EFFECT OF DIRECTED ELECTRON BEAM ON MOVING DISLOCATIONS

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The effect of conduction electrons drifting under the action of an external magnetic field on a dislocation moving in the direction of the drift is studied. General expressions are derived for the force exerted by the electrons on the dislocation in metals and semiconductors. The force is an accelerating one if the drift velocity exceeds the dislocation velocity. Numerical estimates of the accelerating force indicate that in some cases the effect may be observed experimentally.

1. INTRODUCTION

DISLOCATIONS in crystals can be set in motion under the influence of external mechanical stresses. As they move in the crystal, the dislocations are slowed down. If we disregard various obstacles produced by lattice imperfections, then the resistance to the dislocation motion is produced by forces of atomic nature (Peierls barrier) and dissipative forces. The former cannot be considered within the framework of the continual theory of dislocations, and require a microscopic approach for which there is still no consistent theory. However, if the kinetic energy of the dislocation exceeds the potential barrier connected with the discrete nature of the crystal, then forces of atomic nature should have little effect on the dislocation motion. A criterion for this condition is the inequality

$$V \gg s\sqrt{\sigma_s / G}, \qquad (1.1)$$

where V is the dislocation velocity, s the speed of sound, G the shear modulus, and σ_S the starting stress, which characterizes the height of the potential relief (typical values are $\sigma_S \sim 10^{-4}-10^{-5}$ G). When (1) is satisfied, we can use a consistent continual theory of dislocations to consider dissipative forces of a relaxation nature, connected with various mechanisms for transferring the dislocation energy to the crystal.

This raises the following question: is it possible to transfer energy from the crystal to the dislocation by some external action? Obviously, this can be realized only under conditions when any one of the subsystems forming the crystal is taken out of the equilibrium state and then, relaxing towards equilibrium, transfers part of the perturbation energy also to the perturbing moving dislocation. This subsystem may be the free carriers in the crystal (for concreteness we assume that these are electrons).

The electrons produce a unique force decelerating the moving dislocation. The mechanism of this effect is as follows: the moving elastic field perturbs the electron distribution [1)], and the return to equilibrium is accompanied by the appearance of a dissipative force which hinders the dislocation motion[1]. The situation should be different if an electric current is produced in the crystal and the system of electrons is itself not in equilibrium. The absorption of the elastic-wave energy by the electrons then gives way to a stimulated energy transfer to the perturbing field, provided the electron drift velocity exceeds the phase velocity of the elastic wave. The energy is transferred by Cerenkov radiation: the probability of emission of a quantum by the electron becomes larger than the absorption probability. In the case of a dislocation moving with constant speed, the phase velocity of the elastic-perturbation waves is the dislocation velocity.

The foregoing considerations allow us to expect to be able to excite with the aid of an electric current a force that accelerates moving dislocations. It then becomes possible to control the dislocation motion with the aid of nonmechanical forces.

To solve this problem it is necessary to consider simultaneously the kinetic equation for the conduction electrons, Maxwell's equations, and the equations of motion of the medium with the dislocations. Analyzing the variation of the free energy of the entire system, it is possible to reveal the addi-

[1)] We consider here only uncharged dislocations.

tional forces exerted on the lattice by the electrons, and the forces experienced by the dislocation. An analysis of this kind was used for a description of propagation of sound in metals in[2,3]. The dynamics of a medium with dislocations has many unique differences; in addition, it is necessary to take into account the action of the external electric field. In Sec. 2 we derive the main equations of the problem for metals, with account taken of the mutual influence of the system of electrons and the lattice with moving dislocations. In Sec. 3 we calculate the force accelerating the dislocations in the metals. Section 4 contains an analysis of the case of piezoelectric semiconductors, with due allowance for the distinguishing features arising in the interaction between the electrons and the deformation.

2. FUNDAMENTAL EQUATIONS IN METALS

The equations of the dynamics of a medium with moving dislocations is written in the form:

$$\rho\ddot{u}_i = g_i + \partial\sigma_{ik}/\partial x_k, \tag{2.1}$$

$$\dot{w}_{ik} = \partial\dot{u}_k/\partial x_i + J_{ik}. \tag{2.2}$$

Here $\mathbf{u}$ is the vector of geometric displacement of the points of the medium and $\dot{\mathbf{u}} = \partial\mathbf{u}/\partial t$ their velocity. By $\mathbf{g}$ are denoted the additional forces which are exerted by the electrons and are to be determined. The stress tensor is $\sigma_{ik} = \lambda_{iklm}w_{lm}$, where λ are the elastic moduli and w_{ik} the elastic-distortion tensor. We recall that w_{ik} is not expressed in terms of the derivatives of $\mathbf{u}$ with respect to the coordinates[4], for the latter include also the plastic deformation, the rate of change of which is described by the dislocation flux density tensor J_{ik}. For a single dislocation, J_{ik} is of the form[4]

$$J_{ik} = J_{ik}^0\delta(\xi); \qquad J_{ik}^0 = e_{ilm}q_lV_mb_k, \tag{2.3}$$

where e_{ilm} is a completely antisymmetrical tensor, $\mathbf{V}$ and $\mathbf{q}$ are the velocity and vector of the tangent to the dislocation line at a given point, and $\mathbf{b}$ is the Burger's vector; the two-dimensional radius vector $\xi(\mathbf{r}, t)$ describes the position of the dislocation line in space.

The system (2.1) and (2.2) must be supplemented by the equation of motion of the dislocation. For a dislocation-line element we write[5]

$$\int_L \mu_{ik}(l, l')\dot{V}_k(l')\,dl' = \mathcal{F}_i^0(l) + e_{ikm}q_k(l)\sigma_{mp}^e(l)b_p + \mathcal{F}_i. \tag{2.4}$$

Here μ_{ik} is the nonlocal density of the effective mass of the dislocation, and the integration is along the dislocation line L; $\mathcal{F}_i^0$ is the self-tension force connected with the self-energy W_s of the resting dislocation; σ^e is the external stress causing the dislocation to move. In (2.4) is included a still to be determined additional force $\mathcal{F}_i$, expected to result from the interaction with the electrons. (Equation (2.4) should include also other deceleration forces, but these will not be considered here.)

Let us multiply (2.1) by $\dot{u}_i$ and integrate over the volume of the crystal. Using (2.2) and (2.3), we obtain for the rate of change of the elastic-deformation energy $\dot{W}_{elast}$:

$$\dot{W}_{elast} \equiv \frac{1}{2}\frac{\partial}{\partial t}\int dV\,[\rho\dot{u}^2 + \lambda_{iklm}w_{ik}w_{lm}] =$$
$$= \int dV\dot{\mathbf{u}}\mathbf{g} - \int_L dl e_{ikm}q_k\sigma_{mp}b_pV_i + \int dV\frac{\partial}{\partial x_k}(\sigma_{ik}\dot{u}_i). \tag{2.5}$$

The change in the dislocation energy is obtained by multiplying (2.4) by V_i and integrating along the dislocation line. As a result we obtain

$$\dot{W}_d \equiv \frac{\partial}{\partial t}(W_d^{kin} + W_s) = \int_L dl(e_{ikm}q_k\sigma_{mp}^eb_pV_i + \mathcal{F}_iV_i),$$

where the kinetic energy is

$$W_d^{kin} = \frac{1}{2}\int dl\,dl'\mu_{ik}(l, l')V_i(l)V_k(l').$$

The total change in the energy of the elastic medium and of the dislocation is, apart from fluxes through the surface of the crystal (last term in (2.5)), which we shall omit throughout,

$$\dot{W}_{elast} + \dot{W}_d = \int dV\dot{u}_ig_i + \int_L dlV_i[\mathcal{F}_i - e_{ikm}q_k\sigma'_{mp}b_p]. \tag{2.6}$$

We have taken account here of the fact that $\sigma = \sigma^e + \sigma'$ (σ'—internal stresses). As shown by Kosevich and Natsik[6], the force exerted on the dislocation by the stresses σ' produced by it differs from zero only in the presence of dispersion of the elastic moduli. In our investigation we are analyzing in fact the dispersion introduced by the interaction with the electrons.

Let us proceed to consider the electronic part of the system. The electron distribution function f satisfies the kinetic equation

$$df/dt + \hat{\nu}f = 0, \tag{2.7}$$

where

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \frac{\partial\varepsilon}{\partial\mathbf{p}}\frac{\partial}{\partial\mathbf{r}} + \left(e\mathbf{E} - \frac{\partial\varepsilon}{\partial\mathbf{r}}\right)\frac{\partial}{\partial\mathbf{p}}; \tag{2.8}$$

$\hat{\nu}$ is the collision operator; $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$ is the sum of the external field $\mathbf{E}_0$ and the local field $\mathbf{E}_1$ produced upon the deformation of the crystal and determined by Maxwell's equations. We have left out

of (2.8) the Lorentz force $(e/c)[\partial\epsilon/\partial\mathbf{p} \times \mathbf{H}]$, since there is no external magnetic field, and the contribution of the solenoidal parts of $\mathbf{E}_1$ and of the magnetic fields associated with them can be neglected, owing to the smallness of $\partial\epsilon/\partial\mathbf{p}$ and $\dot{\mathbf{u}}$ compared with the speed of light.

In a local coordinate frame moving together with the lattice with velocity $\mathbf{u}$, we can introduce the concept of the conduction-electron dispersion in the undeformed medium[2]:

$$\varepsilon'(\mathbf{p}', \mathbf{r}', t) = \varepsilon_0(\mathbf{p}') + \Lambda_{ik}(\mathbf{p}')w_{ik} - m\dot{\mathbf{u}}\partial\varepsilon/\partial\mathbf{p} + \Delta'\varepsilon, \quad (2.9)$$

$\epsilon_0(\mathbf{p})$ is the electron energy in the undeformed lattice, and $\Lambda_{ik}(\mathbf{p}) = \Lambda_{ik}(-\mathbf{p})$ is the deformation-potential tensor. The term $m\dot{\mathbf{u}}\partial\epsilon/\partial\mathbf{p}$ takes into account the inertia force (the Stuart-Tolman effect, m—mass of free electron); $\Delta'\epsilon$ are terms that are quadratic in the deformation.

We transform to the laboratory frame (l.s.) in which the kinetic equation (2.7) is written, with the aid of the relations[2]

$$\mathbf{r} = \mathbf{r}' + \mathbf{u}(\mathbf{r}, t); \quad \mathbf{p}' = \mathbf{p} + \nabla(\mathbf{up});$$
$$\varepsilon'(\mathbf{r}', \mathbf{p}', t) = \varepsilon(\mathbf{r}, \mathbf{p}, t) - \dot{\mathbf{u}}\mathbf{p}. \quad (2.10)$$

Thus, the Hamiltonian in the l.s. is

$$\varepsilon(\mathbf{r}, \mathbf{p}, t) = \varepsilon_0(\mathbf{p}) + \delta\varepsilon(\mathbf{r}, \mathbf{p}, t);$$
$$\delta\varepsilon(\mathbf{r}, \mathbf{p}, t) = \Lambda_{ik}w_{ik} + \mathbf{v}\nabla(\mathbf{pu}) + \mathbf{p}\dot{\mathbf{u}} - m\dot{\mathbf{u}}\partial\varepsilon/\partial\mathbf{p} + \Delta\varepsilon,$$
$$\mathbf{v} = \partial\varepsilon_0/\partial\mathbf{p}. \quad (2.11)$$

As usual, we represent the distribution function by a sum of the instantaneous equilibrium and non-equilibrium parts:

$$f = f_0(\varepsilon - \mathbf{p}\dot{\mathbf{u}} - \mu) + \frac{\partial f_0}{\partial\varepsilon}\chi(\mathbf{r}, \mathbf{p}, t), \quad (2.12)$$

with $\hat{\nu}f_0 = 0$. In (2.12), the chemical potential has a deformation increment, namely $\mu = \mu_0 + \delta\mu(\mathbf{r}, t)$. It can be assumed that, owing to its large electric conductivity, the metal is electrically neutral also under conditions of variable deformation. Therefore both f_0 and f in (2.12) can be normalized to the instantaneous electron density in the deformed lattice, n, which in the approximation linear in the deformation is equal to $n_0(1 - u_{ii})$, where

$$n_0 = \int d\tau_p f_0(\varepsilon_0 - \mu_0), \quad d\tau_p = \frac{2}{h^3}d\mathbf{p}.$$

From the normalization conditions it follows that

$$\bar{\chi} = 0, \quad \delta\mu = \bar{\Lambda}_{ik}w_{ik}. \quad (2.13)$$

We have introduced here the notation

$$\bar{\varphi} = \int d\tau_p \frac{\partial f_0}{\partial\varepsilon}\varphi \Big/ \int d\tau_p \frac{\partial f_0}{\partial\varepsilon}; \quad \int d\tau_p \frac{\partial f_0}{\partial\varepsilon} \equiv -\frac{\partial n_0}{\partial\mu_0}. \quad (2.14)$$

The total current density in the approximation linear in the deformation is

$$\mathbf{j} = \mathbf{j}_{el} + \mathbf{j}_{ion} = e\int d\tau_p f\frac{\partial\varepsilon}{\partial\mathbf{p}} - en\dot{\mathbf{u}} = e\int d\tau_p \frac{\partial f_0}{\partial\varepsilon}\chi\frac{\partial\varepsilon}{\partial\mathbf{p}}. \quad (2.15)$$

As shown in[2], in the l.s. the average electron energy is connected with the Hamiltonian averaged over the quasi-particle distribution by the equality

$$W_{el} = \int dV\, d\tau_p f\left[\varepsilon - \dot{\mathbf{u}}\left(\mathbf{p} - m\frac{\partial\varepsilon}{\partial\mathbf{p}}\right)\right]. \quad (2.16)$$

The free energy of the electrons is

$$F_{el} = W_{el} - TS_{el}; \quad S_{el} =$$
$$-\int dV d\tau_p[(1-f)\ln f + f\ln f] \quad (2.17)$$

(T—temperature, S—entropy). Assuming that the system is in a thermostat, we get $\dot{F}_{el} = W_{el} - TS_{el}$ We note that if account is taken of (2.8) we obtain, accurate to the fluxes through the surface of the crystal, which we neglect throughout, the following for any quantity φ:

$$\int d\tau_p\dot{\varphi} = \int d\tau_p \frac{d\varphi}{dt}.$$

Using this relation, Eq. (2.7), and formulas (2.8), (2.11), (2.12), and (2.15), we obtain, accurate to terms quadratic in the deformation:

$$\dot{F}_{el} = \int dV\, d\tau_p\left[\frac{df}{dt}\left(\varepsilon - \mathbf{p}\dot{\mathbf{u}} + T\ln\frac{f}{1-f}\right) + f\frac{d}{dt}(\varepsilon - \dot{\mathbf{u}}\mathbf{p})\right.$$
$$\left. + \frac{\partial}{\partial t}\left(fm\dot{\mathbf{u}}\frac{\partial\varepsilon}{\partial\mathbf{p}}\right)\right] = \int dV\, d\tau_p \chi\hat{\nu}f$$
$$+ \int dV\left\{\int d\tau_p\left[\frac{\partial f_0}{\partial\varepsilon}\chi\Lambda_{ik}\dot{w}_{ik}\right.\right.$$
$$\left.\left. + eE_i\left(\frac{\partial\varepsilon}{\partial p_i} - \dot{u}_i\right)f + f\tilde{\Delta\varepsilon}\right] + \frac{m}{e}\dot{u}_i\frac{\partial j_i}{\partial t}\right\}. \quad (2.18)$$

$\tilde{\Delta\epsilon}$ combines the quadratic terms of the type $\dot{u}w$, $\ddot{u}u$, etc., which lead to renormalization of the moduli λ and of the density of the medium ρ, but which do not introduce dispersion. We shall henceforth omit the term with $\tilde{\Delta\epsilon}$.

Transforming the second term of (2.17) with the aid of (2.2) and (2.3), and separating from it the surface integral, we get

$$\dot{F}_{el} = \int dV\, d\tau_p \frac{\partial f_0}{\partial\varepsilon}\chi\hat{\nu}\chi - \int dV\left\{\dot{u}_k\right.$$
$$\left.\times\left[\frac{\partial}{\partial x_i}\int d\tau_p\frac{\partial f_0}{\partial\varepsilon}\chi\Lambda_{ik} - \frac{m}{e}\frac{\partial j_k}{\partial t}\right] - \mathbf{jE}\right\}$$
$$- \int_L dl e_{ikm}q_k b_p V_i \int d\tau_p \frac{\partial f_0}{\partial\varepsilon}\chi\Lambda_{mp}. \quad (2.19)$$

In writing the term with the field E, we took account of (2.15) and (2.13).

Assuming the electromagnetic field and the lattice to be mechanical systems whose entropy does not change, and taking into account the equality

$$\dot{W}_{\text{em}} = -\int dV \left\{ \mathbf{jE} + \frac{c}{4\pi} \operatorname{div} [\mathbf{EH}] \right\},$$

we obtain for the change in the free energy of the crystal

$$\begin{aligned} \dot{F} &= \dot{F}_{\text{el}} + \dot{W}_{\text{em}} + \dot{W}_{\text{elast}} + \dot{W}_{\text{d}} \\ &= \int dV \, d\tau_p \frac{\partial f_0}{\partial \varepsilon} \chi \hat{\nu} \chi + \int dV \dot{u}_k \\ &\times \left[g_k + \frac{m}{e} \frac{\partial j_k}{\partial t} - \int \frac{\partial}{\partial x_i} \chi \frac{\partial f_0}{\partial \varepsilon} \Lambda_{ik} d\tau_p \right] \qquad (2.20) \\ &+ \int_L dl \, V_i \left[\mathcal{F}_i - e_{ikm} q_k b_p \left(\sigma'_{mp} + \int d\tau_p \frac{\partial f_0}{\partial \varepsilon} \chi \Lambda_{mp} \right) \right]. \end{aligned}$$

According to the general rules F should be a decreasing quantity for all processes. By virtue of the inequality

$$\int d\tau_p \frac{\partial f_0}{\partial \varepsilon} \chi \hat{\nu} \chi < 0$$

this requirement will be satisfied if we put

$$g_k = \frac{\partial}{\partial x_i} \int d\tau_p \frac{\partial f_0}{\partial \varepsilon} \chi \Lambda_{ik} - \frac{m}{e} \frac{\partial j_k}{\partial t}, \qquad (2.21)$$

$$\mathcal{F}_i(l) = e_{ikm} q_k(l) b_p \left(\sigma_{mp}'(l) + \int_l d\tau_p \frac{\partial f_0}{\partial \varepsilon} \chi(l) \Lambda_{mj} \right). \qquad (2.22)$$

Thus, the system of equations to be solved has now been completely determined. It consists of the equations of motion of the medium (2.1), (2.2) with (2.3), and (2.21), the equation of motion of the dislocation (2.12) with (2.22), the kinetic equation (2.7) with (2.12), which determines the function χ, and the Maxwell's equations for $\mathbf{E}_f$.

We simplify the problem by confining ourselves to the following case: let a straight-line dislocation parallel to the z axis (with $q_k = q_z = -1$) move along the x axis. The electrons drift in the external field $\mathbf{E}_0$ also along the x axis. We seek the force $\mathcal{F}_i$ (2.22) acting on the dislocation, but neglect the influence of the change of its velocity V on the deformation of the medium, that is, in solving (2.1) and (2.2) we write for the flux ((1, 2, 3) = (x, y, z))

$$J_{ik} = J_{ik}^0 \delta(x - Vt) \delta(y); \quad J_{ik}^0 = J_{2k}^0 = -V b_k, \quad V = \text{const}. \qquad (2.23)$$

Further simplifications involve the solution of the kinetic equation and the determination of $\mathbf{E}_1$. We assume that the collision integral can be written in the approximation of the relaxation-time τ in the form[7]

$$\hat{\nu} f = \frac{1}{\tau} \frac{\partial f_0}{\partial \varepsilon} (\chi - \bar{\chi}) \qquad (2.24)$$

(the requirement $\int d\tau_p \hat{\nu} f = 0$ is satisfied when $\hat{\nu} f$ is written in this form). We represent χ in the form of two parts:

$$\chi = \chi_E + \chi_u, \qquad (2.25)$$

where $\chi_E \sim E_0$ is connected with the disturbance of the equilibrium by the external field $\mathbf{E}_0$, and χ_u is caused by the deformation. Taking (2.12), (2.13), and (2.24) into account, Eq. (2.7) assumes, in the approximation in the deformation and in the field $\mathbf{E}_0$, the form

$$\begin{aligned} \frac{\partial f_0}{\partial \varepsilon} \Big[& \frac{\partial}{\partial t} (\delta\varepsilon - \mathbf{p}\dot{\mathbf{u}} - \delta\mu) + \mathbf{v}\nabla (\delta\varepsilon - \mathbf{p}\dot{\mathbf{u}} - \delta\mu) \\ & + \left(eE_1 - \frac{\partial \varepsilon}{\partial \mathbf{r}} \right) \left(\mathbf{v} + \frac{\partial \chi_E}{\partial \mathbf{p}} \right) + e\mathbf{E}_0 \frac{\partial}{\partial \mathbf{p}} (\varepsilon - \mathbf{p}\dot{\mathbf{u}} + \chi_u) \\ & + \left(\frac{\partial}{\partial t} + \mathbf{v}\nabla + \frac{1}{\tau} \right) \chi_u + \frac{1}{\tau} \chi_E \Big] \\ & + \frac{\partial^2 f_0}{\partial \varepsilon^2} \mathbf{v} \left[\chi_E \left(e\mathbf{E}_1 - \frac{\partial \varepsilon}{\partial \mathbf{r}} \right) + e\mathbf{E}_0 \chi_u \right] = 0. \end{aligned}$$

The influence of the field $\mathbf{E}_0$ on the lattice, which is the important factor in this problem, is described by the terms $\sim E_0 l l$ ($\mathbf{E}_1 \partial \chi_E / \partial \mathbf{p}$ etc.). It can be shown that the main features of the phenomena will still be represented if we retain only the term $[e\mathbf{E}_1 - \nabla(\lambda_{ik} w_{ik})] \partial \chi_E / \partial \mathbf{p}$, and leave out all others. This greatly simplifies the solution of the kinetic equation, without exerting an appreciable influence on the results. Such an approximation was used in[8] to solve the problem of ultrasound amplification. Using (2.11), we write down the equations for χ_E and χ_u in the stipulated approximation

$$\chi_E = -\mathbf{p}_0 \mathbf{v}, \quad \mathbf{p}_0 = e\tau \mathbf{E}, \quad \mathbf{V}_0 = \frac{\partial}{\partial \mathbf{p}} (\mathbf{p}_0 \mathbf{v}); \qquad (2.26)$$

$$\begin{aligned} \left(\frac{\partial}{\partial t} + \mathbf{v}\nabla + \frac{1}{\tau} \right) \chi_u = & -(\Lambda_{ik} - \bar{\Lambda}_{ik}) \dot{w}_{ik} \\ & + \mathbf{v} (m\ddot{\mathbf{u}} - e\mathbf{E}_1 + \nabla \delta\mu) + [e\mathbf{E}_1 - \nabla (\Lambda_{ik} w_{ik})] \mathbf{V}_0, \end{aligned} \qquad (2.27)$$

$\mathbf{V}_0$ is the electron drift velocity.

The field $\mathbf{E}_1$ in (2.27) is determined by the simultaneous solution of (2.27) and Maxwell's equations. It is, however, possible to find $\mathbf{E}_1$ in a simpler manner—with the aid of the already used electroneutrality condition $\bar{\chi} = 0$[7]. It is easy to see that the term $m\mathbf{v} \cdot \ddot{\mathbf{u}}$ in (2.27) can be neglected ($\Lambda \sim \mu_0 \sim mv^2$, and the product of the characteristic frequencies by the characteristic scales of the deformation is much smaller than v). For this reason we can omit $me^{-1} \partial \mathbf{j} / \partial t$ from (2.21).

As a result of the foregoing approximations, the complete system of equations for the simplified problem consists of (2.1), (2.2) with (2.21), (2.23),

(2.27), (2.13), and the expression (2.22) for the force acting per unit dislocation length, where we must put x = Vt and y = 0. In (2.27) and (2.21) we can omit terms of inertial origin (v̈üm and $me^{-1}\partial j/\partial t$).

3. SOLUTION OF EQUATIONS IN METALS

The problem is solved in the Fourier representation, for which all the quantities that depend on r and t are expanded in a two-dimensional Fourier integral

$$\varphi(\mathbf{r},t)=\frac{1}{(2\pi)^2}\int dk_1 dk_2 \varphi^k \exp\left(ik_1x+ik_2y-i\omega t\right);\ \omega=Vk_1 \tag{3.1}$$

(such a dependence on t is determined by the form of J_{ik} in (2.23); by virtue of the symmetry of the problem, there is no z-dependence).

Omitting the intermediate steps, we present the expression obtained from (2.27) for χ_u^k, from which the longitudinal field E_1 is eliminated by the condition (2.13):

$$\chi_u{}^k=\chi_{mn}^k w_{mn}^k;\ \chi_{mn}^k=i\omega'\tau\left\{\frac{1}{1-i\omega\tau+i\mathbf{kv}\tau}\left[\Lambda_{mn}-\overline{\Lambda}_{mn}+\frac{1-i\omega'\tau}{1+i\omega'\tau Y/(1-Y)}L_{mn}\frac{Y}{1-Y}\right]-\frac{Y}{1-Y}L_{mn}\frac{1}{1+i\omega'\tau Y/(1-Y)}\right\}. \tag{3.2}$$

We used the notation: $\omega'=\omega-\mathbf{k}\cdot\mathbf{V}_0=k_1(V-V_0)$,

$$Y=\left(\overline{\frac{1}{1-i\omega\tau+i\mathbf{kv}\tau}}\right),\ L_{mn}=\frac{1}{Y}\left(\overline{\frac{\Lambda_{mn}-\overline{\Lambda}_{mn}}{1-i\omega\tau+i\mathbf{kv}\tau}}\right). \tag{3.3}$$

(We recall that a superior bar denotes averaging over the Fermi surface, see (2.14).)

According to (2.1) and (2.21), the Fourier transform of the effective stress tensor

$$\tilde{\sigma}_{ik}=\lambda_{iklm}w_{lm}+\int d\tau_p\frac{\partial f_0}{\partial\varepsilon}\chi\Lambda_{ik}$$

takes the form

$$\tilde{\sigma}_{jl}{}^k=\lambda_{jlmn}^k w_{mn}^k;\quad \lambda^k=\lambda+\delta\lambda^k,$$
$$\delta\lambda_{jlmn}^k=\int d\tau_p\frac{\partial f_0}{\partial\varepsilon}\chi_{mn}^k\Lambda_{jl}. \tag{3.4}$$

From (2.1) and (2.2) we obtain an expression for w_{jl}^k [6]:

$$w_{jl}{}^k=\frac{i}{\omega}\left(J_{jl}{}^0-k_jk_pG_{ln}^k\lambda_{npqm}^k J_{qm}^0\right), \tag{3.5}$$

where G_{ln}^k is the Fourier transform of the Green's tensor of the dynamic elasticity-theory problem, in which the moduli are λ^k. Relations (3.1), (3.4), and (3.5) determine the effective stress tensor

$$\tilde{\sigma}_{mn}(\mathbf{r},t)=\frac{1}{(2\pi)^2}\int dk_1\,dk_2\exp\left(ik_1x+ik_2y-i\omega t\right)B_{mn}^k, \tag{3.6}$$

$$B_{mn}^k=\frac{i}{\omega_k}\lambda_{mnjl}^k\left[J_{jl}{}^0-k_jk_pG_{li}{}^k\lambda_{ipqs}^k J_{qs}{}^0\right]. \tag{3.7}$$

According to (2.22) and (3.6), we obtain an expression for the force $\mathcal{F}_1$ acting per unit length of the dislocation and the direction of the dislocation motion:

$$\mathcal{F}_1=\frac{1}{(2\pi)^2}\int dk_1\,dk_2\,b_nB_{2n}{}^k. \tag{3.8}$$

For further calculations we need the concrete form of the Green's tensor G_{ik}. In the general case of an anisotropic crystal this is a complicated problem. For order-of-magnitude estimates we resort to further simplifications: we find that part of the force $\mathcal{F}$, which is due to the electronic part of the stress tensor (the second term in (2.22)) without taking into account the influence of $\delta\lambda^k$ on the values of w_{ij}. This means that we neglect the "elastic screening" of the action of the dislocation, which should become manifest here via the dependence of w_{ij} on $\delta\lambda^k$. Estimates made in an approximation linear in $\delta\lambda^k$ for an isotropic medium have shown that such an approach greatly overestimates the result. In addition, the numerical estimates can be greatly simplified by considering only the first term in the square brackets of (3.7). This is permissible with accuracy to several fractional coefficients containing the ratios of the elastic constants, so that the results do not change seriously.

Thus, for numerical estimates let us consider

$$\mathcal{F}_1\sim\frac{1}{(2\pi)^2}\int dk_1\,dk_2\frac{i}{\omega}b_nJ_{jl}{}^0\delta\lambda_{2njl}^k=-\frac{i}{(2\pi)^2}b_nb_l\times\int dk_1\,dk_2\frac{\delta\lambda_{2n2l}}{k_1}. \tag{3.9}$$

The quantity $\delta\lambda_{\mathrm{mnj}l}^{\mathrm{k}}=-\overline{\chi_{\mathrm{mn}}^{\mathrm{k}}\Lambda_{\mathrm{j}l}}\partial n_0/\partial\mu$ (we used (3.4) and (2.14)) is similar to that determining the absorption of ultrasound with wave vector k in metals[7]. Just as in[7], we can show that $\delta\lambda^k$ can be estimated by the expression

$$\delta\lambda^k\sim-i\omega'\tau\frac{\partial n_0}{\partial\mu}\left[\frac{(\Lambda-\overline{\Lambda})(\Lambda-\overline{\Lambda})}{1-i\omega\tau+i\mathbf{kv}\tau}\right]\sim-ik_1\tau(V-V_0)\frac{\partial n_0}{\partial\mu}\Delta^2Y(k). \tag{3.10}$$

Here Δ is the constant of the deformation potential ($\Lambda-\overline{\Lambda}\sim\Delta$). The function Y(k) (3.3) varies from unity when $kl\ll1$ to $\pi/2kl$ when $kl>1$[7] ($l=\tilde{v}\tau$ is the electron mean free path and $\tilde{v}$ the electron velocity on the Fermi surface). Substituting (3.10) in (3.9) we obtain

$$\mathcal{F}_1\sim\frac{1}{2\pi}\left(\frac{V_0}{V}-1\right)V\frac{\partial n_0}{\partial\mu}\Delta^2\tau b^2\int dk\,kY(k). \tag{3.11}$$

The integral in (3.11) diverges at the upper limit, because the methods of elasticity theory are not applicable at distances on the order of the radius of the dislocation nucleus. This nonphysical divergence is eliminated by usual cutoff at $k_0 \sim 1/b$.

We assume that the case

$$k_0 l > 1 \tag{3.12}$$

is realized. Then the main contribution to (3.11) is made by large $k \sim k_0$, that is,

$$\mathcal{F}_1 \sim \frac{b}{4}\left(\frac{V_0}{V}-1\right)\frac{V}{\tilde{v}}\frac{\partial n_0}{\partial \mu}\Delta^2. \tag{3.13}$$

The foregoing estimate is suitable for the case of arbitrary nonspherical Fermi surface, which can be distorted by both compression and shear deformations, causing the appearance of the corresponding Λ_{jl}, which generally speaking are different for different types of deformations. Therefore, although (3.13) is suitable for both edge and screw dislocations, the values of the parameters Δ for the screw dislocation (shear only) and for the edge dislocation (shear and compression) can differ noticeably.

Analogous results for the force of dislocation deceleration in the absence of an external electric field ($V_0 = 0$) were obtained by another method in an earlier paper by the author[1]. According to (3.13), when $V_0 > V$, as expected from general considerations, the deceleration gives way to acceleration.

4. PIEZOELECTRIC SEMICONDUCTORS

The analysis of the electron-lattice coupling in semiconductors differs in many respects from the case of metals. The radius of the Debye screening is here, as a rule, large and therefore the condition for the instantaneous electro-neutrality, $\overline{\chi} = 0$, will not be satisfied for short waves with $k \sim k_0$. In addition to the deformation interaction, a role is played also by other mechanisms of electron couplings with the lattice, which come into play via the field E_1. In piezoelectric semiconductors, the most important is the piezoelectric coupling between the field and the deformation. If the parameter of this coupling is sufficiently large, then in calculating the force acting on the lattice it is possible to disregard completely the deformation change in the law of electron dispersion $\delta\epsilon$ (deformation interaction with electrons in semiconductors, leads, as can be shown to an expression analogous to (3.13) for the force $\mathcal{F}_1$, with "nonscreened" constants of the deformation potential).

We present below an analysis of a piezoelectric situated in an external field, without account of $\delta\epsilon$. We reformulate, to fit this case, the derivation of the expression for the forces g_i in (2.1) and $\mathcal{F}_i$ in (2.4). As is well known, in a piezoelectric[9]

$$\sigma_{jl} = \lambda_{jlmn} w_{mn} + \beta_{m,\,jl} E_m, \tag{4.1}$$

$$D_i = \varkappa_{il} E_l - 4\pi\beta_{j,\,mn} w_{mn}; \tag{4.2}$$

D is the electric induction, κ_{il} is a tensor of the dielectric constant, and $\beta_{i,\,mn}$ is the piezoelectric tensor. Multiplying (2.1) by $\dot{u}_i$ and integrating, we obtain, taking (4.1) into account

$$\dot{W}_{\text{elast}} = \int dV \dot{\mathbf{u}}\mathbf{g} - \beta_{m,\,jl}\int dV\, E_m \dot{w}_{jl} - \int_L dl V_i e_{imn} q_m \sigma_{np} b_p + \int dV \frac{\partial}{\partial x_j}(\sigma_{jl}\dot{u}_l). \tag{4.3}$$

The change in the energy of the electromagnetic field W_{em} with allowance for (4.2) and Maxwell's equations, is represented in the form

$$\dot{W}_{\text{em}} = \frac{\partial}{\partial t}\int dV \frac{1}{8\pi}(\varkappa_{il}E_iE_l + H^2) = \frac{1}{4\pi}\int dV\,[\dot{\mathbf{D}}\mathbf{E} + \dot{\mathbf{H}}\mathbf{H} + 4\pi\beta_{i,\,jl}E_i\dot{w}_{jl}] = \int dV\left\{\beta_{i,\,jl}E_i\dot{w}_{jl} - \mathbf{j}\mathbf{E} - \frac{c}{4\pi}\operatorname{div}[\mathbf{E}\mathbf{H}]\right\}. \tag{4.4}$$

The expression for $\dot{W}_d$ remains the same as in Sec. 2. In the formula for $\dot{F}_{el}$ (2.18) we omit the terms connected with $\delta\epsilon$, but we take into consideration the violation of electro-neutrality ($\overline{\chi} \neq 0$). Then the term of (2.18) containing $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$ is transformed with the aid of (2.15) into

$$-\int d\tau_p \frac{\partial f_0}{\partial \varepsilon}\chi\mathbf{E}_0\dot{\mathbf{u}} + \mathbf{j}\mathbf{E}.$$

As a result the change in the total free energy of the system is written in the form

$$\dot{F} = \int dV\, d\tau_p \chi\hat{\nu}f + \int dV\dot{u}_i\left[g_i - E_{0i}\int d\tau_p \frac{\partial f_0}{\partial \varepsilon}\chi\right] + \int_L dl V_i\,[\mathcal{F}_i - e_{imn}q_m b_p \sigma'_{np}]. \tag{4.5}$$

From this, just as in Sec. 2, it follows that

$$\mathbf{g} = e\mathbf{E}_0\int d\tau_p \frac{\partial f_0}{\partial \varepsilon}\chi, \tag{4.6}$$

$$\mathcal{F}_i(l) = e_{imn}q_m(l)\,b_p\sigma_{np}'(l) = e_{imn}q_m(l)\,b_p[\lambda_{npjs}w_{js}(l) + \beta_{j,\,np}E_{1j}(l)]. \tag{4.7}$$

The physical meaning of (4.6) is clear: this is the force exerted by the external field on local nonequilibrium charge produced by the deformation.

In the same approximations as in Sec. 2, the total system of the equations of the problem consist of (2.1), (2.2) with (2.23), (4.6), and the equations for χ_u and $\mathbf{E}_1$:

*$[\mathbf{E}\mathbf{H}] \equiv \mathbf{E} \times \mathbf{H}$.

$$\left(\frac{\partial}{\partial t}+\mathbf{v}\nabla+\frac{1}{\tau}\right)\chi_u=\frac{\bar{\chi}_u}{\tau}-e\mathbf{E}_1(\mathbf{v}-\mathbf{V}_0), \quad (4.8)$$

$$\operatorname{div}\mathbf{D}=4\pi e\int d\tau_p\frac{\partial f_0}{\partial\varepsilon}\chi_u=-4\pi e\frac{\partial n_0}{\partial\mu}\chi_u \quad (4.9)$$

with allowance for (4.2). The force $\mathcal{F}_i$ is determined by formula (4.7) with x = Vt and y = 0.

Going over to a Fourier representation in accord with (3.1), we get χ_u^k and E_1^k from (4.8) and (4.9):

$$\chi_u{}^k=\frac{ie}{k}E_1{}^k\left[1+\frac{i\omega'\tau}{1-Y}\frac{1}{1-i\omega\tau+i\mathbf{k}\mathbf{v}\tau}\right], \quad (4.10)$$

$$E_1{}^k=\frac{4\pi k\beta_{i,jl}k_iw_{jl}{}^k}{\varkappa_{mn}k_mk_n}\frac{R^2k^2}{1+R^2k^2+i\omega'\tau Y/(1-Y)}$$

$$R^2k^2=\frac{\varkappa_{mn}k_mk_n}{4\pi e^2\partial n_0/\partial\mu}, \quad (4.11)$$

R is the Debye screening radius.

We shall make a numerical estimate of $\mathcal{F}_i$ in the same approximation as in Sec. 3, that is, we shall disregard the dispersion of the effective moduli in determining w_{ik}. Then the first term of (4.7) will make no contribution to F_i[6], whereas allowance for the second term (4.7) using (4.11), leads to the expression

$$\mathcal{F}_1=\frac{1}{(2\pi)^2}\int dk_1\,dk_2\,b_p\delta\lambda_{2pjl}\,w_{jl}^n, \quad (4.12)$$

$$\delta\lambda'_{2pjl}=-i\omega'\tau\frac{\beta_{m,2p}\beta_{n,jl}k_nk_m}{e^2\partial n_0/\partial\mu}z_1(k)Q(k). \quad (4.13)$$

Here

$$Q^{-1}=(1+R^2k^2-\omega'\tau z_2)^2+(\omega'\tau z_1)^2;$$

$$z_1=\operatorname{Re}\frac{Y}{1-Y},\quad z_2=\operatorname{Im}\frac{Y}{1-Y}, \quad (4.14)$$

Y is defined by (3.3) and w^k by (3.5), where dispersion is disregarded in the moduli of λ.

In order not to complicate the derivations, we consider again, just as in Sec. 3, only the first term of (3.5); for an order-of-magnitude estimate this is sufficient, as can be readily verified. Then we must estimate the integral

$$\mathcal{F}_1\sim\frac{1}{(2\pi)^2}\int dk_1\,dk_2\frac{i}{\omega}b_pJ_{jl}{}^0\delta\lambda'_{2pjl}$$
$$=\frac{1}{(2\pi)^2}(V_0-V)\frac{\tau}{e^2\partial n_0/\partial\mu}$$
$$\times\ \beta_{s,2l}\beta_{m,2p}b_pb_l\int dk_1\,dk_2\,k_mk_sz_1Q. \quad (4.15)$$

The complicated integrand in (4.15) depends on the parameters l, R, V, and V_0. Subject to certain readily realizable relations between these parameters, the estimate is relatively easy to obtain. In the case of a spherical equal-energy surface, taking into account the fact that $kl\gg\omega$ always, we obtain for Y:

$$Y=\frac{1}{kl}\operatorname{arctg}kl-\frac{\omega^2\tau^2}{(1+k^2l^2)^2}+i\frac{\omega\tau}{1+k^2l^2}.$$

Analyzing the behavior of Qz_1, which contains ReY and ImY, as a function of kl, we can obtain a fairly good approximation for this function

$$Qz_1\approx\frac{3+\pi kl/2}{k^2l^2(1+k^2R^2)^2}, \quad (4.16)$$

which is valid under the conditions

$$k_0l>1;\quad \alpha\frac{R^2}{l^2}\ll1;\quad \alpha=9\left(1+\left|\frac{V_0}{V}\right|\right)\frac{V^2}{v^2} \quad (4.17)$$

(k_0 is the cutoff value of k, which has already been used in Sec. 3). The calculation of the integral then leads to the estimate

$$\mathcal{F}_1\sim\left(\frac{V_0}{V}-1\right)\frac{\tau V}{\varkappa l^2}\beta^2b^2\left(1+\frac{l}{R}\right), \quad (4.18)$$

which is valid if the following inequality is satisfied

$$Rk_0\gg1. \quad (4.19)$$

We confine ourselves to examination of only the cases when the conditions (4.17) and (4.19) are satisfied (it can be verified that the value of the integral decreases rapidly when any of these inequalities is violated).

5. DISCUSSION

In the case when $V_0 > V$, the accelerating force can modify the dislocation motion observed without the field E_0. The acceleration can become noticeable if a large drift velocity $V_0 = ME_0$ is produced ($M = e\tau/m^*$—carrier mobility, m^*—effective mass). It must be recognized, however, that the acceleration consumes only a small part of the external field energy $\mathbf{j}_0\cdot\mathbf{E}_0 = enV_0^2/M$, which goes almost entirely into Joule heat. Therefore electronic acceleration can appear only if the dissipation does not change the temperature strongly. For an estimate, we confine ourselves to a dissipation which is much smaller than several hundred W/cm^3; then

$$nV_0^2\ll10^{21}M \quad (5.1)$$

(the mobility M is in cm^2/V-sec and the remaining quantities are in cgs esu).

For metals, the condition (5.1) limits the possibility of acceleration. Taking in (3.14) $\Delta\sim\mu\sim1$ eV, $\tilde{v}\sim10^8$ cm/sec, $\partial n/\partial\mu\sim n/\mu$, and $n\sim10^{22}$ cm^{-3}, we obtain the estimate $\mathcal{F}_1\sim bV_0\times10^2$ dyn/cm. From (5.1) with $M\sim10^4$ cm^2/V-sec it follows that

$V_0 < 10^2$ cm/sec, just as in pure metal at low temperatures. Thus $\mathcal{F}_1/b \ll \sigma^S$ (σ^S is the starting voltage, $\sigma^S \sim 10^6-10^7$ dyn/cm^2). Even under conditions of efficient heat removal, when (5.1) can be disregarded, it is difficult to produce large V_0 in metals, since their good electric conductivity prevents production of a large E_0. In semimetals such as bismuth, it is possible to produce large V_0 at low temperatures (the mobility $M = e\tau/m^*$ is high since τ is large and m^* small), but owing to the low concentrations, $n \sim 10^{17}$ cm^{-3}, the values of $\mathcal{F}_1$ remain small.

One can hope to observe the acceleration of dislocations in metals and semimetals because of the following circumstance: $\mathcal{F}_1$ reverses sign and turns into a deceleration force when the direction of the field E_0 is reversed. The difference between the action of $\mathcal{F}_1$ when $E_0 < 0$ and $E_0 > 0$ may turn out to be appreciable. The limitation imposed by the heating of the crystal is also easier to circumvent in this case, for the heating is proportional to E_0^2 and is the same under acceleration and deceleration.

Let us turn to the case of semiconductors. For non-piezoelectric crystals, the estimates are similar to those presented above for the metals, but $\partial n/\partial\mu = n/kT$ and the constants of the deformation potential are usually larger ($\Delta \sim 10$ eV). In the case of large mobility, such as in InSb, the acceleration effect may appear. In piezoelectric semiconductors it is also possible to produce strong fields and cause drift of electrons with $V_0 \gg V$. In a sufficiently wide range of variation of n, the value of F_1 turns out to be almost insensitive to the value of n. Indeed, we note that (5.1) is compatible with the condition (4.19) at realistic values of the mobility M. Therefore, for values of n which are bounded from above by the condition (5.1), and are bounded from below by the value determined from the inequality $R^2 \ll l^2/\alpha$, that is,

$$n \gg \varkappa kTV(V + |V_0|) / e^2 l^2 \tilde{v}^2,$$

it is possible to use the estimate (4.18); the dependence of the concentration n, contained in the second term of (4.18), is inessential, for we have $l \ll R$ when (5.1) is satisfied at room temperatures and large drift velocities.

In crystals with strong piezoelectric coupling, the acceleration can be quite effective. Let us estimate (4.18) numerically for CdS, where $\beta \sim 10^5-10^6$ cgs esu, $\varkappa \sim 10$[10], and $\tau \sim 10^{-13}$ sec (this value of τ at room temperature follows from $M \sim 300$ cm^2/V-sec[11] and the effective mass $m^* = 0.35\, m$[12]). When $l \sim 10^{-6}$ cm and $b \sim 10^{-7}$ cm we obtain $\mathcal{F}_1/b \sim 10^7-10^8$ dyn/cm^2 for $V_0 = 10^5$ cm/sec. The starting voltage can thus be exceeded. The limitations on n are not very stringent here and reduce at room temperature to the inequality $10^{14}\ V \gg n \gg 10^9\ V$. The required electron concentrations can be produced in CdS by illumination, as was done in experiments on amplification of ultrasound[11]. Piezoelectric semiconductors are apparently the most favorable object for observation of electronic acceleration.

The theory presented pertains to the case of sufficiently rapid dislocations. To some degree, the results can be meaningful also for small V, as follows from the physical picture of the phenomenon. But it is hardly possible to count on obtaining quantitative estimates. The theory can be extended also to the case of a dislocation that vibrates under the influence of ultrasound. Then resonance effects are possible in the alternating external field.

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[1] V. Ya. Kravchenko, FTT 8, 927 (1966), Soviet Phys. Solid State 8, 740 (1966).

[2] V. M. Kontorovich, JETP 45, 1638 (1963), Soviet Phys. JETP 18, 1125 (1964).

[3] V. G. Skobov, and É. A. Kaner, JETP 46, 273 (1964), Soviet Phys. JETP 19, 189 (1964).

[4] L. D. Landau and E. M. Lifshitz, Teoriya uprugosti (Theory of Elasticity), Fizmatgiz, 1965.

[5] A. M. Kosevich, JETP 45, 637 (1962), Soviet Phys. JETP 18, 437 (1963).

[6] A. M. Kosevich and V. D. Natsik, FTT 8, 1250 (1966), Soviet Phys. Solid State 8, 993 (1966).

[7] A. I. Akhiezer, M. I. Kaganov, and G. Ya. Lyubarskiĭ, JETP 32, 837 (1957), Soviet Phys. JETP 5, 685 (1957).

[8] R. F. Kazarinov and V. G. Skobov, JETP 43, 1496 (1962), Soviet Phys. JETP 16, 1057 (1963).

[9] L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Fizmatgiz, 1957.

[10] A. R. Hutson, Phys. Rev. Lett. 4, 505 (1960). D. Berlincourt, H. Jaffe, and L. Shiozawa, Phys. Rev. 129, 1009 (1963).

[11] A. R. Hutson, L. H McFee, and D. L. White, Phys. Rev. Lett. 7, 237 (1961).

[12] R. H. Bube, Photoconductivity of Solids, Wiley, 1960.

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