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A MODEL OF WEAK INTERACTIONS BETWEEN BARYONS AND LEPTONS

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A model is considered in which interactions between two fundamental fields—the lepton and the baryon fields—appear as a result of a change in the structure of space-time at small distances. As consequences of the fundamental assumptions of the model one obtains: the universality of weak interactions in Cabibbo form, the selection rule $|\Delta S| \leq 1$ for all the decays and the rule $\Delta S = \Delta Q$ for the leptonic decays of hadrons.

1. INTRODUCTION

ONE of the most interesting and most fundamental properties of weak interactions is, apparently, their universality, which at first manifested itself in the approximate equality of the Fermi vector constant G_V for the β decay of neutrons[1] and the constant G_μ for the decay of muons.[2,3] Later the idea of universality was extended to the weak interactions of other particles[4] and received its most complete form in the paper by Feynman and Gell-Mann.[5] However, it soon became clear that the original idea of the universality of all weak interactions sharply contradicts data on the leptonic decays of hyperons with a change of strangeness.[6] Moreover, further experiments led to the conclusion that the equality of the constants G_V and G_μ is only approximate (cf. the latest review article[7]) while according to the hypothesis of the conservation of vector current[8,5,7] it must hold exactly. Cabibbo[9] has succeeded in overcoming this difficulty by proposing a new formulation of the idea of universality which enabled him to explain both the suppression of hyperon decays, and the lack of agreement between G_V and G_μ by means of a new parameter θ which has received the name of the Cabibbo angle.

Another important property of weak interactions is the nonconservation of parity[10] which in the universal theory of Fermi[5] is described by the V – A interaction.[1)]

Moreover, in all the weak decays of hadrons the following selection rule holds with a high degree of accuracy[11]

$$|\Delta S| \leqslant 1, \tag{1.1}$$

while the leptonic decays of the hadron satisfy the following selection rules with a reasonably good accuracy:

$$|\Delta T_3| = 1 \text{ for } \Delta S = 0,$$
$$\Delta S = \Delta Q, \ |\Delta T_3| = 1/2 \text{ for } \Delta S \neq 0. \tag{1.2}$$

In order to explain these basic properties of the weak interactions various hypotheses were proposed. Thus the universality of the weak interactions (and also the "current" nature of the interaction) can be understood on the basis of the hypothesis about the intermediate vector boson.[12] For the explanation of the V – A interaction considerations of γ_5 invariance were invoked.[13] The selection rules (1.1) and (1.2) can be explained with the aid of composite models of particles (cf., for example, [11]) or by introducing the octet hypothesis of Cabibbo.[9] However, none of these hypotheses unifies all the enumerated properties of weak interactions, and it would be of interest to try to find such a description of weak interactions which would correlate all the above facts from a unified point of view.

The universality of weak interactions and the fact that they contain a small parameter of the dimensions of length $l \sim \sqrt{G/\hbar c} \sim 6 \times 10^{-17}$ cm[14] suggest that the weak interactions might be related to the local curvature of the space-time structure at distances "close to" the particles. In [15], a specific formulation of this idea was proposed as applied to the weak interactions between leptons. We recall the basic assumptions of this model in a somewhat altered and generalized form.[2)]

1) Many facts in the physics of weak interactions agree well with the hypothesis that the interaction can be represented in the form of a product of charged currents [5], but experimental checks of this hypotheses are as yet insufficient.

2) A detailed development of the mathematical formalism is carried out in [15]. Here we consider only those questions which are essential for the formulation of the model proposed below.

We assume that the four-dimensional physical space "near" particles is curved, while at large distances from the particles it becomes flat. The latter condition which will in future be referred to as the "condition of Euclidity at infinity" is quite comprehensible intuitively and in principle enables us to formulate integral laws of energy and momentum conservation. If we were to drop this condition we would encounter difficulties analogous to the difficulties in defining the energy-momentum tensor in the general theory of relativity (cf., for example, [16]).

Further, we assume that the reason for a change in the structure of space is the presence of particles in it. Therefore, quantities which characterize the geometry are associated with certain fundamental fields.[3] First of all we note that the fundamental fields must have spin $1/2$. However, the definition of spinors in curved space is far from being always possible.[17, 18] In [15] in order to solve this problem use was made of the mathematical device of incorporating the physical four-dimensional space V_4 into a many-dimensional pseudoeuclidean space S_m. In this procedure the physical space V_4 is regarded as a certain surface in the space S_m in which one can introduce spinors without difficulty,[17] with the number of components of the simplest spinors for the spaces S_{2n} and S_{2n+1} being equal to 2^n (we shall in future call a spinor in the many-dimensional space a "superspinor"). If in S_{2n} or in S_{2n+1} we pick out a four-dimensional Minkowski subspace, then with respect to the Lorentz transformations in this space a superspinor with 2^n components decomposes into 2^{n-2} ordinary four-component spinors. From this it follows that a superspinor can be used to describe a multiplet consisting of 2^{n-2} particles of spin$^{1/2}$. In order to utilize this possibility we take the values $\psi(x)$ of the superspinor on the surface V_4 and require that the superspinor $\psi(x)$ should satisfy a certain equation on V_4. For such an equation we postulate the simplest generalization of the Dirac equation obtained by replacing the usual derivative $\partial_k\psi$ by the covariant derivative $\psi_{,k}$.

In order to define the covariant derivative it is necessary to utilize in an essential manner the geometry of the space V_4. In order to describe this geometry we define at each point of the space an m-dimensional set of orthogonal unit basis vectors the first four axes of which lie in the space tangential to V_4. The geometry of the surface is locally determined by the conditions for translating this set of basis vectors along V_4. The change in the set of basis vectors in going over to an infinitely close point is determined by the rotation coefficients $\omega_{\alpha\beta k}$ where α and β take on all the m values, while k takes on only the first four values. The rotation coefficients satisfy certain simple geometrical requirements, viz., conditions of symmetry with respect to the indices α, β, k and the conditions of integrability (cf., in greater detail in [15]).

For the definition of the covariant derivative of a spinor use was made in [15] of a generalization of the method proposed in [19] which made it possible to express linearly in terms of the rotation coefficients the spinor connectivity C_l which defines the translation of a spinor. The rotation coefficients in turn can be expressed in terms of bilinear combinations of spinors:

$$\omega_{\alpha\beta k} = a\bar{\psi}(x) B_{\alpha\beta k}\psi(x). \qquad (1.3)$$

Here the matrices $B_{\alpha\beta k}$ are chosen from the algebra of β matrices in m-dimensional space[15, 17] taking into account the conditions of symmetry and the tensor properties of the coefficients $\omega_{\alpha\beta k}$ which, moreover, must satisfy the integrability conditions. We emphasize that the choice of $\omega_{\alpha\beta k}$ in the form (1.3) is the simplest way of giving concrete form to the assumption that space is curved only in the presence of particles.

We now define the covariant derivative in the form

$$\psi_{,k} = \partial_k\psi + C_k\psi, \qquad \bar{\psi}_{,k} = \partial_k\bar{\psi} - \bar{\psi}C_k \qquad (1.4)$$

and with the aid of this derivative we construct the generalized Dirac equation[4] replacing the derivative ∂_k by the expression (1.4) and using instead of the Dirac γ matrices the first four β matrices of the space S_m. For a physical interpretation of this equation we go over to the quasieuclidean approximation, i.e., we consider the space to be flat and retain only terms of order a (cf., (1.3)). Since the connectivity C_k is a linear combination of the coefficients ω contracted with certain matrices, then the generalized Dirac equation contains in the quasieuclidean approximation a nonlinear term decribing the four-fermion interaction. The condition of integrability of the rotation coefficients and the condition of "Euclidity" of the surface V_4 at

[3]Such a point of view is suggested by an analogy with the general theory of relativity which, however, should not be taken too literally. Taking into account that at distances $\sim l$ the gravitational interaction is much smaller than the weak interaction we shall neglect it in subsequent discussion.

[4]We do not discuss the problem of the masses of the particles in this paper and, therefore, we do not consider the generalization of the mass term.

infinity impose essential limitations on the form of this four-fermion interaction. In particular, in [15] it was shown that these conditions necessarily lead to nonconservation of parity, and one can also introduce certain considerations favoring the V + A interactions.

It is important to note that in defining the spinor connectivity C_k in terms of the rotation coefficients there is a certain degree of arbitrariness to which we shall assign a definite physical meaning. Specifically, without changing the geometry it is always possible to add to the spinor connectivity an expression of the form iRB_k where B_k is an arbitrary real vector, while the matrix R commutes with all the matrices β_α contained in (1.3). In the simplest case one can set R = I. in the quasieuclidean approximation the arbitrariness indicated above can be associated with gauge invariance if one chooses the arbitrary vector B_k in the form $B_k = \partial_k\varphi$. In certain papers[19] the vector B_k was associated with an actually existing field, for example the electromagnetic field. However, in order to interpret this vector in precisely this manner one must impose upon it certain auxiliary conditions (Maxwell's equations!)[5] which by no means follow from the geometry. It appears to us to be more consistent to extend the geometric point of view to all the actually existing fields. Therefore, we shall interpret the arbitrariness under discussion specifically as an arbitrariness of gauge. (For us it is important that the gauge invariance mentioned above should guarantee the conservation of a certain quantity; in particular the eigenvalues of the matrix R define the conserved charge.)

On the other hand the actually existing vector fields, and in the first instance the electromagnetic field, are naturally associated with the geometry of space-time and can be introduced also through the rotation coefficients. (Such a point of view is naturally not new and relates back to the ideas of the unified field theories of Weyl, Eddington,[21] and Einstein.[22] In this connection an essential modification of the apparatus proposed in [15] could be required.[6] However, in the present paper we restrict ourselves to an investigation of only the quasieuclidean approximation in which only the effective Lagrangians of the weak and the electromagnetic interactions are taken into account, and this modification will not lead to any changes.

In concluding this section we shall make a few remarks on the choice of the embedding space. In differential geometry the fact is known that a four dimensional Riemann space with a given metric can be locally embedded into a ten dimensional space. In special cases the dimensionality of the embedding space can also be smaller. Since in our case the metric is not given but only the equations and the conditions of integrability are given, there are no considerations available for the choice of the dimensionality of the embedding space, except for correspondence with physical reality and, first of all, with the number of components of the different spinor multiplets. Combining 2^{n-2} particles into one multiplet we take S_{2n+1} as the embedding space. We note that in the proposed scheme for the weak interactions the fundamental spinor field cannot be a triplet (of quarks, trions, etc.).

2. WEAK INTERACTIONS OF LEPTONS

We first of all consider the weak and the electromagnetic properties of leptons.[7] Unifying the four leptons (e^-, ν_e, μ^-, ν_μ) into one superspinor we take a 9-dimensional embedding space. In the quasieuclidean approximation the effective Lagrangian is[15]

$$L_{int} = \frac{G}{4\sqrt{2}}\sum_{\alpha,\beta,k}(\bar{\psi}\gamma_{\alpha\beta}O_k\psi)(\bar{\psi}\gamma_{\alpha\beta}O_k\psi). \qquad (2.1)$$

Here the summation over k refers to the four coordinates of physical space, while α and β are the numbers of excess axes which we shall number in order 0, 1, 2, 3, 4 (we shall not explicitly write out the numbers k). It is usually customary to call the space formed by the 0, ... 4 axes the "inner" space, and we shall also utilize this terminology. In order to obtain this Lagrangian from the Lagrangian (4.1) in [15] one must choose such a representation for the matrices in (4.1) in which they decompose into the direct product of four-rowed matrices of the physical space and the four-rowed matrices of the inner space.[8] In this representation

$$O_n = \tilde{\gamma}_n(1+\tilde{\gamma}_5); \qquad (2.2)$$

$$\gamma_{\alpha\beta} = \frac{1}{2i}[\gamma_\alpha, \gamma_\beta], \qquad (2.3)$$

[5]A similar problem has been investigated by Ogievetskii and Polubarinov[20].

[6]In particular, the space can become twisted.

[7]We note that in the present work we do not set ourselves the aim of carrying out a complete geometrical investigation of the problems that arise. Instead of that we basically use qualitative geometric considerations, and also certain limitations arising from experiment.

[8]In this case the superspinor is represented in the form of a four-component column (ψ_1, ψ_2, ψ_3, ψ_4), each component ψ_i of which is an ordinary Dirac spinor.

where $\tilde{\gamma}_n$, $\tilde{\gamma}_5$ are the usual Dirac matrices operating on usual spinors[9] while γ_α are the four-rowed matrices corresponding to spinors in the five-dimensional Euclidean inner space operating on particles as a whole. The matrices γ_α are defined by the relation

$$\gamma_\alpha\gamma_\beta + \gamma_\beta\gamma_\alpha = 2\delta_{\alpha\beta}; \quad \alpha, \beta = 0, 1, \ldots, 4, \quad (2.4)$$

and we shall utilize the following Hermitian representation:

$$\gamma_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix}, \quad k = 1, 2, 3; \quad \gamma_4 = \begin{pmatrix} 0 & -iI \\ iI & 0 \end{pmatrix};$$

$$\gamma_0 \equiv \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}. \quad (2.5)$$

The interaction (2.1) possesses a high degree of internal symmetry described by the group of five-dimensional rotations R(5). In the algebra of this group there exist two mutually commuting generators whose eigenvalues can uniquely characterize particles. It is natural to associate one of these generators with the electric charge. The other, evidently, can describe the muonic charge.[10] Both these charges are conserved in all the interactions. However, the mechanisms guaranteeing the conservation of these charges have an essentially different appearance. The conservation of electric charge is guaranteed by the very existence of the electromagnetic field. An analogous field associated with the muonic charge is unknown[11] and, therefore, it is reasonable to suppose that the conservation of muonic charge is guaranteed by the arbitrariness of gauge discussed in the Introduction. Moreover, the eigenvalues of the matrix must give the muonic charge of the leptons. For example, let us take for R the matrix

$$F_L = \tfrac{1}{2}(I + \gamma_{34}), \quad (2.6)$$

where the unit matrix is added from considerations of correspondence with the usual values of the muonic charge which are equal to zero or unity. In order that there should in fact be arbitrariness it is necessary that the matrix F_L should commute with all the matrices γ_α appearing in the rotation coefficients. From this it follows that one must exclude from them the matrices $\Gamma_{\alpha 3}$ and $\Gamma_{\alpha 4}$ ($\alpha \neq 3, 4$). Therefore, in the Lagrangian (2.1) there will be no terms containing these matrices. We note here that the Lagrangian (2.1) by itself is so symmetric that it conserves both the electric and the muonic charges and, therefore, the requirement of arbitrariness of gauge may turn out to be superfluous. However, if one includes the interaction of leptons with baryons, then without this requirement the muonic charge will not be conserved in baryon decays. Thus, the breaking of the symmetry introduced above appears to be inevitable.

In the representation under consideration for the γ matrices the electric charge is determined in the following manner:

$$Q_L = \tfrac{1}{2}(\gamma_{12} - I). \quad (2.7)$$

It is now easy to establish correspondence between the spinor components of ψ and the known leptons:

$$\psi = (\nu_\mu, e^-, \nu_e, \mu^-). \quad (2.8)$$

Until now we have nowhere taken into account the requirement of the anticommutativity of the spinors ψ since the whole theory was being constructed essentially in terms of the language of classical and not quantum concepts. From considerations of correspondence between the classical and the quantum pictures we now require that the Lagrangian (2.1) should describe the same interactions both in the case of commuting and in the case of anticommuting spinors. Then utilizing the commutation relations due to Fierz[24] it can be easily verified that the Lagrangian (2.1) must not contain terms involving the matrices Γ_{12} and Γ_{34}, since these terms are essentially different for the commuting and the anticommuting spinors.

On the basis of these considerations we retain in the Lagrangian (2.1) summation only over the pairs of indices (01) and (02) and arrive at the following Lagrangian for the weak interactions between leptons:

$$L_{int} = \frac{G}{4\sqrt{2}} \sum_{\alpha=1,2} (\bar{\psi}\gamma_{0\alpha}\psi)(\bar{\psi}\gamma_{0\alpha}\psi), \quad (2.9)$$

which can be rewritten in the form

$$L_{int} = \frac{G}{\sqrt{2}} (\bar{\nu}_\mu\mu - \bar{\nu}_e e)(\bar{\mu}\nu_\mu - \bar{e}\nu_e). \quad (2.10)$$

This Lagrangian coincides with the Lagrangian due to Feynman and Gell-Mann[5] and differs from the Lagrangian in [15] which does not reduce to a product of currents and which contains "neutral"

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[9] In future we shall sometimes omit the matrices O_n retaining only the inner structure and having in mind the V-A variant.

[10] In our scheme the conservation of lepton charge is guaranteed by the general gauge transformation $\psi \to e^{i\alpha}\psi$, $\bar{\psi} \to e^{-i\alpha}\bar{\psi}$, which corresponds to the always possible choice R = I.

[11] The interaction of μ and ν_μ with a hypothetical neutral vector field has been repeatedly discussed in the literature[23]. With the passage of time the upper bound for the coupling constant continually diminishes, while the lower bound for the meson mass increases.

interactions $(\bar{\nu}_\mu \nu_\mu)(\bar{e}e)(\bar{e}e)$ etc. Such interactions have been excluded by the requirement of correspondence with the classical picture.

3. THE BARYON FIELD

The next step consists of including baryons in our scheme. As we have already noted in the Introduction the scheme does not admit fundamental triplets of particles; this immediately forces us to drop the idea of a minimum number of basic fields. Therefore, in introducing baryons we can start either with four fundamental particles, or with eight. In order to have a possibility of making the fields correspond to actually existing baryons whose number is equal to eight, it is sensible to start with the second possibility (cf., for example, with [25]). This corresponds to choosing the dimensionality of the embedding space m = 11, i.e., to a dimensionality of the ''inner'' space equal to 7.

We shall construct a baryon theory by analogy with leptons. We will not be able to obtain here a complete picture of baryon interactions, since we leave aside the effect of strong interactions which according to our ideas are not associated with local space curvature. Therefore, we, in particular, retain for the baryons also a pure V − A variant; moreover, we shall not claim to explain certain facts for which the effect of strong interactions is essential.[12)]

Thus, we introduce for the description of the baryons a second fundamental field which represents an eight component spinor Ψ of the seven-dimensional ''inner'' Euclidean space.[13)] We shall also construct the corresponding algebra of eight-rowed matrices Γ_μ satisfying the relations

$$\Gamma_\mu\Gamma_\nu + \Gamma_\nu\Gamma_\mu = 2\delta_{\mu\nu}, \quad \mu, \nu = 0, 1, \ldots, 6. \qquad (3.1)$$

In future we shall utilize the following Hermitian representation for these matrices:

$$\Gamma_0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}; \quad \Gamma_\alpha = \begin{pmatrix} 0 & \gamma_\alpha \\ \gamma_\alpha & 0 \end{pmatrix}, \quad \alpha = 1, \ldots, 5;$$
$$\Gamma_6 = i\begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}. \qquad (3.2)$$

The commutators

$$\Gamma_{\mu\nu} = \frac{1}{2i}[\Gamma_\mu, \Gamma_\nu] \qquad (3.3)$$

form a set of generators of the group R(7).

Now the rotation coefficients are determined both by the bilinear combinations of the lepton spinors and also by the bilinear combinations of the baryon spinors, i.e., by expressions of the form

$$\omega_{\alpha\beta k} = a\bar{\psi}B_{\alpha\beta k}\psi + b\bar{\Psi}\bar{B}_{\alpha\beta k}\Psi, \qquad (3.4)$$

where the matrices $\bar{B}_{\alpha\beta k}$ are constructed from the matrices Γ_α in just the same way as the matrices $B_{\alpha\beta k}$ are constructed from γ_α. For the convenience of presentation we introduce the operator

$$P = {}^1/_2(I + \Gamma_{56}) \qquad (3.5)$$

which projects the baryon algebra on the lepton algebra. In particular, we have the following relations:

$$P\Gamma_{\alpha\beta} = \gamma_{\alpha\beta}, \quad \alpha, \beta = 0, \ldots, 4; \quad P\Gamma_{56} = I. \qquad (3.6)$$

Expression (3.4) leads in the quasieuclidean approximation to the covariant derivatives of the spinors ψ and Ψ (cf., [15]):

$$\partial_n\psi + \frac{i}{4}\sum_{\alpha,\beta}\gamma_{\alpha\beta}(1+\tilde{\gamma}_5)\{a\bar{\psi}\gamma_{\alpha\beta}O_n\psi + b\bar{\Psi}\Gamma_{\alpha\beta}{}'O_n\Psi\}\psi;$$
$$\partial_n\Psi + \frac{i}{4}\sum_{\alpha,\beta}\Gamma_{\alpha\beta}{}'(1+\tilde{\gamma}_5)\{a\bar{\psi}\gamma_{\alpha\beta}O_n\psi + b\bar{\Psi}\Gamma_{\alpha\beta}{}'O_n\Psi\}\Psi. \qquad (3.7)$$

(The term contained in brackets in this formula is simply the rotation coefficient of the form (3.4).) Here the representation of the matrices Γ'_α, generally speaking, does not coincide with the representation (3.2). The reason for this will become apparent later. In order to have the possibility of formulating conservation laws (energy, momentum, etc.) we require that the equations obtained utilizing the covariant derivatives (3.7) should follow from a single Lagrangian. It can be easily verified that this requirement leads to the condition of universality:

$$a = b = \sqrt{2}G, \qquad (3.8)$$

with the summation in (3.7) necessarily extending for baryons only to those values of α and β which appear in the leptonic Lagrangian (2.9). Thus, we arrive at the following Lagrangian for the weak interactions between baryons and leptons:

$$L_{int} = \frac{G}{4\sqrt{2}}\sum_{\alpha=1,2}\{\bar{\Psi}\Gamma_{0\alpha}{}'\Psi + \bar{\psi}\gamma_{0\alpha}\psi\}\{\bar{\Psi}\Gamma_{0\alpha}{}'\Psi + \bar{\psi}\gamma_{0\alpha}\psi\}. \qquad (3.9)$$

In order to establish the correspondence between the eight components of the spinor Ψ and the eight baryons p, n, Λ, Σ^+, Σ^-, Σ^0, Ξ^-, Ξ^0, we

12)For example, the rule $|\Delta T| = 1/2$ in nonleptonic decays of hadrons, (V,A) structure of baryon currents etc.

13)Just as in the case of leptons each component of superspinor is a four-component Dirac spinor.

construct operators whose eigenvalues enable us to distinguish between these particles. In the algebra R(7) there are three mutually commuting generators. We take for such generators the matrices Γ_{12}, Γ_{34}, Γ_{56}, which are diagonal in the representation (3.2). We first find the operator for the electric charge Q_B which must be related to the leptonic operator for the electric charge Q_L by the projection relation:

$$PQ_B = Q_L = {}^1/_2(\gamma_{12} - I). \tag{3.10}$$

It is easy to verify that the operator Q_B must have the form

$$Q_B = {}^1/_2(\Gamma_{12} - \Gamma_{56}) + c(I - \Gamma_{56}). \tag{3.11}$$

In order to eliminate the ambiguity existing here we discuss the geometric meaning of the electromagnetic interactions. As we have already noted the electromagnetic interaction must appear in the rotation coefficients. This can be easily done for c = 0. Indeed, we include in the rotation coefficients the terms

$$e[\overset{12}{\varepsilon}_{\alpha\beta} - \overset{56}{\varepsilon}_{\alpha\beta}]A_k, \tag{3.12}$$

where A_k are the components of the electromagnetic potential in an orthogonal system of basis vectors, while $\epsilon^{\alpha'\beta'}_{\alpha\beta}$ is the completely antisymmetric tensor of the two dimensional space defined by the axes α' and β'. Then in the effective interaction Lagrangian the following term will arise for the electromagnetic interaction

$$e(\overline{\Psi}, \widetilde{\gamma}_k Q_B \Psi + \overline{\psi}\widetilde{\gamma}_k Q_L \psi) A_k. \tag{3.13}$$

Noting that for $c \neq 0$ a similar geometric interpretation of the charge operator is impossible we arrive at the conclusion that

$$Q_B = {}^1/_2(\Gamma_{12} - \Gamma_{56}). \tag{3.14}$$

We note that the eigenvalues of this operator are equal to +1, +1, −1, −1, 0, 0, 0, 0, as required (for $c \neq 0$ one does not obtain the correct eigenvalues!).

For a further identification of the particles we define a second quantum number as the operator for the hypercharge Y. We assume that the representation (3.2) is chosen in such a way that the operator Y is also diagonal in it. Then one can assume[14)]

$$Y = -{}^1/_2(\Gamma_{34} + \Gamma_{56}). \tag{3.15}$$

It is easy to verify that this operator has the same eigenvalues as Q_B and that the Gell-Mann–Nishijima relation

$$Q - Y/2 = T_3, \tag{3.16}$$

is satisfied, i.e., the difference of the matrices on the left hand side of formula (3.16) gives the correct distribution of the eigenvalues of the third component T_3 of the isotopic spin for baryons.

Using any pair of the three operators (3.14)–(3.16) we obtain the following distribution of particles in the superspinor:

$$\Psi = (\Xi^0, \Sigma^-, p, Z^0, \Sigma^+, n, Y^0, \Xi^-), \tag{3.17}$$

where Z^0 and Y^0 are certain linearly independent combinations of the particles Λ and Σ^0. In order to find the form of these combinations it is necessary to know the operator for the square of the isotopic spin. Making the additional assumption that among the generators $\Gamma_{\mu\nu}$ there are included the operators for the isotopic spin T_+ and T_- for which

$$[T_+, T_-] = 2T_3, \quad [T_3, T_\pm] = \pm T_3, \tag{3.18}$$

we can find the general form of the operators (analogous expressions were obtained in [25]):

$$T_+ = {}^1/_2 e^{i\varphi_1}(\Gamma_{20} + i\Gamma_{01}) + {}^1/_4 e^{i\varphi_2}[(\Gamma_{36} - \Gamma_{45}) + i(\Gamma_{35} - \Gamma_{64})]; \tag{3.19}$$

$T_- = (T_+)^+$, φ_1, and φ_2 are real. Evaluating the operator T^2 it can be easily verified that it contains in addition to diagonal terms also nondiagonal elements mixing Z^0 and Y^0. Diagonalizing T^2 we obtain up to a nonessential phase factor that[15)]

$$Z^0 = (\Lambda + \Sigma^0)/\sqrt{2}, \quad Y^0 = (\Lambda - \Sigma^0)/\sqrt{2}.$$

The operator introduced above cannot be regarded as completely analogous to the operator for the muonic charge F_L (although it is related to it by a simple projection relation) since the hypercharge is not conserved in weak interactions. We, therefore, define for the baryons a new quantum number[16)]

$$F_B = {}^1/_2(\Gamma_{34} + I), \tag{3.20}$$

which carries over to the baryons the concept of the muonic charge and is in form identical with ex-

14)The obvious ambiguity in the choice of this operator reduces to a renumbering of the particles.

15)This notation thus corresponds to the notation given in the doublet theory of Gell-Mann and Pais [26].

16)Defining this charge by the requirement $PF_B = F_L$ we obtain its general form: $F_B = ½(\Gamma_{34} + I) + c(I - \Gamma_{56})$. However, below we shall go over to a representation in which the matrix Γ_{56} is nondiagonal and, therefore, we at the outset choose c = 0.

pressions (2.6). Just as F_L, the charge F_B is associated with gauge transformations which guarantee the conservation of F_B. Since gauge transformations can be carried out separately for baryons and for leptons the charges F_B and F_L are separately conserved. This property of the charges F_B and F_L essentially differentiates them from the electric charge which is conserved for the totality of all the particles. The reason for this is the difference in the mechanisms discussed above guaranteeing the conservation of these charges.

4. WEAK INTERACTIONS BETWEEN BARYONS AND LEPTONS

In the representation (3.2) utilized until now the operators for the charge, the hypercharge and the F_B-charge are diagonal. Carrying out the baryon-lepton analogy in a consistent manner it is natural to require only the conservation of the charges Q and F. But the operator for the hypercharge introduced only for the classification of the particles is not associated with geometrical concepts and need not be conserved (nor be diagonal). We obtain the most general transformation of the representation (3.2) which conserves the form of the operators for the charges Q_B and F_B.

In order to simplify the calculations we consider the infinitesimal unitary transformations of the matrices Γ_μ:

$$\Gamma_\mu' = U^+\Gamma_\mu U, \quad U = 1 - i\varepsilon\Phi, \quad \Phi^+ = \Phi, \tag{4.1}$$

where ϵ is a real infinitesimal number and Φ is a linear combination of the generators $\Gamma_{\mu\nu}$. The operators for the charges Q and F transform in the following manner:

$$Q' = U^+QU = Q + {}^1/_2 i\varepsilon\{[\Phi, \Gamma_{12}] - [\Phi, \Gamma_{56}]\} + O(\varepsilon^2);$$

$$F' = U^+FU = F + {}^1/_2 i\varepsilon[\Phi, \Gamma_{34}] + O(\varepsilon^2). \tag{4.2}$$

Therefore, the conditions for the conservation of the operators Q and F are

$$[\Phi, \Gamma_{12}] = [\Phi, \Gamma_{56}], \quad [\Phi, \Gamma_{34}] = 0. \tag{4.3}$$

It can be easily verified that the operator

$$\Phi = A(\Gamma_{15} + \Gamma_{62}) + B(\Gamma_{16} + \Gamma_{25}) + C_1\Gamma_{12} + C_2\Gamma_{34} + C_3\Gamma_{56} \tag{4.4}$$

satisfies all the conditions (4.3) with A, B, C being arbitrary constants. The last three terms in expression (4.4) describe nonessential transformations since they commute with all three diagonal operators. We shall use only one of them in order to simplify the nontrivial transformation which is described by the first two terms. In order to do this we carry out in addition to the infinitesimal transformation with the generator

$$\Phi = A(\Gamma_{15} + \Gamma_{62}) + B(\Gamma_{16} + \Gamma_{25}), \tag{4.5}$$

also a finite rotation in the (5, 6) plane. Corresponding to this the generator Φ is transformed into the generator

$$\begin{aligned}\Phi' &= e^{-i\alpha\Gamma_{56}}\{A(\Gamma_{15} + \Gamma_{62}) + B(\Gamma_{16} + \Gamma_{25})\}e^{i\alpha\Gamma_{56}} \\ &= (A\cos 2\alpha - B\sin 2\alpha)(\Gamma_{15} + \Gamma_{62}) \\ &\quad + (A\sin 2\alpha + B\cos 2\alpha)(\Gamma_{16} + \Gamma_{25}).\end{aligned} \tag{4.6}$$

Choosing $\tan 2\alpha = -B/A$ we obtain

$$\Phi' = \pm\sqrt{A^2 + B^2}(\Gamma_{15} + \Gamma_{62}).$$

Going over from infinitesimal transformations to finite ones we obtain the theorem: apart from rotations through finite angles in the (1, 2), (3, 4) and (5, 6) planes the most general unitary transformation which leaves the charges Q_B and F_B invariant can be written in the form

$$\Gamma_\mu' = U_\theta^+\Gamma_\mu U_\theta, \tag{4.7}$$

where

$$U_\theta = \exp\left[i\frac{\theta}{2}(\Gamma_{15} + \Gamma_{62})\right].$$

This transformation can be easily written in the following form (we shall call it the Cabibbo transformation[17]):

$$\Gamma_1' = \cos\theta\,\Gamma_1 + \sin\theta\Gamma_5, \qquad \Gamma_5' = -\sin\theta\Gamma_1 + \cos\theta\Gamma_5,$$

$$\Gamma_2' = \cos\theta\Gamma_2 - \sin\theta\Gamma_6, \qquad \Gamma_6' = \sin\theta\Gamma_2 + \cos\theta\Gamma_6, \tag{4.8}$$

while the remaining matrices are not transformed. This transformation amounts to a rotation through an angle θ in the (1, 5) plane and through an angle $-\theta$ in the (2, 6) plane.

In the new transformation the charges Q and F remain diagonal, while the operator for the hypercharge for $\theta \neq n\pi$, where n is an integer, is nondiagonal and, therefore, for $Q \neq n\pi$ the hypercharge is not conserved in weak interactions. We shall regard the angle θ as arbitrary. Since in any representation there must be three diagonal operators the question arises as to how one should construct in the Cabibbo representation a diagonal operator linearly independent of Q_B and F_B. Using the transformation (4.8) and the definition of the matrices $\Gamma_{\mu\nu}$ it can be easily verified that the matrix

$$^1/_2\{I + \Gamma_{12}'\cos^2\theta - \Gamma_{56}'\sin^2\theta + {}^1/_2(\Gamma_{16}' + \Gamma_{25}')\sin 2\theta\} \tag{4.9}$$

[17]Subsequently it will become clear that θ is the angle introduced by Cabibbo.

is diagonal and in its form coincides with $\frac{1}{2}(I + \Gamma_{12})$.

Thus, in the Lagrangian (3.9) we have introduced a specific representation of the matrices Γ'_μ and we now write it in the final form:

$$L_{int} = \frac{G}{\sqrt{2}}(J_B + J_L)(J_B{}^+ + J_L{}^+) \qquad (4.10)$$

(summation over the spatial indices and the matrices O_n are implied), where

$$J_B = \cos\theta[\bar{\Xi}^0\Xi^- + \bar{p}n - \bar{Y}^0\Sigma^- - \bar{\Sigma}^+Z^0] - \sin\theta[\bar{p}Y^0 + \bar{Z}^0\Xi^- + \bar{\Sigma}^+\Xi^0 + \bar{n}\Sigma^-], \quad J_L = (\bar{\nu}_\mu\mu - \bar{\nu}_e e). \qquad (4.11)$$

This Lagrangian for the weak interactions between four leptons and eight baryons has the form of a product of charged currents and guarantees the selection rules (1.1) and (1.2). Moreover, the current conserving the hypercharge appears with the factor $\cos\theta$ while the current which alters the hypercharge appears with the factor $\sin\theta$, i.e., we have obtained the universality of weak interactions in the form proposed by Cabibbo.[9] The selection rules (1.1) and (1.2) and the universality modified in this manner agree well with the experimental data.[27]

We note that in the vector part of the current (4.11) the terms $\bar{\Lambda}\Sigma^-$ and $\bar{\Sigma}^+\Lambda$ appear which are usually excluded by the hypothesis regarding the conservation of vector currents.[28] Experimental data do not allow us as yet to determine the V, A structure in the corresponding decays.[27] An experimental determination of this structure is very important for checking the proposed scheme (the corresponding experiments are proposed in [28]). The present theory can lay no claims to explain the finer details of weak interactions essentially associated with the effects of strong interactions.

5. CONCLUSION

It is important to note that the selection rules (1.1) and (1.2) follow from the conservation of the charge F_B and from the rule $|\Delta Q| = 1$. Indeed, it can be easily seen that the charge F_B divides all the baryons into two quartets:

$$F_B = 0:\ \Sigma^-,\ p,\ n,\ Y^0 = (\Lambda - \Sigma^0)/\sqrt{2}; \qquad (5.1)$$

$$F_B = 1:\ \Sigma^+,\ \Xi^-,\ \Xi^0,\ Z^0 = (\Lambda + \Sigma^0)/\sqrt{2}, \qquad (5.2)$$

and one can directly verify that the selection rules (1.1) and (1.2) are satisfied. Thus, a simple generalization of the law of conservation of the muonic charge, which in this scheme is automatically extended to baryons, enables one to obtain all the selection rules. Until now these rules were explained either within the framework of the hypothesis regarding the fundamental triplet of fields ("sakatons," quarks, trions etc.[11]), or by means of the hypothesis that the "weak" hadron current belongs to the octet of the SU_3 group.[9] Quarks cannot be included in our scheme in a natural manner, while the classification of currents according to the representations of the SU_3 group is not considered at all. Therefore, a new point of view arises regarding the origin of the selection rules in weak interactions.[18)] It is interesting to note that from this point of view one can have a new interpretation of the origin of the Cabibbo angle θ which appears naturally without any additional hypotheses. We also recall that the Lagrangian for weak interactions was obtained in the form of a product of charged currents.

In the present paper we have restricted ourselves to a discussion of weak and electromagnetic interactions in the quasieuclidean approximation. All the conservation laws obtained in this approximation (selection rules, CP conservation etc.) are valid in any order of perturbation with respect to the effective interaction Lagrangian. However, already for terms of order Ge not only the higher order approximations of perturbation theory are essential, but also the geometrical effects which we have not considered here. There is no basis for expecting that the conservation laws obtained above must also hold when higher geometrical effects are taken into account. In particular, it is quite possible that in the order Ge both CP-invariance and the selection rules (1.1) and (1.2) are violated, and, therefore, one can assert that the conservation laws found above are satisfied, generally speaking, with an accuracy of 1%. It is tempting to relate these remarks to the observed effects of nonconservation of CP[29] [19)] and to the indications of an observation of decays with $\Delta Q = -\Delta S$.[3] However, in order to make more definite predictions it is necessary to investigate further the effects due to the curvature of space-time.

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[1] E. Fermi, Nuovo Cimento **11**, 1 (1934).
[2] M. Conversi, M. Pancini, and O. Piccioni,

18)It is appropriate to note that as soon as the quantum number F has been introduced and its definite values (5.1) and (5.2) have been ascribed to the baryons the selection rules (1.1) and (1.2) can be obtained without any reference at all to the geometric nature of the charge F.

19)In an earlier paper [30] we have made such an attempt.

Phys. Rev. 71, 209 (1947). B. Pontecorvo. Phys. Rev., 72, 246, (1947).

[3] O. Klein, Nature, 161, 897, (1948); G. Puppi, Nuovo Cimento 5, 587, (1948).

[4] N. Dallaporta, Nuovo Cimento 1, 962, (1955).

[5] R. P. Feynman and M. Gell-Mann, Phys. Rev., 11, 362, (1958).

[6] R. H. Dalitz, Rev. Mod. Phys., 31, 823, (1955).

[7] C. S. Wu, Rev. Mod. Phys., 36, 618, (1964).

[8] S. S. Gershteĭn and Ya. B. Zel'dovich, JETP 29, 698 (1955), Soviet Phys. JETP 2, 576 (1956).

[9] N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[10] T. D. Lee, C. N. Yang, Phys. Rev., 104, 254, (1956).

[11] L. B. Okun', Slaboe vzaimodeĭstvie elementarnykh chastits (Weak Interaction Between Elementary Particles), Fizmatgiz, 1963.

[12] H. Yukawa, Proc. Phys. Math. Soc. Japan, 17, 48, (1935). J. Schwinger, Ann. Phys. (N. Y.) 2, 407 (1957).

[13] A. Salam, Nuovo Cimento 5, 209, (1957). E. C. G. Sudarshan and R. E. Marshak, Phys. Rev. 109, 1860, (1958).

[14] W. Heisenberg, Z. Physik 101, 533, (1936).

[15] B. A. Arbuzov, JETP 46, 1285 (1964), Soviet Phys. JETP 19, 872 (1964).

[16] V. A. Fock, Teoriya prostranstva, vremeni i tyagoteniya (Theory of Space, Time and Gravitation), Fizmatgiz, 1961.

[17] E. Cartan, Theory of Spinors (Russ. Transl. IIL, 1957).

[18] A. Lichnerowicz, Ann. Inst. Henri Poincaré, Al, 1964, p. 233.

[19] V. A. Fock, D. D. Ivanenko, Compt. Rend. 188, 1470, (1929); Physik Z. 30, 648, (1929), V. A. Fock, Z. Physik 57, 261, (1929).

[20] V. I. Ogievetskiĭ and I. V. Polubarinov, Ann. Phys. (N. Y.) 25, 358 (1963).

[21] A. S. Eddington, The Mathematical Theory of Relativity (Russ. Transl. DNTVU, Khar'kov, 1933).

[22] A. Einstein, The Meaning of Relativity (Russ. Transl. IIL, 1955).

[23] L. B. Okun' and I. Yu. Kobzarev, JETP 41, 1205 (1961), Soviet Phys. JETP 14, 859 (1962).

[24] M. Fierz, Z. Physik 102, 572, (1936).

[25] F. Gursey, Ann. Phys. (N. Y.) 12, 91, (1961).

[26] M. Gell-Mann, Phys. Rev., 106, 1296, (1957); A. Pais, Phys. Rev., 110, 574, (1958).

[27] R. Dalitz, Properties of the Weak Interactions (cf. collection of articles ''Poluprostye gruppy i sistematika elementarnykh chastits'' (Semisimple Groups and the Systematics of Elementary Particles), vol. II, Dubna, 1965).

[28] N. Cabibbo and R. Gatto, Nuovo Cimento 15, 304, (1960). N. Cabibbo and P. Franzini, Phys. Letters 3, 217 (1963).

[29] J. H. Christenson et al. Phys. Rev. Letters 13, 138, (1964).

[30] B. A. Arbuzov, A. T. Filippov, Phys. Letters 20, 537, (1966).

[31] A. Barbaro-Galtieri et al. Phys. Rev. Letters 9, 26, (1962).

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168