SPIN-LATTICE RELAXATION OF WEAKLY INHOMOGENEOUS CONDUCTION-ELECTRON DISTRIBUTIONS IN A STRONG MAGNETIC FIELD

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The longitudinal relaxation lifetimes for spin magnetization of carriers in a quantizing magnetic field are calculated in the Born scattering approximation. The transverse diffusion coefficients of spatially inhomogeneous magnetic-moment distributions are also determined. All conduction electron spin-lattice relaxation mechanisms of any significance in conducting crystals are considered. A simple relation is established between the spin-diffusion coefficient and dissipative part of the transverse electric conductivity coefficient for strong magnetic fields. A concrete calculation is carried out of the spin-lattice relaxation time for carriers interacting with magnetic impurities in the quantum limit.

1. The spin-lattice relaxation of carriers in conducting crystals in weak magnetic fields has been investigated quite thoroughly. At the present time, however, there is no theory of this phenomenon suitable for strong "quantizing" magnetic fields. Qualitative estimates of the expected temperature and field dependence of the spin-lattice relaxation time when scattering by acoustic phonons in the quantum limits are given in the review of Yafet[1]. Yet it is precisely the region of quantizing magnetic fields which is of greatest interest for the study of different resonance effects in the conduction-electron system of a solid. An important role is played in these effects by the diffusion of the spin magnetization, which calls for an account of spatial inhomogeneities in the distribution of the density of the magnetic moment of the carrier[2]. In the limit of strong magnetic fields in which we are interested $\omega_0\tau \gg 1$ ($\omega_0 = eH/mc$ is the cyclotron frequency and τ is the average momentum scattering time) and weak spatial inhomogeneities (Larmor radius r_L much smaller than the characteristic dimensions of the inhomogeneities L in the particle distribution), it is possible to neglect the difference between the coordinates of the electron and the coordinates of the center of its cyclotron orbit. Then the diffusion processes in the conduction-electron system reduce to the diffusion of the centers of the cyclotron orbits. This approximation turned out to be quite fruitful both in the study of transport phenomena in solids and in a magnetized plasma[3].

In this paper we study the spin-lattice relaxation and the transverse diffusion of the longitudinal component of spin magnetization of the conduction electrons in a strong magnetic field (00H). We obtain general expressions for the spin-lattice relaxation time and the coefficient of spin diffusion in the Born approximation in the scattering; all the main types of spin-lattice interactions that are essential for the carriers in the conducting crystals are taken into account.

2. The kinetic equation for a weakly inhomogeneous nonequilibrium system of conduction electrons interacting in a strong magnetic field with different branches of the vibrational spectrum of the lattice, with static elastically-scattering impurities or defects, and also with magnetic impurities can be written in the form

$$\frac{\partial}{\partial t} f_{np_z\sigma}(x, y) = \frac{2\pi}{\hbar^2} \sum_{n'\sigma'} \int_{-\infty} \frac{dp_z}{(2\pi\alpha)^2} \int \frac{d\mathbf{q}}{(2\pi)^3}$$

$$\times \left[1 - P_{n'\sigma', n\sigma} \exp\left\{X\partial_x - Y\partial_y - \hbar q_z \frac{\partial}{\partial p_z}\right\}\right]$$

$$\times \left\{\left| V_{sph}\begin{pmatrix} p_z + \hbar q_z & n'\sigma' & x - X & y + Y \\ p_z & n\sigma & x & y \end{pmatrix}\right|^2\right.$$

$$\times [(N_q + 1) f_{n'p_z+\hbar q_z\sigma'}(x - X, y + Y) [1 - f_{np_z\sigma}(x, Y)]$$

$$- N_q f_{np_z\sigma}(x, y) [1 - f_{n'p_z+\hbar q_z\sigma'}(x - X, y + Y)]]$$

$$\times \delta(\varepsilon_{n'p_z+\hbar q_z\sigma'} - \varepsilon_{np_z\sigma} - \hbar\omega_q)$$

$$+ \left| V_{si}\begin{pmatrix} p_z + \hbar q_z & n'\sigma' & x - X & y + Y \\ p_z & n\sigma & x & y \end{pmatrix}\right|^2$$

$$\times N_i [f_{n'p_z+\hbar q_z\sigma'}(x-X, y+Y) - f_{np_z\sigma}(x, y)]\,\delta$$

$$\times (\varepsilon_{n'p_z+\hbar q_z\sigma'} - \varepsilon_{np_z\sigma})$$

$$+ \left| V_{sd}\begin{pmatrix} p_z+\hbar q_z & n'\sigma'\sigma & x-X & y+Y \\ p_z & n\sigma\sigma' & x & y \end{pmatrix}\right|^2$$

$$\times [n_\sigma f_{n'p_z+\hbar q_z\sigma'}(x-X, y+Y)[1-f_{np_z\sigma}(x, y)]$$

$$- n_{\sigma'} f_{np_z\sigma}(x, y)[1-f_{n'p_z+\hbar q_z\sigma'}(x-X, y+Y)]]$$

$$\times \delta(\varepsilon_{n'p_z+\hbar q_z\sigma'} - \varepsilon_{np_z\sigma} - (\sigma'-\sigma)\Delta_d)\};$$

$$P_{1,2}(1, 2) = (2, 1); \quad \varepsilon_{np_z\sigma} = p_z^2/2m$$

$$+ \hbar\omega_0(n + 1/2) + \sigma\Delta_s; \quad \alpha^2 = c\hbar/eH; \quad \sigma, \sigma' = \pm 1; \quad (1)$$

V_{sph}, V_{si}, and V_{sd} are respectively the matrix elements of the scattering of the electrons by phonons, elastically-scattering impurities, and magnetic impurities, including processes with change in the spin orientation; x and y are the coordinates of the center of the electron orbit (it is known that only one of the two quantities, x or y, is a good quantum number[4]; the dependence of the diagonal density-matrix element $f_{np_z\sigma}(x, y)$ on both coordinates x and y simultaneously is tantamount to neglecting their non-commutativity, which is important only at distances of the order of $\alpha \ll L$); X and Y are the displacements of the center of the orbit in the scattering of the electron by the harmonic component of the interaction potential, having a wave vector **q**:

$$X = x' - x = \alpha^2 q_y, \qquad Y = y' - y = \alpha^2 q_x;$$

N_q is the phonon distribution function (the index of the phonon-spectrum branch has been left out), N_i is the concentration of the non-magnetic impurities

$$n_\sigma = n_d\, e^{-\sigma\Delta_d/T} / 2\,\mathrm{ch}\frac{\Delta_d}{T}$$

are the populations of the Zeeman sublevels of the paramagnetic impurities, having a concentration n_d; $\Delta_s = \mu_s H$ and $\Delta_d = \mu_d H$ are the Zeeman energies and the magnetic moments of the conduction electron and of the impurity respectively.

We assume that the non-equilibrium nature of the electron distribution in (1) is due only to perturbations of the longitudinal component of the spin magnetization

$$\delta M(x, y) = \chi H S(x, y),$$

$$\chi = \frac{\mu_s}{\hbar(2\pi\alpha)^2}\frac{\partial}{\partial H}\sum_{n\sigma}\sigma\int_{-\infty}^{+\infty} dp_z\, f_{np_z\sigma}, \quad (2)$$

the relaxation of which can be interpreted as relaxation of the electron Zeeman temperature[5]. Then, in an approximation linear in the magnetization fluctuation (ζ = chemical potential)

$$f_{np_z\sigma}(x, y) = f_{np_z\sigma} + \frac{1}{T} f_{np_z\sigma}(1 - f_{np_z\sigma})$$

$$\times\left\{\sigma\Delta_s + H\frac{\partial\zeta}{\partial H}\right\} S(x, y) \quad (3)$$

we obtain from (1) the following balance equation for the longitudinal magnetic moment

$$\delta M(x, y) = \frac{\mu_s}{\hbar(2\pi\alpha)^2}\sum_{n\sigma}\sigma\int_{-\infty}^{+\infty} dp_z\{f_{np_z\sigma}(x, y) - f_{np_z\sigma}\};$$

$$\frac{\partial}{\partial t}\,\delta M(x, y) = \frac{2\pi\mu_s^2}{\hbar^2 T\chi}\sum_{\sigma\sigma'}\int\frac{d\mathbf{q}}{(2\pi)^3}[(\sigma-\sigma')$$

$$+ \sigma'(1 - e^{X\partial_x - Y\partial_y})]\sum_{nn'}\int_{-\infty}^{+\infty}\frac{dp_z}{(2\pi\alpha)^2}$$

$$\times\left\{\left| V_{sph}\begin{pmatrix} p_z+\hbar q_z & n'\sigma' & x-X & y+Y \\ p_z & n\sigma & x & y \end{pmatrix}\right|^2\right.$$

$$\times N_q f_{np_z\sigma}(1 - f_{n'p_z+\hbar q_z\sigma'})\,\delta(\varepsilon_{n'p_z+\hbar q_z\sigma'} - \varepsilon_{np_z\sigma} - \hbar\omega_{\mathbf{q}})$$

$$+ \left| V_{si}\begin{pmatrix} p_z+\hbar q_z & n'\sigma' & x-X & y+Y \\ p_z & n\sigma & x & y \end{pmatrix}\right|^2 N_i f_{np_z\sigma}(1 - f_{np_z\sigma})$$

$$\times \delta(\varepsilon_{n'p_z+\hbar q_z\sigma'} - \varepsilon_{np_z\sigma})$$

$$+ \left| V_{sd}\begin{pmatrix} p_z+\hbar q_z & n'\sigma'\sigma & x-X & y+Y \\ p_z & n\sigma\sigma' & x & y \end{pmatrix}\right|^2$$

$$\times n_{\sigma'} f_{np_z\sigma}(1 - f_{n'p_z+\hbar q_z\sigma'})\,\delta(\varepsilon_{n'p_z+\hbar q_z\sigma'} - \varepsilon_{np_z\sigma}$$

$$- (\sigma'-\sigma)\Delta_d)\Big\}\left[-(\sigma-\sigma') + \left(\sigma' + \frac{1}{\mu_s}\frac{\partial\zeta}{\partial H}\right)\right.$$

$$\left.\times(e^{-X\partial_x + Y\partial_y} - 1)\right]\delta M(x, y). \quad (4)$$

Expanding in (4) the differential operators $\exp(\pm X\partial_x \mp Y\partial_y)$ in powers of the operation $(X\partial_x - Y\partial_y)$, corresponding to expansion in increasing powers of $r_L/L \ll 1$, we obtain

$$\frac{\partial}{\partial t}\delta M(x, y) + \mathrm{r}\frac{\delta M(x, y)}{\tau_s} - \begin{pmatrix}\partial_x\\ \partial_y\end{pmatrix}_i D_{ik}\begin{pmatrix}\partial_x\\ \partial_y\end{pmatrix}_k \delta M(x, y) = 0, \quad (5)$$

where the relaxation time of the spin magnetization τ_s is determined by the formulas

$$\tau_s^{-1} = \tau_{sph}^{-1} + \tau_{si}^{-1} + \tau_{sd}^{-1}; \quad (6)$$

$$\tau_{sph}^{-1} = \frac{2\pi\mu_s^2}{\hbar^2 T\chi}\sum_{\sigma\sigma'}(\sigma-\sigma')^2\sum_{nn'}\int_{-\infty}^{+\infty}\frac{dp_z}{(2\pi\alpha)^2}$$

$$\times\int\frac{d\mathbf{q}}{(2\pi)^3}\left| V_{sph}\begin{pmatrix} p_z+\hbar q_z & n'\sigma' & x-X & y+Y \\ p_z & n\sigma & x & y \end{pmatrix}\right|^2$$

$$\times N_q f_{np_z\sigma}(1 - f_{n'p_z+\hbar q_z\sigma'})\,\delta(\varepsilon_{n'p_z+\hbar q_z\sigma'} - \varepsilon_{np_z\sigma} - \hbar\omega_q), \quad (7)$$

$$\tau_{si}^{-1} = \frac{2\pi\mu_s^2}{\hbar^2 T\chi}\sum_{\sigma\sigma'}(\sigma-\sigma')^2\sum_{nn'}\int_{-\infty}^{+\infty}\frac{dp_z}{(2\pi\alpha)^2}$$

$$\times\int\frac{d\mathbf{q}}{(2\pi)^3}\left| V_{si}\begin{pmatrix} p_z+\hbar q_z & n'\sigma' & x-X & y+Y \\ p_z & n\sigma & x & y \end{pmatrix}\right|^2$$

$$\times N_i f_{np_z\sigma}(1 - f_{np_z\sigma})\,\delta(\varepsilon_{n'p_z+\hbar q_z\sigma'} - \varepsilon_{np_z\sigma}), \quad (8)$$

$$\tau_{sd}^{-1} = \frac{2\pi\mu_s^2}{\hbar^2 T\chi}\sum_{\sigma\sigma'}(\sigma-\sigma')^2\sum_{nn'}\int_{-\infty}^{+\infty}\frac{dp_z}{(2\pi\alpha)^2}$$
$$\times\int\frac{d\mathbf{q}}{(2\pi)^3}\left|V_{sd}\begin{pmatrix}p_z+\hbar q_z & n'\sigma'\sigma & x-X & y+Y\\ p_z & n\sigma\sigma' & x & y\end{pmatrix}\right|^2$$
$$\times n_{\sigma'}f_{np_z\sigma}(1-f_{n'p_z+\hbar q_z\sigma'})\,\delta(\varepsilon_{n'p_z+\hbar q_z\sigma'}$$
$$-\varepsilon_{np_z\sigma}-(\sigma'-\sigma)\Delta_d), \tag{9}$$

and the spin-magnetization diffusion coefficient takes the form

$$D_{ik} = D_{ik}^{(sph)} + D_{ik}^{(si)} + D_{ik}^{(sd)}; \tag{10}$$

$$D_{ik}^{(sph)} = \frac{2\pi\mu_s^2}{\hbar^2 T\chi}\sum_{\sigma\sigma'}\int\frac{d\mathbf{q}}{(2\pi)^3}R_{ik}\sum_{nn'}\int_{-\infty}^{+\infty}\frac{dp_z}{(2\pi\alpha)^2}$$
$$\times\left|V_{sph}\begin{pmatrix}p_z+\hbar q_z & n'\sigma' & x-X & y+Y\\ p_z & n\sigma & x & y\end{pmatrix}\right|^2$$
$$\times N_q f_{np_z\sigma}(1-f_{n'p_z+\hbar q_z\sigma'})\delta(\varepsilon_{n'p_z+\hbar q_z\sigma'}-\varepsilon_{np_z\sigma}-\hbar\omega_q), \tag{11}$$

$$D_{ik}^{(si)} = \frac{2\pi\mu_s^2}{\hbar^2 T\chi}\sum_{\sigma\sigma'}\int\frac{d\mathbf{q}}{(2\pi)^3}R_{ik}\sum_{nn'}\int_{-\infty}^{+\infty}\frac{dp_z}{(2\pi\alpha)^2}$$
$$\times\left|V_{si}\begin{pmatrix}p_z+\hbar q_z & n'\sigma' & x-X & y+Y\\ p_z & n\sigma & x & y\end{pmatrix}\right|^2$$
$$\times N_i f_{np_z\sigma}(1-f_{np_z\sigma})\,\delta(\varepsilon_{n'p_z+\hbar q_z\sigma'}-\varepsilon_{np_z\sigma}), \tag{12}$$

$$D_{ik}^{(sd)} = \frac{2\pi\mu_s^2}{\hbar^2 T\chi}\sum_{\sigma\sigma'}\int\frac{d\mathbf{q}}{(2\pi)^3}R_{ik}\sum_{nn'}\int_{-\infty}^{+\infty}\frac{dp_z}{(2\pi\alpha)^2}$$
$$\times\left|V_{sd}\begin{pmatrix}p_z+\hbar q_z & n'\sigma'\sigma & x-X & y+Y\\ p_z & n\sigma\sigma' & x & y\end{pmatrix}\right|^2$$
$$\times n_{\sigma'}f_{np_z\sigma}'(1-f_{n'p_z+\hbar q_z\sigma'})\,\delta(\varepsilon_{n'p_z+\hbar q_z\sigma'}-\varepsilon_{np_z\sigma}$$
$$-(\sigma'-\sigma)\Delta_d), \tag{13}$$

$$R_{ik} = \begin{pmatrix}X^2 & -XY\\ -YX & Y^2\end{pmatrix}_{ik}\left[1+\sigma'(\sigma-\sigma')+\frac{\sigma+\sigma'}{2\mu_s}\frac{\partial\zeta}{\partial H}\right],$$

$$\chi = \frac{\mu_s^2}{T}\sum_{n\sigma}\int_{-\infty}^{+\infty}\frac{dp_z}{\hbar(2\pi\alpha)^2}f_{np_z\sigma}(1-f_{np_z\sigma})\left\{1+\frac{\sigma}{\mu_s}\frac{\partial\zeta}{\partial H}\right\},$$

$$\frac{\partial\zeta}{\partial H} = -\mu_s\left\{\sum_{n\sigma}\sigma\int_{-\infty}^{+\infty}dp_z f_{np_z\sigma}(1-f_{np_z\sigma})\right\}$$
$$\times\left\{\sum_{n\sigma}\int_{-\infty}^{+\infty}dp_z f_{np_z\sigma}(1-f_{np_z\sigma})\right\}^{-1}. \tag{14}$$

Recognizing that the displacement of the center of the electron orbit upon scattering is of the order of the Larmor radius, we can neglect in the region $r_L \ll L$ the influence of the inhomogeneities on the matrix elements of the scattering. Comparing (11)—(13) with the expressions for the transverse conductivity of the carriers in quantizing magnetic fields[6], we can readily establish after a number of estimates the following connection between the diagonal elements of the tensors of the magnetic spin diffusion D_{ii} and the electric conductivity σ_{ii}, which takes place when $\Delta_s < \bar{\epsilon}$ ($\bar{\epsilon}$ is the average kinetic energy of the conduction electrons):

$$D_{ii} = \frac{\mu_s^2}{\chi e^2}\sigma_{ii}. \tag{15}$$

It follows therefore that for nondegenerate statistics, when $\omega_0\tau \gg 1$ and $T \gg h\omega_0$, the spin diffusion coefficient coincides with the electron-density diffusion coefficient, since in this region $\chi = \mu_s^2 n_0/T$, and (15) goes over into Einstein's relation between the electric conductivity and the density of diffusion coefficient. Formulas (6)—(9) for the spin-lattice relaxation time are suitable both in quantizing fields and in the classical region (in the latter case the quantum numbers of the electron in the magnetic field must be replaced by the components of the quasi-momentum p_α).

3. Let us calculate the spin-lattice relaxation time τ_{sd} for the case of scattering of nondegenerate carriers by magnetic impurities in the quantum limit $\hbar\omega_0 > T$. Assuming, as usual, that the Fourier transform J of the interaction Hamiltonian, which leads to a simultaneous spin flip of the electron and of the impurity, does not depend on the state of the conduction electron, we obtain

$$\left|V_{sd}\begin{pmatrix}p_z+\hbar q_z & n'\sigma'\sigma & x-X & y+Y\\ p_z & n\sigma\sigma' & x & y\end{pmatrix}\right|^2$$
$$\approx J^2\exp\left\{-\frac{\alpha^2 q_\perp^2}{2}\right\}\left[\frac{\bar{n}!}{\sqrt{n!n'!}}L_{\bar{n}}^{|n'-n|}\left(\frac{\alpha^2 q_\perp^2}{2}\right)\right.$$
$$\left.\times\left(\frac{\alpha^2 q_\perp^2}{2}\right)^{|n'-n|}\right]^2, \qquad \bar{n} = \min(n, n');$$

$$L_s^r(x) = \sum_{t=0}^{s}\binom{s+r}{s-t}\frac{(-x)^t}{t!}, \qquad q_\perp^2 = q_x^2+q_y^2. \tag{16}$$

Substituting this expression in (9) and calculating the encountered integral, we obtain the following expression for $\tau_{sd}^{-1}(\hbar\omega_0 > T)$:

$$\tau_{sd}^{-1}(\hbar\omega_0 > T) = \tau_{sd}^{-1}(\hbar\omega_0 < T)\frac{\hbar\omega_0}{4T}\frac{\cosh(\Delta_s/T)}{\cosh(\Delta_d/T)}$$
$$\times K_0\left(\left|\frac{\Delta_s-\Delta_d}{T}\right|\right), \tag{17}$$

where $\tau_{sd}(\hbar\omega_0 < T)$ is the time of relaxation of the electron spin on the magnetic impurities in the classical region of variation of the magnetic field:

$$\tau_{sd}^{-1}(\hbar\omega_0 < T) = \frac{2^{5/2}}{\pi^{3/2}}\frac{J^2 n_d m^{3/2}T^{1/2}}{\hbar^4},$$

$K_0(x)$ is the Macdonald function. Thus, when Δ_s or $\Delta_d > T$, the dependence of τ_{sd}^{-1} on the temperature and on the field takes the form

$$\tau_{sd}^{-1} \sim T^{-1/2}H\exp\left\{\frac{H(\mu_s-\mu_d)}{T}\right\}K_0\left(\left|\frac{\Delta_s-\Delta_d}{T}\right|\right).$$

When $\mu_d \sim \mu_s$ we have

$$K_0\left(\left|\frac{\Delta_s-\Delta_d}{T}\right|\right)\approx\ln\left|\frac{\Delta_s-\Delta_d}{2T}\right|^{-1},$$

so that τ_{sd}^{-1} diverges logarithmically if the magnetic moments of the conduction electron and of the impurity coincide[1)]. This divergence is essentially of the same nature as the divergences of the thermogalvanomagnetic coefficients in the quantum limit; it can be eliminated by taking into account one of the cutoff mechanisms at small longitudinal-momentum transfers[6].

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1)A similar line-width resonance is observed in processes of inelastic scattering by optical phonons, if the magnitude of the Zeeman splitting of the conduction electrons approaches the energy of the optical phonon [7].

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