frac{\Delta\omega}{\omega_{12}}\right)^2 \frac{(\Delta k)^2}{\omega_{0*}{}^2(\nu_2 - \nu_1)}, \tag{8}$$

i.e., W_{max}^t decreases with decreasing $\Delta\omega$.

If $W_0^l > W_{max}^l$, the statistical description of the wave interaction is not applicable. In a dynamic description of the interaction of waves with negative energy it is advantageous to use the methods employed in [3, 4], thus providing an estimate for the characteristic time of the process

$$t_0' = \left(\frac{n m_e c^2}{W_0{}^l}\right)^{1/2} \frac{2\omega_{12}}{\omega_{0e}{}^2} \frac{\Delta k}{\omega_{0*}\sqrt{\nu_2 - \nu_1}} \ln \frac{16 W_0{}^l \omega_{12}}{\omega_{0e}(W_{10}{}^t + W_{20}{}^t)},$$

$$W_0{}^l \gg \frac{\omega_{0e}}{\omega_{12}}(W_{10}{}^t + W_{20}{}^t). \tag{9}$$

As $t \to t_0'$, the solution is of the form $\text{const}/(t_0' - t)^2$. The limits of applicability of the results are connected with the smallness of the saturation effects, which equalize the populations and eliminate the region of negative energy of the waves.

2. Notice should be taken of a number of peculiarities of the foregoing instability mechanism and a number of possible applications of the theory.

First, it must be emphasized that plasma oscillations can be excited relatively rapidly within times much shorter than $1/\gamma$, reaching an order of magnitude ω_{0e}^{-1}. Therefore rapidly excited plasma oscillations can serve as a mechanism for rapid de-excitation of the system, thus increasing the generation power. We note that an increase in the generation power was observed experimentally by Kulagin et al.[5] for a powerful pulsed pinch discharge in a plasma, when an intense excitation of the plasma wave at the instant when the discharge had a singularity could be assumed.

Second, inasmuch as it is difficult for the negative-energy waves to leave the plasma, the frequency of the generated waves should be shifted relative to ω_{12} in the red direction by an amount of the order of ω_{0e}. We note that excitation of frequencies that differ from the transition frequency by an amount of the order ω_{0e} were observed when these frequencies were generated in a semiconductor plasma excited by an electron beam.[6] In the latter case the electron beam can, in accordance with rough estimates, intensely excite plasma oscillations.

Third, attention should be called to possible astrophysical consequences. We note that the anomalously large radiation by the OH molecules at frequencies 1665, 1667, and 1720 MHz[7,8] was recently observed in nebulas in which the existence of plasma oscillations could be assumed.

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