DRIFT WAVES IN A FINITE-PRESSURE PLASMA

A. B. MIKHAĬLOVSKIĬ and A. M. FRIDMAN

Novosibirsk State University

Submitted to JETP editor April 30, 1966

J. Exptl. Theoret. Phys. (U.S.S.R.) **51**, 1430–1444 (November, 1966)

It is shown that the low-frequency oscillations of a collisionless, inhomogeneous, finite-pressure plasma comprise two wave types; these are similar to the Alfvèn waves and the slow magnetosonic waves in a homogeneous plasma. Instabilities due to spontaneous excitation of these oscillations in a finite-pressure plasma are considered. As in a homogeneous plasma, there is no interaction between resonance particles and the Alfvèn waves in the approximation in which the ion Larmor radius is taken to be zero. However, these waves can be associated with a hydrodynamic instability if the plasma pressure is comparable with that of the magnetic field and if the temperature and density gradients are in opposite directions. A large class of plasma instabilities is associated with magnetosonic waves, these instabilities arising from resonance particle effects. Certain new plasma instabilities of this kind are considered. In general, these instabilities arise when the relative temperature gradient is comparable to, or greater than, the density gradient and is of opposite sign. The ion temperature instability of a finite-pressure plasma, known earlier for a low-pressure plasma in which $\partial \ln T/\partial \ln n > 1$, is also discussed. It is shown that this instability disappears when the plasma pressure is greater than the magnetic pressure.

1. INTRODUCTION

MAGNETOHYDRODYNAMICS (MHD) reveals that there are two waves that can propagate in a uniform conducting medium in a fixed magnetic field, these being the magnetosonic and the Alfvèn waves.[1] These rather simple wave modes have analogs in media for which the MHD description does not apply, for example, in a uniform collisionless plasma. In the present work we consider the waves that can propagate in an inhomogeneous collisionless plasma. As in the conventional MHD approach, we take the gas pressure P to be comparable with the magnetic pressure $B_0^2/8\pi$. A plasma of this kind will be called a "finite-pressure" plasma (in contrast with a plasma with small $\beta = 8\pi P/B_0^2$, which is usually called a "low-pressure" plasma).

The question of wave propagation in an inhomogeneous plasma is closely related to the problem of plasma stability. This is due to one extremely interesting feature of these waves, the fact that they can be excited spontaneously. A number of earlier authors have considered the stability of an inhomogeneous finite-pressure plasma in papers which are reviewed in various surveys.[2,3] These authors have investigated plasma stability in the presence of current flow, anisotropy, curvature and other features whose role is fairly well known at the present time. In this connection it is of interest to consider the so-called "microinstabilities" of a finite-pressure plasma which have been investigated in [4–8].

In the work cited above a number of simplifying assumptions[1)] have been made in order to overcome certain formal difficulties[2)] which arise in the analysis of a finite-pressure plasma. Because of the assumptions that have been made, the results that have been obtained are useful for only relatively few cases in practice; specifically, the cases are a pinch in the absence of an external magnetic field, a long pinch in a magnetic field with lines of force that are straight and parallel, and a plasma in a curved magnetic field in which there is no shear.

In the present work the investigation of oscillations in a finite-pressure plasma is carried out

[1)]Waves for which $\mathbf{k}\cdot\mathbf{B}_0 = 0$ are considered in [4-7]. In [8] waves for which $\mathbf{k}\cdot\mathbf{B}_0 \neq 0$ are considered but the temperature gradient is not introduced and it is also assumed that $\omega \gg \omega_M$; $\mathbf{k}$ is the wave vector and ω_M is the magnetic drift frequency (cf. Sec. 2).

[2)]The basic difficulty in the analysis of a finite-pressure plasma is the fact that the number of unknown perturbation functions is greater than in a low pressure plasma.

for arbitrary temperature and density gradients, arbitrary oscillation frequency ω, and arbitrary magnetic drift frequency ω_M. The direction of the wave vector $\mathbf{k}$ with respect to the unperturbed magnetic field $\mathbf{B}_0$ is also arbitrary.

Although a large number of authors have considered waves in an inhomogeneous plasma, there are a number of reasons for which the problem we consider here has not yet been treated. One of these is the fact that in most present-day experiments in plasma physics it is only possible to obtain plasmas at extremely low pressures; for example, in Tokomak, according to [9] $\beta = 8\pi P/B_0^2 \sim 10^{-5}$ whereas in the experiments carried out by Ioffe and his colleagues[10] we find $\beta \lesssim 3\times 10^{-4}$. Thus, in most theoretical investigations as well as in other present-day experiments the parameter β is taken to be very small (cf. for example, one of the recent reviews[11] and the literature cited therein). Another factor that has tended to inhibit the interest of theorists in plasmas with finite β is the fact that the equations for this case are extremely complicated as compared with the equations that apply for a low-pressure plasma. However, recent experimental work[12, 13] indicates that plasmas with finite β are becoming available.

Recently, Mikhaĭlovskiĭ has carried out an analysis[14] in which waves in a plasma with finite β were investigated in the hydrodynamic approximation. It was shown in this work that the low-frequency long-wave oscillations of an inhomogeneous plasma ($\omega \ll \omega_{Bi}$, $k^2\rho_i^2 \ll 1$ where ω_{Bi} is the ion Larmor frequency and ρ_i is the ion Larmor radius) can be divided into two wave classes which are independent of each other and which are the analogs of the usual magnetoacoustic waves and the Alfvèn waves. Mathematically, this procedure corresponds to separation of the general plasma oscillation equation into two independent equations in the limiting case $\omega \ll \omega_{Bi}$, $k^2\rho_i^2 \ll 1$. The analysis in [14] is based on the use of the two-fluid hydrodynamic equations[15] which, in general, do not apply for the description of a collisionless plasma. Thus, in the final analysis, in order to investigate waves in an inhomogeneous finite-pressure plasma it is necessary to use a kinetic investigation.

In the following sections of the present work we show that as in a uniform plasma, in an inhomogeneous collisionless plasma at arbitrary pressure there are two wave modes which are the analogs of the magnetosonic waves and the Alfvèn waves. The analogy stems from the fact that the collisionless inhomogeneous plasma exhibits two oscillation branches. When the angle between the wave vector and the magnetic field is close to a right angle the dispersion relation is determined by the inhomogeneity of the plasma and the magnetic field; in other directions of propagation the oscillations become the usual magnetosonic and Alfvèn waves.

By virtue of the fact that the general oscillation equation in the present problem can be divided into two independent equations, it is possible to carry out a limiting analysis of each wave mode, thereby simplifying the problem considerably. In this case one sees a less direct analogy between the "Alfvèn" wave observed in our case and the usual Alfvèn wave. It is found that the dispersion equation for these and other waves is quadratic in the oscillation frequency and does not contain effects due to resonant interactions between the waves and particles. For this reason the analysis of the equation that describes the Alfvèn-type wave is rather simple (Section 3).

The equation for the "magnetosonic" wave is found to be more complicated. As in the equation for the magnetosonic waves in a uniform plasma, we find that this equation is transcendental in ω. The analysis of instabilities due to excitation of magnetosonic waves (Secs. 4–6) shows that in general a finite value of β exerts a stabilizing effect. Nevertheless, in certain cases the finite pressure can lead to the excitation of magnetosonic waves.

2. GENERAL ANALYSIS OF LOW-FREQUENCY LONG-WAVE OSCILLATIONS

We assume that the plasma is located in a magnetic field whose lines of force are parallel and straight $\mathbf{B}_0 \parallel \mathbf{z}$. Particle collisions are neglected. The equilibrium distribution function for each particle species (ion and electrons) is assumed to be approximately Maxwellian. The magnetic field, density, and temperature of the plasma are inhomogeneous in the $\mathbf{x}$ direction. We shall investigate the oscillations of a plasma of this kind assuming that all perturbed quantities depend on coordinates and time in accordance with the relation

$$\exp\left(-i\omega t + i\int k_x(x)\,dx + ik_y y + ik_z z\right).$$

The oscillation frequency ω is assumed to be small compared with the ion cyclotron frequency ω_{Bi} and the wave number k_x is assumed to be large compared with the reciprocal scale of the plasma inhomogeneity. As is well known, these restrictions simplify the problem considerably and allow the use of the WKB method.[16-18]

When the conditions listed above are introduced we can relate the frequency and wave vector by an equation which has been derived earlier in [8]. This equation is

$$\begin{vmatrix} -k^2\varepsilon_0 & \alpha_2 & \alpha_3 \\ \alpha_2 & c^2k^2\omega^{-2}-\varepsilon_2 & -\varepsilon_{23} \\ \alpha_3 & -\varepsilon_{23} & c^2k^2\omega^{-2}-\varepsilon_3 \end{vmatrix} = 0. \quad (2.1)$$

The following notation has been used:

$$\varepsilon_0 = \frac{4\pi e^2}{k^2 T_i}\hat{l}_i n_0 [1 - I_0(Z_i)e^{-Z_i}] - \sum_{i,e} \frac{4\pi e^2}{k^2 T}\hat{l} n_0 \langle \zeta J_0^2 (k_z v_z + \omega_M \varepsilon)\rangle,$$

$$\varepsilon_2 = -\sum_{i,e} \frac{4\pi e^2}{\varkappa_B^2 T\omega}\hat{l} n_0 \omega_M^2 \left\langle \zeta\varepsilon^2 \frac{4J_1^2}{\xi^2}\right\rangle$$

$$\varepsilon_3 = -\sum_{i,e} \frac{4\pi e^2}{T\omega}\hat{l} n_0 \langle \zeta v_z^2 J_0^2\rangle,$$

$$\alpha_2 = \sum_{i,e} \frac{4\pi e^2}{\varkappa_B T}\hat{l} n_0 \omega_M \left\langle \zeta\varepsilon \frac{2J_1 J_0}{\xi}\right\rangle,$$

$$\alpha_3 = -\sum_{i,e} \frac{4\pi e^2}{T}\hat{l} n_0 \langle \zeta v_z J_0^2\rangle,$$

$$\alpha_{23} = \sum_{i,e} \frac{4\pi e^2}{T\omega\varkappa_B}\hat{l} n_0 \omega_M \left\langle \zeta\varepsilon v_z \frac{2J_1 J_0}{\xi}\right\rangle. \quad (2.2)$$

The summation in (2.2) is taken over the ions and electrons and the subscripts have been omitted on the summation signs. The operator $\hat{l}$ operates on the density and temperature functions and is given by

$$\hat{l} = 1 - \frac{k_y T}{M\omega\omega_B}\left(\frac{\partial n_0}{\partial x}\frac{\partial}{\partial n_0} + \frac{\partial T}{\partial x}\frac{\partial}{\partial T}\right). \quad (2.3)$$

The symbol $\langle \ldots \rangle$ means averaging over a Maxwellian distribution function normalized to unity, i.e.,

$$\langle \ldots \rangle = \left(\frac{M}{2\pi T}\right)^{1/2} \int_{-\infty}^{\infty} \exp\left(-\frac{Mv_z^2}{2T}\right) dv_z \int_0^{\infty} e^{-\varepsilon} d\varepsilon (\ldots), \quad (2.4)$$

where ϵ, as in (2.2) is $Mv_\perp^2/2T$. The other notation in (2.2) is as follows:

$$\zeta = (\omega - k_z v_z - \omega_M \varepsilon)^{-1}, \quad \xi = \frac{kv_\perp}{\omega_B} \equiv k_\perp \rho_i,$$

$$Z_i = \frac{k^2 T_i}{M_i \omega_{Bi}^2},$$

ω_M is the magnetic drift frequency

$$\omega_M = \varkappa_B kT / M\omega_B, \quad \varkappa_B = \partial \ln B / \partial x,$$

J is the Bessel function with argument ξ, I_0 is the Bessel function with imaginary argument, n_0 is the unperturbed density, T is the temperature, M is the particle mass, v_z and $v_\perp$ are the longitudinal and transverse components of the particle velocity, e is the charge of the electron and the subscripts e and i refer to electrons and ions respectively. Below we make use of the notation $v_T^2 = 2T/M$.

Using some familiar properties of determinants, we replace the first row of the determinant in (2.1) by its sum with the second row multiplied by κ_B and the third row multiplied by k_z. The analogous operation is then carried out with the first column. As a result, (2.1) becomes

$$\begin{vmatrix} N_1 & N_2 & N_3 \\ N_2 & c^2k^2\omega^{-2}-\varepsilon_2 & -\alpha_{23} \\ N_3 & -\alpha_{23} & c^2k^2\omega^{-2}-\varepsilon_3 \end{vmatrix} = 0. \quad (2.5)$$

Here

$$N_1 = \frac{c^2 k_z^2}{\omega^2}k^2 - \frac{4\pi e^2}{T_i}\hat{l}_i n_0 \times \left[(1 - I_0 e^{-Z_i}) - \frac{\omega_{Mi}}{\omega}\left(1 - \frac{2I_1 e^{-Z_i}}{Z_i}\right)\right] + \frac{4\pi e^2}{T_i\omega}\hat{l}_i n_0 \left\langle \zeta_i \varepsilon\omega_{Mi}(\omega - k_z v_z)\left(J_0 - \frac{2J_1}{\xi}\right)^2\right\rangle, \quad (2.6)$$

$$N_2 = -\frac{4\pi e^2}{\varkappa_B T_i \omega}\hat{l}_i n_0 \omega_{Mi}\left(1 - \frac{2I_1 e^{-Z_i}}{Z_i}\right) \times \frac{4\pi e^2}{\varkappa_B T_i\omega}\hat{l}_i n_0 \omega_{Mi}\left\langle \zeta\varepsilon(\omega - k_z v_z)\left(\frac{2J_1 J_0}{\xi} - \frac{4J_1^2}{\xi^2}\right)\right\rangle,$$

$$N_3 = k_z \frac{c^2 k^2}{\omega^2} - \frac{4\pi e^2}{T_i\omega}\hat{l}_i n_0 \omega_{Mi}\left\langle \zeta\varepsilon v_z J_0\left(J_0 - \frac{2J_1}{\xi}\right)\right\rangle.$$

It follows from (2.5) and (2.6) that so long as $k^2\rho_i^2 \ll 1$ (and obviously $k^2c^2 \ll \omega_{Pe}^2$ which is a consequence of $k^2\rho_i^2 \ll 1$, $\beta \gg M_e/M_i$), the elements N_1, N_2 and N_3 are small, going as $k^2\rho_i^2$ (or as k^2c^2/ω_{Pe}^2 if $\omega > k_z v_{Te}$) as compared with the elements of the 2×2 matrix in the lower right of the determinant of (2.5). Hence, in the lower-order approximation in the parameter $k^2\rho_i^2$ the dispersion equation (2.5) divides into two equations:

$$N_1^{(0)} = 0, \quad (2.7)$$

$$\begin{vmatrix} c^2k^2/\omega^2 - \varepsilon_2^{(0)} & -\alpha_{23}^{(0)} \\ -\alpha_{23}^{(0)} & -\varepsilon_3^{(0)} \end{vmatrix} = 0, \quad (2.8)$$

where, in the elements ϵ_2, α_{23}, ϵ_3 and N_1 we keep only the lower-order terms in $k^2\rho_i^2$ (this is denoted by the superscript zero).

The splitting of the dispersion equation for the low-frequency longwave oscillations into two equations is well known in the theory of a uniform plasma.[19] One of these equations describes the

Alfvèn waves and the other describes the magnetosonic waves. This kind of splitting of the dispersion equation into two equations was indicated earlier for the analysis of oscillations in a low-pressure inhomogeneous plasma $\beta \ll 1$ (cf. [20]). The analysis given above shows that this property of the oscillations holds for an arbitrary value of the parameter β (this result was first obtained through the use of a hydrodynamic analysis in [14]).

It has been assumed in the derivation of (2.1) that the oscillations exhibit a low phase velocity $\omega/k_\perp \ll v_T$, c_A so that we have omitted terms of order $\omega/k_\perp c_A$, $\omega/k_z v_T$. (If this is not the case, $\omega \sim k_\perp v_T$, $k_\perp c_A$ the drift effects are not important.) For this reason (2.8) does not contain branches that correspond to the fast magnetosonic wave in a uniform plasma. (In particular, in order to exhibit this wave it is necessary to retain the term c^2/c_A^2 in ϵ_2.) Hence, in neglecting effects due to the inhomogeneity Eq. (2.8) describes only the slow magnetosonic waves (frequently called ion sound). The wave in (2.8) for which the inhomogeneity is important will be called the drift magnetosonic wave.

In concluding this section we show that there is another form of the equation which describes the drift magnetosonic wave; this will be used together with (2.8), below. We replace the third column in the determinant in (2.1) by its sum with the first column divided by k_z. Then the third row is replaced by its sum with the first row of the determinant divided by k_z. We then multiply the second row of the determinant obtained in this way by $-\kappa_B/k_z$ and add it to the third row. As a result we obtain the determinant $|a_{ik}|$, in which, in the approximation $k^2\rho_i^2 \to 0$, the terms $a_{13} = a_{23} = 0$ as a consequence of the relations

$$-\alpha_2^{(0)}\frac{\varkappa_B}{k_z} + \alpha_3^{(0)} - \frac{k^2\varepsilon_0^{(0)}}{k_z} = 0,$$
$$-\frac{\varkappa_B}{k_z}\left(\frac{c^2k^2}{\omega^2} - \varepsilon_2^{(0)}\right) + \left(\frac{\alpha_2^{(0)}}{k_z} - \alpha_{23}^{(0)}\right) = 0. \qquad (2.9)$$

Thus, we see that the dispersion equation divides into two independent equations, one of which corresponds to the magnetosonic wave

$$\begin{vmatrix} -k^2\varepsilon_0 & \alpha_2 \\ \alpha_2 & c^2k^2\omega^{-2} - \varepsilon_2 \end{vmatrix} = 0. \qquad (2.10)$$

It will be evident that (2.8) and (2.10) are two ways of writing the same equation.

3. ALFVÈN-LIKE WAVES

It is clear from (2.7) and (2.8) that the simplest wave is that in (2.7). Writing (2.7) in explicit form we have

$$\omega^2 - \omega(\omega_{Mi} + \omega_{Pi}^*) + \omega_{Mi}\omega_{Pi}^*(1+\tau_i) - k_z^2c_A^2 = 0; \qquad (3.1)$$

where

$$c_A = \frac{B_0}{(4\pi M_i n_0)^{1/2}}, \qquad \tau_{i,e} = \frac{\partial \ln T_{i,e}}{\partial \ln P_{i,e}},$$

c_A is the Alfvèn velocity and

$$\omega_{Pi}^* = \frac{k_y T_i}{M_i\omega_{Bi}}\varkappa_{Pi}, \qquad \varkappa_{Pi} = \frac{\partial \ln P_i}{\partial x}.$$

It is evident that if k_z is not too small these are the usual Alfvèn waves of a uniform plasma; when k_z approaches zero the frequency of these waves is modified appreciably by drift effects. For this reason, the wave in (2.7) may be called the drift Alfvèn wave. In the limit $k_z = 0$ the equation in (3.1) coincides with the corresponding equation obtained by Tserkovnikov.[4] For any value of k_z this equation describes stable oscillations so long as

$$\tau_i > -\frac{1}{2\beta}\left(1+\frac{\beta}{2}\right)^2, \qquad (3.2)$$

this relation having also been obtained in [4] for the case $k_z = 0$.

In the limit $k_z = 0$ and $\nabla T = 0$ Eq. (3.1) has two roots: the first coincides with the ion drift frequency ω_{ni}^*:

$$\omega_{ni}^* = \frac{k_y T_i}{M_i\omega_{Bi}}\varkappa_{ni}, \qquad \varkappa_{ni} = \frac{\partial \ln n_i}{\partial x},$$

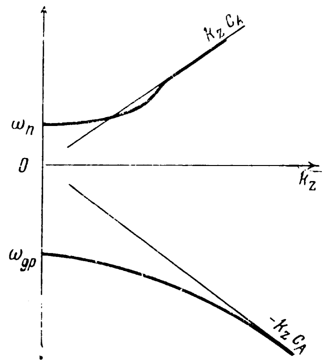
and the second coincides with the magnetic drift frequency ω_{Mi}. In what follows the oscillations corresponding to the first root will be called the drift ion wave and the oscillations described by the second root will be called the magneto-drift ion wave.

As k_z increases, the frequency of the drift ion waves increases, reaching a value of order $k_z c_A$; the frequency of the magneto-drift ion waves increases to a quantity of order $k_z c_A$. The dependence of the frequency of the Alfvèn-like wave on k_z for $\nabla T = 0$ is given in the figure, where ω_{dr} is the ion drift frequency, which is taken to be negative.

4. MAGNETOSONIC-LIKE WAVES FOR $|\omega - \omega_{Me}\epsilon| > k_z v_{Te}$

When $|\omega - \omega_{Me}\epsilon| > k_z v_{Te}$ Eq. (2.8) divides into two simpler equations:

$$c^2k^2/\omega^2 - \varepsilon_2^{(0,0)} = 0, \qquad (4.1)$$
$$\varepsilon_3^{(0,0)} = 0, \qquad (4.2)$$



where the superscripts on ϵ_2 and ϵ_3 indicate that these elements are taken at $k_z = 0$ and $k^2\rho_i^2 = 0$.

Equation (4.1) was obtained and studied for arbitrary values of

$$\partial \ln T / \partial \ln n \equiv \eta$$

in [4]. In particular, when $\beta < 1$ the oscillations described by this equation are unstable if

$$|\eta| > 1. \quad (4.3)$$

Equation (4.2) has been treated by Krall and Rosenbluth for the case $\nabla T = 0$.[6] In this case the equation has a solution that corresponds to unstable oscillations characterized by

$$\operatorname{Im}\omega / \operatorname{Re}\omega \sim M_e e^{-2/\beta} / \beta M_i.$$

This instability is due to resonance ions, the contribution of which in $\epsilon_3^{(0,0)}$ is small (going as M_e/M_i). Thus, taking account of terms of order $(k_z v_{Te}/\omega)^2$ for small but finite k_z [as for $k_z v_{Te}/\omega \gtrsim (M_e/M_i)^{1/2}$] should lead to an important change in the growth rate. For small but finite values of β and $k_z v_{Te}/\omega \ll 1$, the introduction of terms of order $k_z^2 v_{Te}^2/\omega^2$ in Eq. (4.2) leads to the following equation:

$$\hat{l}_e n_0 T_e = i\pi\omega \frac{T_e}{T_i}\left\{\frac{M_e}{M_i}\hat{l}_i n_0 T_i \frac{1}{|\omega_{Mi}|} e^{-\omega/\omega_{Mi}} + \frac{k_z^2}{\omega^2 M_e}\left[\frac{\hat{l}_e(\omega_{Me} T_e n_0)}{\hat{l}_e(\omega_{Me} n_0)}\right]^2 \hat{l}_i \frac{n_0}{|\omega_{Mi}|} e^{-\omega/\omega_{Mi}}\right\} \quad (4.4)$$

When $k_z v_{Te}/\omega \gtrsim M_e/M_i$ in which case terms containing k_z become more important, the instability criterion assumes the form

$$\tau > 0. \quad (4.5)$$

Under these conditions the growth rate is small compared with the frequency, going as

$$\frac{1}{\beta}\left(\frac{k_z v_T}{\omega}\right)^2 e^{-2/\beta}.$$

Thus, the instability criterion for perturbations characterized by $|\omega - \omega_{Me}\epsilon| > k_z v_{Te}$ is represented by the combination of (4.3) and (4.5). The branches in (4.1) and (4.2) do not separate at larger values of k_z. Thus, by writing $k_z \sim \omega/v_{Te}$ one can obtain an estimate of the effect of shear in the lines of force, which has an important effect on the instabilities treated above. To introduce shear in the present equations we must make the substitution $k_z \to k_\parallel \equiv \mathbf{k}\cdot\mathbf{B}_0/B_0$ and take $k_\parallel$ to be a function of the coordinates, which vanishes at some point $x = x_0$.[16, 17] In the vicinity of this point

$$k_\parallel(x) \approx k_\perp \frac{x - x_0}{a}\Theta,$$

where a is the characteristic scale size of the plasma inhomogeneity while Θ is a dimensionless parameter which characterizes the shear of the magnetic field. Under these conditions, for waves characterized by $\omega \sim \omega_M$ the criterion $\omega < k_z v_{Te}$ means

$$\frac{u_M}{v_{Te}} \lesssim \frac{x - x_0}{a}\Theta \quad (4.6)$$

($u_M = \omega_M/k_y$ is the magnetic drift velocity). Everywhere, the quantity $x - x_0$ is to be replaced by the smallest possible value of the localization dimension of the perturbation.[2, 11] When $\gamma \sim \omega$ this dimension is of the order of $1/k_\perp$. At the limits of applicability of the present analysis $k_\perp\rho_i \sim 1$. Hence, a rough criterion for the effect of shear on instabilities such as those in (4.1) is

$$\Theta > (M_e/M_i)^{1/2}\beta. \quad (4.7)$$

5. MAGNETOSONIC-LIKE WAVES FOR $k_z v_{Ti} \ll |\omega - \omega_M\epsilon| \ll k_z v_{Te}$

We now consider (2.8) under the conditions

$$|\omega - \varepsilon\omega_{Me}| \ll k_z v_{Te}, \qquad |\omega - \omega_{Mi}\varepsilon| \gg k_z v_{Ti}.$$

Effects due to resonance electrons are small, going as $\omega/k_z v_{Te}$, and can be neglected. Under these conditions Eq. (2.8) can be written in the form

$$\hat{l}_e n_0 \hat{l}_e \frac{n_0\omega_{Me}}{\omega} + \frac{T_e}{T_i}\hat{l}_i\left(n_0\int_0^\infty \frac{\varepsilon\omega_{Mi}e^{-\varepsilon}d\varepsilon}{\omega - \omega_{Mi}\varepsilon}\right) \times \hat{l}_e n_0\left(1 - \frac{\omega_{Me}}{\omega}\right) = 0. \quad (5.1)$$

At small values of β one of the roots of this equation is approximately the same as the one associated with the electron drift branch which is well-known in the theory of a low-pressure plasma:[16]

$$\omega = \omega_{ne}^*. \quad (5.2)$$

It is also known that taking account of resonance-electron interactions with this wave leads to damping when $\partial \ln T_e/\partial \ln n_0 > 0$ and to growth when $\partial \ln T_e/\partial \ln n_0 < 0$,[21] the growth rate being of order

$$\gamma \approx -\frac{\omega_{ne}^{*2}}{k_z v_{Te}}\eta. \tag{5.3}$$

We now wish to evaluate the role of small terms [of order β] in the oscillations described by (5.2). The chief effect is the resonance interaction of ions with the oscillations. We find

$$\gamma = -\pi\frac{\omega_{ne}^{*2}}{|\omega_{Me}|}\frac{1+T_i/T_e+2\tau/\beta}{(1+T_i/T_e)^2}\exp\left\{-\frac{2}{\beta}\frac{T_e}{T_i}\tau\right\}. \tag{5.4}$$

It is evident that the ions can exhibit a resonance interaction with the waves when $\partial \ln T/\partial \ln n_0 > -1$. Under these conditions the oscillations are damped for any positive $\partial \ln T/\partial \ln n_0$ and also when $-\beta < \eta < 0$; the oscillations can only be excited when $-1 < \eta < -\beta$.

It is of interest to compare this result with (4.5) which indicates growth of the analogous drift wave for $\eta > 0$ and $\omega > k_z v_{Te}$. Physically, these two waves differ in the sign of the energy.[22] For the waves in (4.4)

$$\mathrm{Re}\,\varepsilon_3^{(0)} = -\frac{\omega_{Pe}^2}{\omega^2}\left(1-\frac{\omega_{Pe}^*}{\omega}\right), \quad \omega_{Pe}^2 = \frac{4\pi e^2 n_0}{M_e},$$

so that the wave energy is negative, $w \sim \omega\, \partial\, \mathrm{Re}\, \epsilon_3^{(0)}/\partial\omega < 0$, whereas in the case described by (5.2)

$$\mathrm{Re}\,\varepsilon_3^{(0)} = \frac{1}{k^2 d_e^2}\left(1-\frac{\omega_{ne}^*}{\omega}\right), \quad d_e^2 = \frac{T_e}{4\pi e^2 n_0}$$

and $w > 0$. Thus, in both cases the ions exchange energy with the wave, causing excitation in the first case and damping in the second.

If $\beta \gg 1$, Eq. (5.1) reduces to the following ($\omega \ll \omega_{Mi}, \omega_{Me}$):

$$\hat{l}_e(n_0\omega_{Me}) \approx \frac{i\pi\omega^2}{4n_0^2}\frac{T_e}{T_i}\hat{l}_i\left(\frac{n_0}{|\omega_{Mi}|}\right)(\hat{l}_e n_0)^2\frac{1}{(1+T_e/T_i)^2}. \tag{5.5}$$

It then follows that

$$\mathrm{Re}\,\omega = \omega_{Pe}^*, \tag{5.6}$$

$$\gamma = \mathrm{Im}\,\omega = -\pi\left(\frac{2}{\beta}\right)^2\frac{|\omega_{Pe}^*|}{(1+T_i/T_e)^2}\tau^2 \times\left(1+\frac{T_i}{T_e}\frac{\varkappa_n-\varkappa_T}{\varkappa_n+\varkappa_T}\right), \tag{5.7}$$

where $\kappa_T = \partial \ln T/\partial x$. If $T_i = T_e = T$, it is easily shown by means of (5.7) that the oscillations are unstable only when $\eta < -1$. When $T_i > T_e$ it is interesting to note that the growth rate (5.7) can be positive (instability) even for $\eta > 0$, so long as

$$\eta > \frac{1+T_i/T_e}{T_i/T_e-1}. \tag{5.8}$$

We now consider the other solutions of Eq. (5.1). We shall show that even when $\beta \ll 1$ this equation can have one root whose frequency depends on the magnetic drift frequency ω_M. This result is shown in the following analysis, which applies for arbitrary values of β.

We assume that $\omega \ll \omega_{Mi}$. In this case the integral in (5.1) is given approximately by

$$\int_0^\infty \frac{\varepsilon\omega_{Mi}e^{-\varepsilon}d\varepsilon}{\omega-\omega_{Mi}\varepsilon} \approx -\left\{1+\frac{\omega}{\omega_{Mi}}\left(\ln\frac{\omega_{Mi}}{\omega}+i\pi\,\mathrm{sign}\,\omega_{Mi}\right)\right\}. \tag{5.9}$$

Substituting (5.9) in (5.1) we have

$$\omega = -\omega_{Mi}\exp\left\{\frac{T_i}{T_e}\frac{\varkappa_P}{\varkappa_n-\varkappa_T}\left[\frac{\beta}{2}\left(1+\frac{T_e}{T_i}\right)+\frac{\varkappa_n^2}{\varkappa_P^2}\right]\right\}. \tag{5.10}$$

The condition for the existence of these branches reduces to the requirement

$$1 \gg (\eta-1)(\eta+1) > 0. \tag{5.11}$$

It is evident that $\omega/\omega_{Mi} < 0$ so that there is no resonance interaction between the ions and these waves. In order to find the growth rate or the damping rate it is necessary to consider the interaction with resonance electrons. In this case, (5.10) is replaced by

$$\omega = -\omega_{Mi}\exp\left\{\frac{T_i}{T_e}\frac{\varkappa_P}{\varkappa_n-\varkappa_T}\left[\frac{\varkappa_n^2}{\varkappa_P^2} + \frac{\beta}{2}\left(1+\frac{T_e}{T_i}-i\sqrt{\pi}\frac{\omega_{ne}^*-\omega_{Te}^*/2}{|k_z|v_{Te}}\right)\right]\right\}. \tag{5.12}$$

This branch is stable when $\eta \gtrsim 1$ and unstable when $\eta < -1$.

The oscillation frequency becomes comparable with ω_{Mi} at the limits of applicability of Eq. (5.10). Hence one might expect that when $\eta \gg 1$ this branch will exhibit a frequency greater than ω_{Mi}. That this is actually the case can be demonstrated by the following analysis.

Writing $\omega \gg \omega_{Mi}$, we find that the integral in (5.1) is approximately

$$\int_0^\infty \frac{\varepsilon\omega_{Mi}e^{-\varepsilon}}{\omega-\omega_{Mi}\varepsilon}d\varepsilon \approx \frac{\omega_{Mi}}{\omega}. \tag{5.13}$$

Substituting this result in (5.1) and writing $\omega \ll \omega^*_{Pe}$, ω^*_{Pi}, $\beta \ll 1$, $\eta \gg 1$, we obtain the quadratic equation

$$\omega^2 - \omega\omega_{ne}^* - \frac{\omega^*_{Pe}\omega_{Mi}}{1 + T_i/T_e} = 0. \quad (5.14)$$

When $\eta < 2/\beta$ the larger root of this equation yields the branch in (5.2) while the smaller is

$$\omega = -\frac{\partial \ln P}{\partial \ln n}\frac{\omega_{Mi}}{1 + T_i/T_e}. \quad (5.15)$$

It is evident that this last branch is large compared with ω_{Mi} because when $\eta \gg 1$ and $\eta \sim 1$ it is of the order of (5.10). This means that the oscillations in (5.10) and (5.15) derive from the same branch. It is interesting to note that the branch in (5.15) can also be obtained on the basis of a macroscopic analysis.

If resonance electrons are taken into account, using (2.8) we obtain the following equation in place of (5.14)

$$\omega^2\left(1 + i\sqrt{\pi}\frac{\omega_T^{e*}}{2k_z v_{Te}}\right) - \omega\omega_{ne}^* - \frac{\omega_{Mi}^*\omega_{Pi}^*}{1 + T_i/T_e} = 0. \quad (5.16)$$

Thus, the imaginary part of the frequency (5.15) is

$$\operatorname{Im}\omega = \frac{(\operatorname{Re}\omega)^2\sqrt{\pi}}{|k_z| v_{Te}}\frac{\partial \ln T_e}{\partial \ln n}, \quad (5.17)$$

that is to say, this branch is unstable for large positive values of $\partial \ln T_e/\partial \ln n$.

6. ION TEMPERATURE INSTABILITY OF A FINITE-PRESSURE PLASMA

In carrying out the analysis above for magnetosonic waves we have assumed that the ions do not move along the magnetic field. We now wish to introduce effects associated with the longitudinal motion of the ions. In this case, assuming $\omega \ll k_z v_{Te}$ in (2.8) and neglecting terms of order $\omega/k_z v_{Te}$ we have

$$2 + i\sqrt{\pi}\frac{1}{n_0}\hat{l}_i\frac{\omega n_0}{|k_z| v_{Ti}}\langle W\rangle$$

$$- i\sqrt{\pi}\frac{1}{\omega_M^2}\frac{\beta}{2}\frac{1}{n_0}\hat{l}_i n_0\omega_M\frac{\omega}{|k_z| v_{Ti}}\langle \varepsilon^2 W\rangle$$

$$+ \pi\frac{\beta}{4}\frac{\omega^2}{\omega_M^2}\frac{1}{n_0^2}\left\{\hat{l}_i n_0\frac{1}{|k_z| v_{Ti}}\langle W\rangle\hat{l}_i n_0\frac{\omega_M^2}{|k_z| v_{Ti}}\langle \varepsilon^2 W\rangle\right.$$

$$\left. - \left[\hat{l}_i n_0\frac{\omega_M}{|k_z| v_T}\langle \varepsilon W\rangle\right]^2\right\} = 0, \quad (6.1)$$

where $W = W(|\omega - \epsilon\omega_M|/k_z v_{Te})$ is the Cramp function.[23]

The imaginary terms in Eq. (6.1) describe the resonance interaction between the ions and the waves as in Sec. 5. Here, we shall only be interested in effects which derive from the longitudinal motion of the ions; we rewrite Eq. (6.1) neglecting imaginary terms and also assuming that $\beta \ll 1$ and $\omega \gg \omega_{Mi}$:

$$\begin{vmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{vmatrix} = 0;$$

$$X_{11} = 1 + \frac{\omega_n^*}{\omega} - \left(\frac{\omega_{Mi}}{\omega} + \frac{k_z^2 v_{Ti}^2}{2\omega^2}\right)\left(1 - \frac{\omega_n^* + \omega_T^*}{\omega}\right),$$

$$X_{22} = \frac{\omega_{Mi}}{\omega}\left[2\frac{\omega_n^* + \omega_T^*}{\omega} - 2\frac{\omega_{Mi}}{\omega}\right.$$

$$\left. \times\left(1 + \frac{k_z^2 v_{Ti}^2}{2\omega^2}\right)\left(1 - \frac{\omega_n^* + 2\omega_T^*}{\omega}\right)\right], \quad (6.2)$$

$$X_{12} = X_{21} = \frac{\omega_{Mi}}{\omega}\left[1 - \frac{\omega_n^* + \omega_T^*}{\omega}\right.$$

$$\left. + \left(2\frac{\omega_{Mi}}{\omega} + \frac{k_z^2 v_{Ti}^2}{2\omega^2}\right)\left(1 - \frac{\omega_n^* + 2\omega_T^*}{\omega}\right)\right] \quad (6.3)$$

In general terms that describe the longitudinal ion motion are small, going as $k_z^2 v_{Ti}^2/\omega^2$; however, the contribution of these terms in the dispersion equation can be important in the presence of a large temperature gradient, such that

$$\omega_T^* k_z^2 v_{Ti}^2 / 2\omega^3 \sim 1. \quad (6.4)$$

On the other hand, the presence of a large temperature gradient not only enhances the relative importance of the longitudinal ion motion but also increases the effect of magnetic drift. The latter phenomenon is important when $\omega^*_T\omega_M/\omega^2 \sim 1$ [cf. Eq. (5.14)].

Assuming that $\omega^*_T \gg \omega^*_n$ we can neglect unity in the curved brackets in (6.3). In this case Eq. (6.2) reduces to the cubic equation

$$\omega^3 + \omega\frac{\omega_T^*\omega_{Mi}}{2} + \frac{k_z^2 v_{Ti}^2\omega_T^*}{2} = 0. \quad (6.5)$$

Equation (6.5) differs from the corresponding dispersion equation obtained by Rudakov and Sagdeev,[2] which describes the ion temperature instability, in that the present equation contains the magnetic drift. The latter reduces the instability region. The stabilization effect appears when the following condition is satisfied:

$$\beta > 3\cdot 4^{1/3}(k_z v_{Ti}/\omega_T^*)^{4/3}. \quad (6.6)$$

Up to this point we have been investigating the magnetosonic-like wave in the region $|\omega - \omega_{Mi}\epsilon|$

$> k_z v_{Ti}$. Now, making use of Eq. (6.1) we wish to consider the effect of finite β in the frequency region $|\omega - \omega_{Mi}| \ll k_z v_{Ti}$. For zero values of β ($\omega_{Mi} \approx 0$) Eq. (6.1) becomes the dispersion equation obtained by Galeev, Oraevskiĭ, and Sagdeev[24] in the region $\omega/k_z v_{Ti} < 1$. It follows from [24] that in investigating this instability one must retain second-order terms in $\omega/k_z v_{Ti}$. In this approximation Eq. (2.10) becomes

$$-2-\frac{i\sqrt{\pi}}{n_0}\hat{l}_i n_0\frac{\omega}{k_z v_{Ti}}+i\frac{16\pi^{3/2}}{B_0^2 T n_0}\hat{l}_i\left(T^2 n_0\frac{\omega}{k_z v_{Ti}}\right)$$
$$-\frac{8\pi^2}{T n_0 B_0^2}\hat{l}_i\left(n_0\frac{\omega}{k_z v_{Ti}}\right)\hat{l}_i\left(\frac{n_0 T^2\omega}{k_z v_{Ti}}\right)$$
$$-\frac{32\pi}{n_0 T B_0^2}\hat{l}_i n_0\frac{T^2(\omega-3\omega_{Mi})}{(k_z v_{Ti})^2}$$
$$+\frac{2}{n_0}\hat{l}_i n_0\frac{\omega(\omega-\omega_{Mi})}{(k_z v_{Ti})^2}+\frac{4\pi^2}{n_0 T B_0^2}\left(\hat{l}_i\frac{n_0 T\omega}{k_z v_{Ti}}\right)^2=0. \qquad (6.7)$$

As in [21] we assume $\beta \ll 1$ but take account of the effects due to finite β.

We use the condition

$$\omega_n^* \gg k_z v_{Ti}, \qquad (6.8)$$

which, as will be shown below [Eqs. (6.9) and (6.10)] is a necessary condition for the existence of a solution of Eq. (6.7). This solution is of the form

$$\omega=\frac{k_z^2 v_{Ti}^2}{\omega_T^*-\omega_n^*}\left[1-\frac{\omega_n^*\omega_{Mi}}{k_z^2 v_{Ti}^2}-i\sqrt{\pi}\frac{\omega_n^*}{|k_z|v_{Ti}}\left(1-\frac{\eta}{2}\right)\right]$$
$$-\frac{\beta\pi}{4}\left(\omega_n^{*2}+\omega_n^*\omega_T^*-\frac{7}{4}\omega_T^{*2}\right).$$

As in the case $\beta \to 0$, this equation holds only when $\eta \approx 2$ because if this condition is not satisfied the original assumption $\omega \ll k_z v_{Ti}$ is violated. Making use of this situation we can write Eq. (6.9) in the following approximate form

$$\omega=\frac{k_z^2 v_{Ti}^2}{\omega_n^*}+\omega_{Mi}(\pi-3)-i\sqrt{\pi}\,|k_z|v_{Ti}\left(1-\frac{\eta}{2}\right). \qquad (6.10)$$

It then follows that magnetic drift is not important when $\beta < 1$ in the frequency region $|\omega - \omega_M| \ll k_z v_{Ti}$. The absence of a stabilizing effect due to magnetic drift in the present case follows because of the relatively small contribution made by this term in the dispersion equation ($\sim\omega_{Mi}/k_z v_{Ti} \ll 1$). However, it is reasonable to expect that there will be a stabilization due to the magnetic drift[24] when $\omega_{Mi} \gg k_z v_{Ti}$.

It is found that the oscillation frequency computed from Eq. (6.1) in the case $\omega \ll k_z v_{Ti}$, $\omega_{Mi} \gg k_z v_{Ti}$, $\beta < 1$ does not satisfy the original assumption (this frequency is found to be equal to the magnetic drift frequency, violating the condition $\omega_{Mi} \gg \omega$). Consequently, in this region there are no stable or unstable oscillations. The relatively large value of the magnetic drift $\omega_{Mi} \gg k_z v_{Ti}$ leads to stabilization of the instability[24] in the region $\omega \ll k_z v_{Ti}$, which corresponds to a value of β that is not too small, more precisely

$$\beta > k_z v_{Ti}/\omega_n^*. \qquad (6.11)$$

This last result indicates the possibility of stabilization of the instability[24] when $\beta > 1$.

Writing $\omega < \max(k_z v_{Ti}, \omega_{Mi})$ in Eq. (6.1) we see that terms with ω can be neglected in the Kramp function W. The quadratic terms in W can be omitted since they appear with a small weighting factor $\sim \omega^2/(k_z v_{Ti})^2$. As a result we obtain an equation which is linear in ω:

$$\omega=\frac{i4}{\sqrt{\pi}\,\beta}\frac{k_z v_{Ti}}{\langle\varepsilon^2 W\rangle}+\frac{k_y T_i}{M_i\omega_{Bi}}\frac{\partial}{\partial x}\ln\left(\frac{n_0\omega_M^2}{v_{Ti}}\langle\varepsilon^2 W\rangle\right). \qquad (6.12)$$

The case $k_z v_{Ti} \gg \omega$ corresponds to oscillations of a homogeneous plasma with damping rate

$$\gamma=-4\pi^{-1/2}|k_z|v_{Ti}/\beta. \qquad (6.13)$$

We note that the damping rate given by this formula $\gamma \sim \omega_n^*$. Hence, in analyzing oscillations characterized by $\omega \sim \omega_n^*$, in Eq. (6.12) we must expand the function W in the small argument $k_z v_{Ti}/\omega_M \ll 1$ so that

$$\langle\varepsilon^2 W\rangle=\frac{ik_z v_T}{\sqrt{\pi}\,\omega_M}(1+i\pi\,\mathrm{sign}\,\omega_{Mi}), \qquad (6.14)$$

where $\mathrm{sign}\,\omega_{Mi} = \omega_{Mi}/|\omega_{Mi}|$. Substituting (6.14) in (6.12) we have

$$\omega=-\frac{\omega_{Pi}^*(1-i\pi\,\mathrm{sign}\,\omega_{Mi})}{1+i\pi\,\mathrm{sign}\,\omega_{Mi}}. \qquad (6.15)$$

It then follows that $\mathrm{Im}\,\omega < 0$, that is to say, the oscillations are damped. We also see that the ion temperature instability disappears in a plasma in which $\beta > 1$.

Let us summarize the results of the present section. As we have noted above, the existence of a large temperature gradient enhances the relative importance of effects such as magnetic drift and motion of the ions along the lines of force. In the case of the ion temperature instability these effects appear in a competitive way: the longitudinal ion motion favors the instability while the magnetic drift is a stabilizing factor. As a consequence of the competition between these two effects the case $\omega \gg k_z v_{Ti}$ exhibits a stabilization condition given by (6.6) while the case $\omega \ll k_z v_{Ti}$ exhibits a stabilization condition given by (6.11).

In a plasma in which $\beta \ll 1$ effects due to magnetic drift are small, going as β, and as a consequence these effects can be important only when there is another small parameter in addition to β in the problem. (For example, in the region $\omega \ll k_z v_{Ti}$ this small parameter is the quantity $k_z v_{Ti}/\omega_{Ti}$.) However, if $\beta \gtrsim 1$ the ion temperature instability disappears for any k_z.

7. CONCLUSION

The results obtained in the present work may be summarized as follows:

1. Possible longwave instabilities of a finite-pressure plasma can be associated with the excitation of two wave modes—Alfvèn-like drift waves or drift waves associated with the magnetosonic branch.

2. In the approximation used here $k^2\rho_i^2 \rightarrow 0$ the Alfvèn-like waves are not damped and are not excited for any η if $\beta < 1$; in a plasma in which $\beta \gtrsim 1$ these waves are associated with a hydrodynamic instability if τ satisfies the condition opposite to that given in (3.2).

3. From the point of view of plasma stability theory the most interesting result concerns drift magnetosonic waves. In particular, these waves are to be associated with the ion temperature instability in a plasma for which $\beta \ll 1$ and $\eta > 2$.[21, 24]

The analysis shows that as β increases the instability remains up to $\beta \sim 1$; the instability disappears when $\beta > 1$.

4. In a plasma in which $\beta \ll 1$ it is possible to have oscillations which are sensitive to the magnetic drift velocity of the particles. The characteristic frequency for these oscillations is of order

$$\omega_M \sim \frac{cT}{eB_0} k_\perp \frac{\partial \ln B_0}{\partial x}.$$

These oscillations can lead to an instability if $\partial \ln T/\partial \ln n_0 \neq 0$.

5. The plasma instability indicated in [4] for the case $\mathbf{k} \parallel \mathbf{B}_0$ is evidently unimportant in the case in which the magnetic field exhibits shear if the criterion in (4.7) is satisfied.

[1] L. Landau and E. M. Lifshitz, Electrodynamics of Continuous Media, Addison-Wesley, 1957.

[2] B. B. Kadomtsev, Reviews of Plasma Physics M. A. Leontovich, ed., Consultants Bureau, New York, 1966, Vol. 2.

[3] A. A. Vedenov, E. P. Velikhov and R. Z. Sagdeev, UFN **73**, 701 (1961), Soviet Phys. Uspekhi **4**, 332 (1961).

[4] Yu. A. Tserkovnikov, JETP **32**, 69 (1957), Soviet Phys. JETP **5**, 58 (1957).

[5] L. K. Rudakov and R. Z. Sagdeev, JETP **37**, 1337 (1959), Soviet Phys. JETP **10**, 952 (1960).

[6] N. Krall and M. Rosenbluth, Phys. Fluids **6**, 254 (1963).

[7] L. V. Mikhaĭlovskaya and A. B. Mikhaĭlovskiĭ, Nuclear Fusion **3**, 276 (1963).

[8] L. V. Mikhaĭlovskaya and A. B. Mikhaĭlovskiĭ, JETP **45**, 1566 (1963), Soviet Phys. JETP **18**, 1077 (1963).

[9] L. A. Artsimovich, Controlled Thermonuclear Reactions, Gordon and Breach, New York, 1963.

[10] Yu. V. Gott, M. S. Ioffe, and E. E. Yushmanov, Second International Conference on Plasma Physics and Controlled Thermonuclear Research, Culham, 1965, Paper CN 21/143.

[11] A. A. Galeev, S. S. Moiseev, and R. Z. Sagdeev, Atomnaya énergiya **15**, 451 (1963).

[12] N. V. Filippov and T. I. Filippova, Second International Conference on Plasma Physics and Controlled Thermonuclear Research, Culham, Paper CN 21/250.

[13] A. A. Galeev, A. M. Iskol'dskiĭ, V. N. Luk'yanov, Yu. E. Nesterikhin and A. G. Ponomarenko, Second International Conference on Plasma Physics and Controlled Thermonuclear Research, Culham, Paper CN 21/215.

[14] L. V. Mikhaĭlovskaya, ZhTF **37**, No. 6 (1967), Soviet Phys. Techn. Phys., in press.

[15] S. I. Braginskiĭ, Reviews of Plasma Physics, M. A. Leontovich, ed., Consultants Bureau, New York, 1965, Vol. 1.

[16] A. B. Mikhaĭlovskiĭ, Reviews of Plasma Physics, M. A. Leontovich, ed., Consultants Bureau, New York, 1967, Vol. 3.

[17] A. A. Galeev, JETP **44**, 1920 (1963), Soviet Phys. JETP **17**, 1292 (1963).

[18] V. P. Silin, JETP **44**, 1271 (1963), Soviet Phys. JETP **17**, 857 (1963).

[19] V. D. Shafranov, Reviews of Plasma Physics, M. A. Leontovich, ed., Consultants Bureau, New York, 1967, Vol. 3.

[20] A. B. Mikhaĭlovskiĭ and L. I. Rudakov, JETP **44**, 912 (1963), Soviet Phys. JETP **17**, 621 (1963).

[21] L. I. Rudakov and R. Z. Sagdeev, DAN SSSR **138**, 561 (1961), Soviet Phys. Doklady **6**, 415 (1961).

[22] B. B. Kadomtsev, A. B. Mikhaĭlovskiĭ and A. V. Timofeev, JETP **47**, 2266 (1964), Soviet Phys. JETP **20**, 1517 (1965).

[23] V. N. Fadeeva and N. M. Terent'ev, Tables of Values of the Probability Integral with Complex Argument, Gostekhizdat, 1954.

[24] A. A. Galeev, V. N. Oraevskiĭ, and R. Z. Sagdeev, JETP **44**, 903 (1963), Soviet Phys. JETP **17**, 615 (1963).

Translated by H. Lashinsky
174