THEORY OF THE ANISOTROPY OF RESISTANCE IN FERROMAGNETIC METALS

VU DINH KY

Moscow State University

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The anisotropy of resistance of ferromagnetic metals is explained by interaction between the orbital moment of the conduction electrons and the spin system. The cases of high and low temperatures are considered.

1. INTRODUCTION

It is known that the resistance of an isotropic polycrystalline ferromagnetic specimen magnetized to saturation depends on the angle ζ between the direction of the current and the direction of the magnetization, according to the formula

$$\rho = \rho_\perp + \Delta\rho_s \cos^2 \zeta, \qquad (1)$$

where $\Delta\rho_S = \rho_\| - \rho_\perp$; $\rho_\|$ and $\rho_\perp$ are the resistivities in the directions parallel and perpendicular, respectively, to the magnetization.

In papers of Vonsovskiĭ, Rodionov, and Shavrov[1, 2] this anisotropy of resistance was explained on the basis of the s–d model. In consequence of spin-spin and exchange interaction between the s and d electrons, the effective mass of a conduction electron becomes anisotropic, and therefore the resistance is also anisotropic.

Another explanation of the anisotropy of resistance of ferromagnetic metals was given in papers of Smit,[3] Marsocci,[4] and Berger.[5] The conductivity model used was that of Mott for transition metals. Account was taken of spin-orbit interaction for d electrons in the field of the lattice. Because of this interaction, the probability of transition into the d band is not the same for s electrons with different directions of motion. This leads to anisotropy of resistance. As was noted by Kasuya,[6] however, the Mott mechanism cannot give a correct explanation of the peculiarities of the conduction process in ferromagnetic metals, because in them an important role is played by scattering on spin inhomogeneities.

Despite this, use of a spin-orbit interaction mechanism in the theory of resistance anisotropy is well-founded. In fact, the anomalous Hall effect in ferromagnetic metals has recently been successfully explained by asymmetry of scattering due to spin-orbit interaction of electrons.[7, 8] Thus great interest attaches to a theory in which a mechanism of spin-orbit interaction is the common cause of the Hall effect and of the anisotropy of resistance. Kondo[9] considered exchange interaction between s and d electrons, taking account of spin-orbit interaction inside the ions. This author obtained both the Hall effect and the resistance anisotropy in, respectively, the first and second Born approximations. An assumption made by him, however, is unconvincing: that the ground state of the ions with an orbital moment is nondegenerate. Furthermore, the mechanism of spin-orbit interaction considered by Kondo turns out to be less important than other mechanisms of spin-orbit interaction, considered by Luttinger[7] and by Kagan and Maksimov.[8]

In [10] we considered a theory of the resistance anisotropy of ferromagnetic metals that took account, as in [7], of spin-orbit interaction for the electrons basically responsible for the spontaneous magnetization. It is obvious that such a point of view is possible only for d-ferromagnetic metals,[11] since it is in them that the electrons in question can take part in the conduction process.

The present paper treats resistance anisotropy that is due to interaction between the conduction-electron orbit and the spin system; that is, due to the mechanism of spin-orbit interaction studied in [8]. The influence of this interaction mechanism upon the electrical conductivity of ferromagnetic metals at low temperatures was studied by Turov.[12] Starting from an interaction between the conduction electrons and the spin system by way of the electromagnetic field produced by the spin waves, he derived an expression for the corresponding part of the conductivity:

$$\rho = a_1[1 + \ln (T / T_0)]T + a_2 T, \qquad (2)$$

where a_1 and a_2 are certain temperature-independent coefficients. Because no account was taken of the anisotropic character of the scattering of conduction electrons on magnons, in consequence of the influence of spin-orbit interaction, no anisotropy of resistance was obtained.

We consider below the Hamiltonian of interaction between s and d electrons, including not only exchange but also spin-orbit interaction. This enables us to derive the anisotropy of resistance both at low and at high temperatures, with application of the spin-wave and molecular-field approximations, respectively.

2. DERIVATION OF A GENERAL FORMULA FOR THE ANISOTROPY OF RESISTANCE

Following Vonsovskiĭ and Izyumov,[11] we can write the complete Hamiltonian of the system of conduction electrons and the spin system in the form

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{H}_{sd}, \quad (3)$$

where

$$\mathscr{H}_0 = \sum_{\mathbf{k}\sigma} \varepsilon_{\mathbf{k}}{}^{\sigma} a_{\mathbf{k}\sigma}{}^{+} a_{\mathbf{k}\sigma} - \sum_{j,\,j'=1}^{N} J(jj')(\mathbf{S}_j\mathbf{S}_{j'}), \quad (4)$$

N is the number of equivalent lattice sites occupied by atomic spins $\mathbf{S}_j$, and $\mathscr{H}_{sd}$ is the interaction between s and d electrons.

If we take account both of exchange and of spin-orbit interactions, then $\mathscr{H}_{sd}$ has the form

$$\mathscr{H}_{sd} = \mathscr{H}^{ex} + \mathscr{H}^{s\text{-}o}. \quad (5)$$

In the second-quantization representation, $\mathscr{H}^{ex}$ is given by the formula

$$\mathscr{H}^{ex} = -\frac{1}{N}\sum_{\mathbf{kk}'j} \exp\{i(\mathbf{k}'-\mathbf{k})\mathbf{R}_j\}$$
$$\times(\mathbf{kk}')[S_j^- a_{\mathbf{k}-}{}^{+} a_{\mathbf{k}'+} + S_j^+ a_{\mathbf{k}+}^{+} a_{\mathbf{k}'-}$$
$$+ (S_j^z - \langle S^z\rangle)(a_{\mathbf{k}-}{}^{+} a_{\mathbf{k}'-} - a_{\mathbf{k}+}{}^{+} a_{\mathbf{k}'+})\} \quad (5a)$$

(where z denotes the direction of the spontaneous magnetization).

The spin-orbit interaction of one conduction electron with the spin system has the form[8]

$$H^{s\text{-}o} = -\sum_j \frac{e^2\hbar}{2m^2c^2}\left(\left[\mathbf{S}_j\nabla\frac{1}{\mathbf{r}-\mathbf{R}_j}\right]\mathbf{p}\right). \quad (5b)$$

If we suppose that the free wave function

$$\psi_k = e^{i\mathbf{k}\cdot\mathbf{r}}, \quad (5c)$$

holds for the conduction electrons (crystal volume V = 1), then we easily find in the second-quantization representation

$$\mathscr{H}^{s\text{-}o} = -\frac{1}{N}\sum_{\mathbf{kk}'j\sigma} \exp\{i(\mathbf{k}'-\mathbf{k})\mathbf{R}_j\}[L^+(\mathbf{kk}')S_j^-$$
$$+ L^-(\mathbf{kk}')S_j^+ + L^z(\mathbf{kk}')(S_j^z - \langle S^z\rangle)](a_{\mathbf{k}\sigma}{}^{+} a_{\mathbf{k}'\sigma}), \quad (5d)$$

where

$$L^+ = {}^1/_2(L^x + iL^y), \qquad L^- = {}^1/_2(L^x - iL^y),$$
$$\mathbf{L}(\mathbf{kk}') = iL_0\frac{[\mathbf{kk}']}{|\mathbf{k}-\mathbf{k}'|^2}, \qquad L_0 = 8\pi N\left(\frac{e\hbar}{2mc}\right)^2. \quad (6)^*$$

For $\mathscr{H}^{s\text{-}o}$ there are no diagonal matrix elements. This follows from the assumption that the mean charge density of the electrons is compensated by the positive charge of the lattice ions. In (5a) and (5d) the periodic part of the interaction, proportional to $\langle S^z\rangle$, is included after averaging in $\mathscr{H}_0$. Thus, as also in [11], the energy of a conduction electron is given by the formula

$$\varepsilon_{\mathbf{k}}{}^{\sigma} = \varepsilon_{\mathbf{k}}{}^{0} - (s \pm \langle S^z\rangle)J(\mathbf{kk}). \quad (7)$$

In order to calculate the resistivity in the present case, it is convenient to use the method of Kubo and Nakano,[13, 14] according to which ρ_{sd} is given by the expression

$$\rho_{sd}{}^{\alpha\alpha} = \frac{\chi T}{\hbar^2\langle j_\alpha{}^2\rangle^2}\operatorname{Re}\int_0^\infty \langle[j_\alpha, \mathscr{H}_{sd}(\tau)][\mathscr{H}_{sd}, j_\alpha]\rangle\, d\tau, \quad (8)$$

where

$$\mathscr{H}_{sd}(\tau) = \exp\left(\frac{i\mathscr{H}_0\tau}{\hbar}\right)\mathscr{H}_{sd}\exp\left(-\frac{i\mathscr{H}_0\tau}{\hbar}\right), \quad (9)$$

and

$$j_\alpha = \frac{e}{\hbar}\sum_{\mathbf{k}\sigma}\frac{\partial\varepsilon_{\mathbf{k}}{}^{\sigma}}{\partial\mathbf{k}_\alpha} a_{\mathbf{k}\sigma}{}^{+} a_{\mathbf{k}\sigma}. \quad (10)$$

From (4), (5), (5a), (5d), and (10) we have:

$$\langle[j_\alpha, \mathscr{H}_{sd}(\tau)][\mathscr{H}_{sd}, j_\alpha]\rangle = A_\alpha + B_\alpha; \quad (11)$$

$$A_\alpha = \left(\frac{e}{\hbar}\right)^2\frac{1}{N^2}\sum_{\mathbf{k}'j_1j_2}\exp[i(\mathbf{k}'-\mathbf{k})(\mathbf{R}_{j_1}-\mathbf{R}_{j_2})]|J(\mathbf{kk}')|^2.$$
$$\times\left\{\exp\left[\frac{i\tau}{\hbar}(\varepsilon_{\mathbf{k}}{}^{-} - \varepsilon_{\mathbf{k}'}{}^{+})\right]\left(\frac{\partial\varepsilon_{\mathbf{k}}{}^{-}}{\partial k_\alpha} - \frac{\partial\varepsilon_{\mathbf{k}'}{}^{+}}{\partial k_\alpha{}'}\right)^2\right.$$
$$\times n_{\mathbf{k}}{}^{-}(1-n_{\mathbf{k}'}{}^{+})\langle S_{j_1}{}^{-}(\tau)S_{j_2}{}^{+}\rangle$$
$$\times\exp\left[\frac{i\tau}{\hbar}(\varepsilon_{\mathbf{k}}{}^{+} - \varepsilon_{\mathbf{k}'}{}^{-})\right]\left(\frac{\partial\varepsilon_{\mathbf{k}}{}^{+}}{\partial k_\alpha} - \frac{\partial\varepsilon_{\mathbf{k}'}{}^{-}}{\partial k_\alpha{}'}\right)^2$$
$$\times n_{\mathbf{k}}{}^{+}(1-n_{\mathbf{k}'}{}^{-})\langle S_{j_1}{}^{+}(\tau)S_{j_2}{}^{-}\rangle$$

*$[\mathbf{kk}'] \equiv \mathbf{k}\times\mathbf{k}'$.

$$+\exp\left[\frac{i\tau}{\hbar}(\varepsilon_{k}^{+}-\varepsilon_{k'}^{+})\right]\left(\frac{\partial\varepsilon_{k}^{+}}{\partial k_\alpha}-\frac{\partial\varepsilon_{k'}^{+}}{\partial k_\alpha'}\right)^2 n_k^{+}(1-n_{k'}^{+})$$
$$\times\langle(S_{j_1}^z-\langle S^z\rangle)(\tau)(S_{j_2}^z-\langle S^z\rangle)\rangle$$
$$+\exp\left[\frac{i\tau}{\hbar}(\varepsilon_{k}^{-}-\varepsilon_{k'}^{-})\right]\left(\frac{\partial\varepsilon_{k}^{-}}{\partial k_\alpha}-\frac{\partial\varepsilon_{k'}^{-}}{\partial k_\alpha'}\right)^2 n_k^{-}(1-n_{k'}^{-})\cdot$$
$$\times\langle(S_{j_1}^z-\langle S^z\rangle)(\tau)(S_{j_2}^z-\langle S^z\rangle)\rangle\Big\},$$

$$B_\alpha=\left(\frac{e}{\hbar}\right)^2\frac{1}{N^2}\sum_{kk'j_1j_2\sigma}\exp[i(k'-k)(R_{j_1}-R_{j_2})]. \quad (11a)$$

$$\{{}^1/_4[|L^x(kk')|^2+|L^y(kk')|^2][\langle S_{j_1}^-(\tau)S_{j_2}^+\rangle+\langle S_{j_1}^+(\tau)S_{j_2}^-\rangle]$$
$$+|L^z(kk')|^2\langle(S_{j_1}^z-\langle S^z\rangle)(\tau)(S_{j_2}^z-\langle S^z\rangle)\rangle\}$$
$$\times\exp\left[\frac{i\tau}{\hbar}(\varepsilon_k^\sigma-\varepsilon_{k'}^\sigma)\right]\left(\frac{\partial\varepsilon_k^\sigma}{\partial k_\alpha}-\frac{\partial\varepsilon_{k'}^\sigma}{\partial k_\alpha'}\right)^2 n_k^\sigma(1-n_{k'}^\sigma), \quad (11b)$$

and

$$S_j^{\pm,z}(\tau)=\exp\left(\frac{i\mathcal{H}_0\tau}{\hbar}\right)S_j^{\pm,z}\exp\left(-\frac{i\mathcal{H}_0\tau}{\hbar}\right), \quad (12a)$$

$$n_k^\sigma=\langle a_{k\sigma}^+a_{k\sigma}\rangle=\left[\exp\left(\frac{\varepsilon_k^\sigma-\varepsilon_F}{\chi T}\right)+1\right]^{-1}. \quad (12b)$$

From (11), (11a), and (11b) it is evident that the contribution to the resistance from the exchange and from the spin-orbit interactions are additive. We can therefore write

$$\rho_{sd}=\rho_{ex}+\rho_{s\text{-}o}$$

The part of the resistivity that is due to exchange interaction, ρ_{ex}, was considered in [6, 11, 15]. In the case of an isotropic ferromagnet, ρ_{ex} does not depend on the direction of the magnetization vector with respect to the current. As will be shown below, however, $\rho_{s\text{-}o}$ depends on the direction of the magnetization vector; that is, $\rho_{s\text{-}o}$ includes the magnetic anisotropy of the resistivity of ferromagnetic metals,

$$\Delta\rho_s=\rho_\parallel-\rho_\perp=\rho_{sd}^{zz}-\rho_{sd}^{xx}=\rho_{s\text{-}o}^{zz}-\rho_{s\text{-}o}^{xx}, \quad (13)$$

where

$$\rho_{c.o}^{\alpha\alpha}=\frac{\chi T}{\hbar^2\langle j_\alpha^2\rangle^2}\operatorname{Re}\int_0^\infty B_\alpha d\tau. \quad (14)$$

The system of formulas (11b), (12a), (12b) and (13), (14) enables us to obtain $\Delta\rho_s$. Below, we consider the special cases of low and high temperatures.

3. THE CASE OF LOW TEMPERATURES

At low temperatures, we can use the spin-wave approximation to calculate the correlator operators of the spins. We notice furthermore that in the low-temperature region, the terms containing the correlators $\langle(S_{j_1}^z-\langle S^z\rangle)(S_{j_2}^z-\langle S^z\rangle)\rangle$ can be neglected, since they correspond to two-magnon processes, and these give a contribution, according to Rösler,[16] smaller by a factor of about $5S^{-1}(T/T_c)^{3/2}$ than the contribution of a single-magnon process.

By means of the known representation of spin operators in terms of Bose operators b_q^+ and b_q, according to the formulas

$$S_j^+=\left(\frac{2S}{N}\right)^{1/2}\sum_q\exp(iR_jq)b_q,$$
$$S_j^-=\left(\frac{2S}{N}\right)^{1/2}\sum_q\exp(-iR_jq)b_q^+, \quad (15)$$

we have from (4), (9), (11b), and (12):

$$B_\alpha=\pi\hbar\left(\frac{e}{\hbar}\right)^2\frac{2S}{N}\sum_{kq\sigma}\Big\{\frac{1}{4}[|L^x(k,k-q)|^2+|L^y(k,k-q)|^2]$$
$$\times\left(\frac{\partial\varepsilon_k^\sigma}{\partial k_\alpha}-\frac{\partial\varepsilon_{k-q}^\sigma}{\partial(k-q)_\alpha}\right)^2 n_q n_k^\sigma$$
$$\times(1-n_{k-q}^\sigma)\delta(\varepsilon_k^\sigma-\varepsilon_{k-q}^\sigma-\varepsilon_q)+\frac{1}{4}[|L^x(k,k+q)|^2$$
$$+|L^y(k,k+q)|^2]\left(\frac{\partial\varepsilon_k^\sigma}{\partial k_\alpha}-\frac{\partial\varepsilon_{k+q}}{\partial(k+q)_\alpha}\right)^2$$
$$\times n_k^\sigma(1-n_{k+q}^\sigma)(1+n_q)\delta(\varepsilon_k^\sigma-\varepsilon_{k+q}^\sigma+\varepsilon_q)\Big\}, \quad (16)$$

$$n_q=\langle b_q^+b_q\rangle=[\exp(\varepsilon_q/\chi T)-1]^{-1}. \quad (16a)$$

In the effective-mass approximation, we get from (6), (14), and (16)

$$\rho_{c.o}^{\alpha\alpha}=\frac{\pi\chi T}{\hbar\langle j_\alpha^2\rangle^2}\left(\frac{e}{\hbar}L_0\right)^2\frac{S}{N}\left(\frac{\hbar}{m^*}\right)^2$$
$$\times\sum_{kq}\Big\{\frac{[kq]_x^2+[kq]_y^2}{q^4}q_\alpha^2[n_k(1-n_{k-q})$$
$$\times(n_q+1)\delta(\varepsilon_k-\varepsilon_{k-q}-\varepsilon_q)$$
$$+n_k(1-n_{k+q})n_q\delta(\varepsilon_k-\varepsilon_{k+q}+\varepsilon_q)]\Big\}. \quad (17)$$

After a standard calculation from (13) and (17), supposing that $T\gg T_0$, we have

$$\Delta\rho_s=\frac{3\pi}{160}\frac{S}{Ne^2}L_0^2\frac{\hbar}{\varepsilon_F^2}\Big\{\Big[\frac{\chi}{I}\ln\frac{2T}{T_0}+\frac{1}{4k_F^2}$$
$$\times\left(1+\left[\frac{2Im^*}{\hbar^2}\right]^2\right)\left(\frac{\chi}{I}\right)^2T_0\Big]T$$
$$-\frac{1}{k_F^2}\frac{\pi^2}{8}\left(1+\left[\frac{2Im^*}{\hbar^2}\right]^2\right)\left(\frac{\chi}{I}\right)^2T^2\Big\}. \quad (18)$$

In the derivation of (18) it was supposed that spin waves obey the quadratic dispersion law

$$\varepsilon_{\mathbf{q}} = I\mathbf{q}^2, \qquad I = \chi T_C a^2, \tag{19}$$

where T_C is the Curie temperature and a is the lattice constant. The characteristic temperature T_0, according to the estimate of Abel'skiĭ and Turov,[15] is of order 1° K for typical ferromagnetic metals.

4. THE CASE OF HIGH TEMPERATURES

To calculate the spin correlator operators in this case, we can use the molecular-field approximation:

$$\langle S_{j_1}^{\pm}(\tau) S_{j_2}^{\mp}\rangle = e^{\mp i\lambda\tau/\hbar}\langle S_j^{\pm}S_j^{\mp}\rangle\delta_{j_1j_2},$$
$$\langle (S_{j_1}^z - \langle S^z\rangle)(\tau)(S_{j_2}^z - \langle S^z\rangle)\rangle$$
$$= \langle (S_j^z - \langle S^z\rangle)(S_j^z - \langle S^z\rangle)\rangle\delta_{j_1j_2}. \tag{20}$$

From (11b) we have

$$\mathrm{Re}\int_0^\infty B_\alpha\, d\tau = 2\left(\frac{e}{\hbar}\right)^2\frac{\pi\hbar}{N}\sum_{\mathbf{kk}'}\Big\{\frac{1}{4}[|L^x(\mathbf{kk}')|^2 + |L^y(\mathbf{kk}')|^2]\cdot$$
$$\times\langle S^-S^+\rangle[n_{\mathbf{k}}(1-n_{\mathbf{k}'}) + n_{\mathbf{k}'}(1-n_{\mathbf{k}})e^{\lambda/\chi T}]$$
$$\times\left(\frac{\partial\varepsilon_{\mathbf{k}}}{\partial k_\alpha} - \frac{\partial\varepsilon_{\mathbf{k}'}}{\partial k_\alpha'}\right)^2\delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'} + \lambda)$$
$$+ |L^z(\mathbf{kk}')|^2 n_{\mathbf{k}}(1-n_{\mathbf{k}'})\left(\frac{\partial\varepsilon_{\mathbf{k}}}{\partial k_\alpha} - \frac{\partial\varepsilon_{\mathbf{k}'}}{\partial k_\alpha'}\right)^2$$
$$\times\langle (S^z - \langle S^z\rangle)^2\rangle\delta(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}'})\Big\}. \tag{21}$$

On substituting (21) into (13) and (14), we get in the effective-mass approximation

$$\Delta\rho_s = \frac{3\pi}{64}\frac{m^*L_0^2}{Ne^2\hbar\varepsilon_F}\left\{\frac{17}{15}\langle S^-S^+\rangle\frac{1}{1+e^{-y}} - \frac{8}{5}\langle (S^z - \langle S^z\rangle)^2\rangle\right\}, \tag{22}$$

where $y = \lambda/\chi T$ and

$$\langle(\ldots)\rangle = \sum_{m=-S}^{m=+S} e^{my}(\ldots) \Big/ \sum_{m=-S}^{m=+S} e^{my}. \tag{23}$$

If we suppose that $\langle (S^Z)^2\rangle = (\langle S^Z\rangle)^2$, then from (22) there follows

$$\Delta\rho_s = \frac{17\pi}{320}\frac{m^*L_0^2}{Ne^2\hbar\varepsilon_F}(S - \langle S^z\rangle)(S + \langle S^z\rangle + 1)\frac{1}{1+e^{-y}}. \tag{24}$$

In general the second term in braces in formula (22) is smaller than the first.

5. DISCUSSION OF RESULTS

It is known that for nonferromagnetic metals, the resistivity in the direction of the field is always less than the resistivity in a direction perpendicular to the field. On the other hand, for typical ferromagnetic metals (Fe, Ni, Co) and their alloys it is always observed that $\Delta\rho_S > 0$ $(\rho_{\parallel} > \rho_{\perp})$[3] over a wide temperature interval (from room temperature to the temperature of liquid hydrogen). This fact can be explained on the basis of formulas (24) and (18). At high temperatures it is clear from (24) that $\Delta\rho_S$ is always greater than zero. At low temperatures (but $T \gg T_0$), the condition that $\Delta\rho_S$ should be greater than zero follows from (18):

$$\ln\frac{2T}{T_0} - \alpha\frac{2T}{T_0} > 0, \qquad \alpha = \frac{\pi^2 T_0}{16k_F^2}\left(1 + \left[\frac{2Im^*}{\hbar^2}\right]^2\right)\frac{\chi}{I}. \tag{25}$$

The expression (25) makes it possible to discuss the dependence of the sign of $\Delta\rho_S$ on temperature. If $\alpha \geq 1/2.7$, then $\Delta\rho_S < 0$. For $\alpha \ll 1/2.7$, $\Delta\rho_S > 0$ at low temperatures. For intermediate values of α, a change of sign of $\Delta\rho_S$ can be observed. In the case of typical ferromagnetic metals ($T_C \approx 10^3$ °K, $k_F^2 \approx 10^{15}$ cm^{-2}), $\alpha \approx 10^{-2}$; it is therefore expected that $\Delta\rho_S > 0$, as was observed in [3]. For ferromagnetic rare earths, however, where T_1 is small, there was observed a complicated dependence of the value of $R_{\parallel} - R_{\perp}$ on temperature and on field direction.[17]

From (18) and (24) it is clear that the temperature dependences of the resistivity anisotropy $\Delta\rho_S$, due to a spin-orbit interaction mechanism, and of the resistivity ρ_{ex} due to exchange interaction between s and d electrons[6, 11, 15] resemble each other. In magnitude, however, ρ_{ex} is several orders larger than $\Delta\rho_S$, since J_0 $(J_0 = J(k_F, k_F))$ is enormously larger than L_0. We can estimate L_0 according to (6). For $N \approx 10^{23}$ cm^{-3}, the value of L_0 is about 2.5×10^{-16} erg. But this was based on the approximation of a free wave function for the conduction electrons; therefore, as was mentioned in [8], the value obtained above for L_0 may be appreciably below its real value, based on use of a Bloch wave function, in view of the appreciable localization of the latter near lattice sites. Thus if $J_0 \approx 10^{-14}$ erg for typical ferromagnetic metals, then $(L_0/J_0)^2 \approx 10^{-2}$ to 10^{-4}. It is just in this interval that the value of $\Delta\rho_S/\rho$ for ferromagnetic metals lies.

We can estimate $\Delta\rho_S$ at low temperatures according to (18). If $a \approx 10^{-8}$ cm and $T_C \approx 10^3$ deg, then $I \approx 10^{-29}$ erg cm^2 and $\chi/I \approx 10^{13}$ deg^{-1} cm^{-2}. If, furthermore, $\epsilon_F \approx 10^{-12}$ erg, then $\Delta\rho_S \approx 10^{-26}\,T$ [sec]; that is, of the same order as in the work of

Turov.[12] Thus it is no wonder that in [12], in a comparison with the experimental data of Sudovtsev and Semenenko,[18] a large discrepancy (the theoretical value smaller by a factor 1000) was obtained. The fact is that this part of the resistance, due to spin-orbit interaction, corresponds basically only to the resistance anisotropy, whereas a value of about 10^{-3} for $\Delta\rho_S/\rho$ is entirely possible.

Unfortunately, at present there are still no data for temperatures below the temperature of liquid hydrogen. At liquid-hydrogen temperature the value $\Delta\rho_S/\rho \approx 3\times10^{-2}$ for nickel was obtained in [3].

From (21)–(24) it follows that at temperatures close to the Curie temperature T_C, the value of $\Delta\rho_S$, like that of ρ_{ex}, rises rapidly and thereafter remains constant. Here we must make more precise the concept of $\Delta\rho_S$ at a temperature near T_C, where the spontaneous magnetization vanishes. The resistance then depends not only on the direction, but also strongly on the intensity of the field that is acting. Only extrapolation to $H = 0$ gives us $\Delta\rho_S$. On substituting the values quoted above for the parameters in (24), we get $\Delta\rho_S \approx 10^{-22}$ sec at $T \approx T_C$.

It is interesting to note that for measurement of $\Delta\rho_S$ in polycrystalline specimens, it is much more convenient to use the plane Hall effect.[19, 20]

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