

SECOND SOUND AND THE ABSORPTION OF ORDINARY SOUND IN DIELECTRICS

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A set of equations is obtained describing the heat transfer in dielectrics at low temperatures when the characteristic time of the normal processes is small compared with the characteristic time of the umklapp processes. A theory of second-sound absorption is developed by means of these equations, as well as theories of the interaction of ordinary sound with second sound and of the absorption of ordinary sound.

1. INTRODUCTION

IN recent years and in a number of theoretical papers, it has been established that temperature waves can exist in solids at sufficiently low temperatures—the so-called second sound^[1-6]. Not long ago, they were discovered experimentally.^[7] We would like to demonstrate a new method of investigation of second sound and phenomena related to it. This method is based on the interaction between second sound and ordinary sound. In the present paper it is shown that the measurement of the absorption coefficient (and velocity) of ordinary sound at sufficiently high frequencies serves as a means for the investigation of the region of existence of second sound and its different quantitative characteristics.

As is well known, phonon collisions are of two types—N-processes, in which the phonon quasi-momentum is conserved, and U-processes, which take place with non-conservation of the phonon quasi-momentum. Here we shall understand by U-processes not only the proper umklapp processes, but also any processes of scattering of phonons by lattice defects.

At low temperatures, in sufficiently pure dielectrics, the characteristic time of N-processes τ_N is much less than the time of U-processes τ_U . However, the U-processes determine the finite thermal conductivity of a solid. On the other hand, the N-processes play a no less important role. Because of them, the distribution function of the phonons (with accuracy up to small terms proportional to τ_N/τ_U) take the form of a “Planck distribution with drift”

$$N_{kj} = \left[\exp \frac{\hbar\omega_{kj} - \hbar\mathbf{k}\mathbf{V}}{T} - 1 \right]^{-1}. \quad (1.1)$$

Here ω_{kj} is the phonon frequency with quasi-momentum $\hbar\mathbf{k}$, belonging to the vibration branch j ; T is the temperature (in energy units), and \mathbf{V} is the drift velocity of the phonon gas. Inasmuch as the relaxation of the drift velocity \mathbf{V} because of U-processes occurs slowly, the state with distribution (1.1) can be regarded as a state of incomplete thermodynamic equilibrium, while the quantity \mathbf{V} can be regarded as a thermodynamic variable characterizing the state of the system. The transfer of energy is associated with the drift velocity; therefore, for drift velocity \mathbf{V} different from zero, the energy flux density \mathbf{Q} will also be different from zero. Still another thermodynamic variable is the volume of the system or, in a general case, the deformation tensor u_{ijk} .

In the present research, we want to obtain the macroscopic equations connecting these quantities.

Usually the thermal and mechanical properties of a solid are described by means of a system of two coupled equations—the equation of motion of elasticity theory and the equation of heat conductivity (see the book of Landau and Lifshitz,^[8] Part II, Secs. 6 and 27). The quantities u_{ijk} and T play the role of variables in them. The derivation of these equations is based on an assumption that the energy flux density at the given point at a given instant of time is linearly related with the temperature gradient at that point and at that time:

$$\mathbf{Q} = -\kappa\nabla T, \quad (1.2)$$

where κ is the thermal conductivity tensor.

We now assume that ∇T is a function of time. If the gradient changes in a time $\lesssim \tau_U$, then the simple connection of the type (1.2) no longer occurs. In other words, for such variation frequencies, time dispersion of the tensor κ begins. Similarly, for a sufficiently sharp spatial change of ∇T , a

spatial dispersion of the tensor κ can appear. In order to describe these effects, we substitute the relation (1.2) by a differential equation for the quantity \mathbf{V} , obtaining a system of three equations with three unknowns. Evidently the applicability of this system of equations is limited to frequencies which are small in comparison τ_N^{-1} and to characteristic scales of spatial change that are great in comparison with the free path of the phonon l_N . However, there is a rather wide range of phenomena which are described by such a set of equations. These include a number of effects already considered earlier: second sound in a solid^[1-6] and the theory of thermal conductivity of dielectric rods and plates.^[9]

Our theory makes it possible to consider all these phenomena from a single viewpoint. The new results obtained by us are: a quantitative theory of the damping of second sound, a theory of interaction of first and second sound, and a theory of damping of first sound in a region of frequencies where the already mentioned effects of thermal conductivity dispersion begin to appear.

2. DERIVATION OF THE MACROSCOPIC EQUATIONS WITHOUT ACCOUNT OF DISPERSION AND THERMAL EXPANSION OF THE BODY

If the N-processes take place significantly more frequently than the U-processes, then the state of the phonon gas having a drift velocity \mathbf{V} different from zero can be considered as a state of incomplete thermodynamic equilibrium. It can be characterized by giving the drift velocity \mathbf{V} and the temperature T at each point of the medium. We obtain a basic thermodynamic identity which characterizes this state. Here we shall neglect deformations of the body resulting from thermal expansion.

In addition to the velocity \mathbf{V} , let us consider also the value of the quasi-momentum per unit volume.

$$\mathbf{P} = \sum_j \int d\tau_h \hbar \mathbf{k} N_{hj} \left(d\tau_h = \frac{d^3\mathbf{k}}{(2\pi)^3} \right). \quad (2.1)$$

The internal energy E per unit volume is defined in the following fashion:

$$E = \sum_j \int d\tau_h \hbar \omega_{hj} N_{hj}. \quad (2.2)$$

By using an explicit form for the distribution function (1.1), one can obtain

$$dE = T dS + \mathbf{V} d\mathbf{P}, \quad (2.3)$$

where S is the entropy per unit volume.

We now obtain a closed system of equations for these quantities. We limit ourselves to the linear approximation, assuming the value of \mathbf{V} to be small.

Then a linear relation exists between \mathbf{P} and \mathbf{V} , of the form

$$P_i = \rho_{ik} V_k, \quad (2.4)$$

where $(\rho^{-1})_{ik} = \partial^2 E / \partial P_i \partial P_k$ is a symmetric tensor.

In this section, we shall generally not take into account dispersion processes, including U-processes. In this approximation, the quasi-momentum is a conserved quantity and

$$\frac{\partial P_i}{\partial t} - \frac{\partial F_{ik}}{\partial x_k} = 0, \quad (2.5)$$

where F_{ik} is the quasi-momentum flux density tensor.

We shall show that without account of dispersion

$$F_{ik} = F_{ik}^{(0)} = F \delta_{ik}, \quad (2.6)$$

where F is the density of free energy of the phonon gas. Actually, $F_{il}^{(0)}$ is, by definition,

$$F_{il}^{(0)} = - \sum_j \int d\tau_h \hbar k_i \frac{\partial \omega_{hj}}{\partial k_i} N_{hj}. \quad (2.7)$$

In our approximation,

$$N_{hj} = \left[\exp \frac{\hbar \omega_{hj}}{T} - 1 \right]^{-1},$$

since $F_{il}^{(0)}$ does not contain terms linear in \mathbf{V} . After some transformations, one can obtain

$$F_{il}^{(0)} = -T \sum_j \int d\tau_h \ln(1 + N_{hj}) \cdot \delta_{il} = F \delta_{il}. \quad (2.8)$$

By taking into account that $\partial F / \partial T = -S$, one can put (2.5) in the form

$$\partial \mathbf{P} / \partial t + S \mathbf{V} T = 0. \quad (2.9)$$

Another equation establishing a connection between the quantities we have introduced is the law of conservation of energy:

$$dE / dt + \text{div} \mathbf{Q} = 0. \quad (2.10)$$

In the state of incomplete equilibrium under discussion,

$$\mathbf{Q} = L \mathbf{V}. \quad (2.11)$$

We shall show that $L = TS$. For this purpose, we make use of the fact that the derivative $\partial S / \partial t$ must equal the divergence of some vector (since we neglect dissipation processes). We have from (2.3)

$$\frac{\partial S}{\partial t} = \frac{1}{T} \frac{\partial E}{\partial t} - \mathbf{V} \frac{\partial \mathbf{P}}{\partial t}. \quad (2.12)$$

Taking into account (2.4) and (2.9)–(2.11), we get

$$\frac{\partial S}{\partial t} = - \frac{\text{div} L \mathbf{V}}{T} - S T \mathbf{V} \nabla \frac{1}{T}. \quad (2.13)$$

In order that the expression on the right hand side reduce to a divergence, it is necessary that

$$L = TS, \quad Q = TSV. \quad (2.14)$$

Then $\partial S/\partial t + \text{div } SV = 0$.

Thus, in the linear approximation, the following set of equations is obtained:

$$\partial P_i / \partial t + S \partial T / \partial x_i = 0, \quad (2.15a)$$

$$C_V \partial T / \partial t + ST \text{div } V = 0, \quad (2.15b)$$

where $C_V = T \partial S / \partial T$ is the heat capacity at constant volume.

The system (2.15) has, in particular, a solution of the form of undamped waves with dispersion loss^[6]

$$\omega^2 = TS^2 C_V^{-1} \rho_{ik}^{-1} q_i q_k, \quad (2.16)$$

where q is a wave vector. These indeed are the temperature waves, which have been named second sound in a solid. The wave vector in this wave is directed along P , while the group velocity $\partial \omega / \partial q_i$ is directed along V_i , that is, along the direction of energy flow.

3. MACROSCOPIC EQUATIONS WITH ACCOUNT OF DISSIPATION

The dissipation of energy can take place for two reasons. The first are U-processes. At small P , their role can be taken into account by changing Eq. (2.9) in the following way:

$$\partial P_i / \partial t + S \partial T / \partial x_i + D_{ik} P_k = 0, \quad (3.1)$$

where $D_{ik} \sim \tau_U^{-1}$. In the static case, this equation should give the usual expression for the heat flow, $Q_i = -\kappa_{ik} \partial T / \partial x_k$, where κ is the thermal conductivity tensor. Then, with account of (2.4) and (2.14), we get

$$\kappa^{-1} = D\rho / TS^2. \quad (3.2)$$

For much higher frequencies, if the quantities P , V , and Q change with time like $e^{-i\omega t}$, Eq. (3.1) gives the following connection between the temperature gradient and Q :

$$\nabla T = -\tilde{\kappa}^{-1} Q, \quad \tilde{\kappa}^{-1} = \kappa^{-1} - \frac{i\omega}{TS^2} \rho. \quad (3.3)$$

This expression describes the time dispersion of the thermal conductivity.

The second reason for dissipation is connected with N-processes and takes place if P and T depend on the coordinates. If the density of the quasi-momentum depends on the coordinates, then the irreversible process of its diffusion takes place,

brought about by the normal collisions. This process can be described by an additional component in F_{ik} , which has the form

$$F_{ik}^{(4)} = \gamma_{iklm} \partial V_l / \partial x_m. \quad (3.4)$$

Moreover, the expression for the energy flux density has a dissipation contribution proportional to ∇T :

$$Q_i - TSV_i = -\chi_{ik} \partial T / \partial x_k. \quad (3.5)$$

We now write down the complete system of equations with account of dissipative terms:

$$\frac{\partial P_i}{\partial t} + S \frac{\partial T}{\partial x_i} - \gamma_{iklm} \frac{\partial^2 V_l}{\partial x_k \partial x_m} + TS^2 (\kappa^{-1})_{ik} V_k = 0,$$

$$C_V \partial T / \partial t + \text{div } Q = 0, \quad Q_i = TSV_i - \chi_{ik} \partial T / \partial x_k, \quad P_i = \rho_{ik} V_k. \quad (3.6)$$

We now obtain the Onsager relations for the tensors γ and χ . For this purpose, it is necessary to compute the derivative $\partial S / \partial t$ just as in Sec. 2, but by using Eq. (3.6). As a result, we get

$$\gamma_{iklm} = \gamma_{lmik}, \quad \chi_{ik} = \chi_{ki}. \quad (3.7)$$

We now determine the absorption coefficient of second sound Γ in the anisotropic case. For this purpose, it is necessary to find a solution of the set (3.6) proportional to $e^{i(q \cdot r - \omega t)}$, in which the terms describing the damping can be regarded as a small perturbation. After simple calculations, we obtain

$$\Gamma = \frac{TS^2}{2C_V \omega^2 w_g} \left[\frac{\omega^2}{TS^2} \chi_{ik} q_i q_k + \gamma_{iklm} (\rho^{-1})_{lp} (\rho^{-1})_{is} q_h q_m q_p q_s + TS^2 (\rho^{-1} \kappa^{-1} \rho^{-1})_{ik} q_i q_k \right], \quad (3.8)$$

where ω^2 is connected with q by Eq. (2.16), and $w_g = |\partial \omega / \partial q|$ is the group velocity of second sound.

It is also of interest to explain what form the set (3.6) takes in the static case. We obtain

$$S \frac{\partial T}{\partial x_i} - \gamma_{iklm} \frac{\partial^2 V_l}{\partial x_k \partial x_m} + TS^2 (\kappa^{-1})_{ik} V_k = 0, \quad (3.9)$$

$$\text{div } Q = 0.$$

In the isotropic case, these equations have a very simple form

$$S \frac{\partial T}{\partial x_i} - \gamma \Delta V_i + \frac{TS^2}{\kappa} V_i = 0, \quad (3.10)$$

$$\text{div } Q = 0.$$

In Eq. (3.5) for Q , one can in this case neglect the term with $\chi_{ij} l$, since it makes a small correction to the thermal conductivity, lying outside the limits of accuracy of our present calculation.

Equations (3.10) were first obtained and investigated by Gurzhi, Sussman and Thellung^[9] and a

theory of thermal conductivity of thin rods and plates was based on them. It is seen, however, that the time of umklapp processes which figure in these equations is expressed in terms of the thermal conductivity and the thermodynamic characteristics of the system, while the coefficient γ is connected with the absorption coefficient of second sound.

As a conclusion to this section, we shall enumerate all the conditions of applicability of the given theory. As has already been noted, one of these takes the form of the inequality $\tau_U \gg \tau_N$. Furthermore, the characteristic lengths and times of change of the drift velocity V must be much greater than the corresponding lengths and times of the free flight of phonons relative to the normal processes. Finally, the given theory is a linear theory and is therefore applicable when the drift velocity V is not too large. In particular, it is necessary (but, generally speaking, not sufficient) that it be much smaller than the velocity of sound w .

The nonlinear effects reduce, first, to a change in Eq. (2.15), which does not take into account dissipation, and in the second place to the appearance of a dependence of the coefficients for the dissipation terms on the drift velocity. A number of interesting and important effects can be considered only on the basis of the nonlinear theory. Thus, these effects could lead to the generation of shock waves in the propagation of second sound. Furthermore, the process of the stationary transfer of heat into dielectric plates or rods can be shown (for sufficiently large V) to be unstable, and in this case a situation can arise similar to turbulence in ordinary hydrodynamics. In experiments, this should appear as an increase in the coefficient of the thermal conductivity upon increase in the temperature gradient.

4. THEORY OF DISSIPATIVE PROCESSES WITH ACCOUNT OF DEFORMATION OF THE CRYSTAL

The equations derived in the preceding sections are valid if the thermal expansion of the body is neglected. Allowance for thermal expansion leads to some new effects. For example, the temperature wave is accompanied by a deformation wave. Conversely, in the propagation of a deformation wave, that is, ordinary sound, a temperature wave is generated. Its presence leads to a specific absorption of the sound because of the effects of thermal conductivity. The theory constructed in the previous sections can be regarded as the theory of spatial and temporal dissipation of the thermal

conductivity, in which the temporal dispersion begins to play a role at frequencies $\omega \simeq \tau_U^{-1}$. Evidently, the frequency dependence of the thermal-conductivity absorption coefficient of ordinary sound must change at such frequencies.

We obtain a set of macroscopic equations by means of which we can consider this effect as well as other similar effects. These include the equations expressing the law of energy conservation

$$\partial E / \partial t + \operatorname{div} Q = 0 \quad (4.1)$$

and conservation of the quasi-momentum

$$\frac{\partial P_i}{\partial t} - \frac{\partial F_{il}}{\partial x_l} + D_{il} P_l = 0. \quad (4.2)$$

To these it is necessary to add the equation of motion of the continuous medium

$$\rho_L \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{il}}{\partial x_l}. \quad (4.3)$$

Here ρ_L is the density of the crystal, u the displacement vector at the continuous medium, and σ_{il} the stress tensor.

We shall first show in what system of coordinates these equations are written. Let us consider some point of a continuous medium. Before deformation, its coordinates were x_i ; after deformation, this point takes the position $x'_i = x_i + u_i(x_l, t)$.

We shall assume that Eqs. (4.1)–(4.3) are written in the coordinates x_i and t , that is, in a frame connected with the moving lattice. This means, for example, that E is the internal energy of the amount of material which, before deformation, was included in a unit volume (and after deformation can naturally take a somewhat different volume).

For the determination of the tensor σ_{il} in the absence of dissipation (we shall denote this by $\sigma_{il}^{(0)}$), we write down an expression for the free energy of the deformed body with account of thermal expansion:^[8]

$$F = F_0(T) + \frac{1}{2} \lambda_{ilmn} u_{il} u_{mn} - \lambda_{ilmn} \alpha_{il} u_{mn} (T - T_0), \quad (4.4)$$

where $F_0(T)$ is the free energy of the body in the absence of deformation, λ_{ilmn} is the tensor of the isothermal elastic moduli, α_{il} the tensor of the coefficients of thermal expansion, T_0 the temperature of the undeformed crystal, and u_{il} the deformation tensor. Then,

$$\sigma_{il}^{(0)} = (\partial F / \partial u_{il})_T = \lambda_{ilmn} u_{mn} - \lambda_{ilmn} \alpha_{mn} (T - T_0), \quad (4.5)$$

while the equation of motion (4.3) takes the form

$$\rho_L \frac{\partial^2 u_i}{\partial t^2} = \lambda_{ilmn} \frac{\partial u_{mn}}{\partial x_l} - \lambda_{ilmn} \alpha_{mn} \frac{\partial T}{\partial x_l}. \quad (4.6)$$

Equation (4.6) is the ordinary equation of motion of a medium in the presence of thermal expansion.^[8]

In Eq. (4.1) $\partial E/\partial t$ can be expressed in terms of the time derivative of the entropy: $\partial E/\partial t = T \partial S/\partial t$ (this equation is valid in the linear approximation since the term $\mathbf{P} \cdot \dot{\mathbf{V}}$ is of second order of smallness). The entropy S can be obtained from (4.4):

$$S = -(\partial F/\partial T)_{u_i, \mathbf{P}} = S_0(T) + \lambda_{ilmn} \alpha_{il} u_{mn}, \quad (4.7)$$

where $S_0(T) = -(\partial F_0/\partial T)_{\mathbf{P}}$. Transforming (4.1) with account of (4.7), we get

$$C_V \partial T/\partial t + T \lambda_{ilmn} \alpha_{il} \dot{u}_{mn} + \text{div } \mathbf{Q} = 0. \quad (4.8)$$

If it is assumed that

$$\mathbf{Q} = -\kappa \nabla T, \quad (4.9)$$

then Eqs. (4.6) and (4.8) form a closed system which describes the motion of a continuous medium and the propagation of heat in it.^[8] However, in the general case, the relation (4.9) does not hold but (neglecting dispersion associated with N processes) is replaced by the differential equation (3.1) and the equations (2.4) and (2.14). As we have seen in Sec. 3, they describe the dispersion of the thermal conductivity. This indeed is the basic feature of the set of equations obtained in the present work.

As example of application, let us consider the interaction of ordinary longitudinal sound and second sound propagating along any symmetry axis of the crystal (the x axis). We shall assume that the sound frequency is $\omega \tau_{\text{U}} \gg 1$ and neglect the last term in (3.1) (that is, we shall not take the damping of the sound into account).

By assuming that all the quantities in the wave change with time like $e^{-i\omega t + iqx}$ and setting $\omega = wq$, we obtain the following dispersion equation:

$$w^4 - w^2 \left[\frac{\lambda}{\rho_L} + \frac{TS^2}{\rho C_V} + \frac{(\lambda a)^2 T}{C_V \rho_L} \right] + \frac{\lambda TS^2}{\rho_L \rho C_V} = 0, \quad (4.10)$$

where $\lambda = \lambda_{xxxx}$, $\rho = \rho_{xx}$, and $\lambda a = \lambda_{xxil} \alpha_{il}$.

It is easy to obtain the tensor of the adiabatic moduli λ^S , which is connected with the tensor of the isothermal elastic modulus λ by the relation:

$$\lambda_{ilmn}^S = \lambda_{ilmn} + T C_V^{-1} \lambda_{ilpq} \lambda_{rsmn} \alpha_{pq} \alpha_{rs}. \quad (4.11)$$

Therefore, in our case,

$$\lambda^S = \lambda + T C_V^{-1} (\lambda a)^2, \quad (4.11a)$$

where the second term as a rule is small in comparison with the first. Then (4.10) takes the form

$$w^4 - w^2 \left(\frac{\lambda^S}{\rho_L} + \frac{TS^2}{\rho C_V} \right) + \frac{\lambda TS^2}{\rho_L \rho C_V} = 0. \quad (4.12)$$

This equation is identical in accuracy with the dispersion equation of superfluid helium at very low

temperatures, when all the excitations in it are phonons and the density of the superfluid component ρ_S is virtually identical with the total density of the liquid. If we neglect thermal expansion, that is, assume that $\lambda^S = \lambda$, then, as is seen from (4.12), the first and second sound do not interact.

We now take into account the dispersion which is connected with the normal processes. For this purpose, we first need to replace the relation $\mathbf{Q} = T \mathbf{S} \mathbf{V}$ by Eq. (3.5). Next, we must write down the dissipation parts for the tensor \mathbf{F} and σ :

$$F_{il}^{(4)} = \gamma_{ilmn} \partial V_m / \partial x_n + \mu_{ilmn} \dot{u}_{mn}, \quad (4.13)$$

$$\sigma_{il}^{(4)} = \eta_{ilmn} \dot{u}_{mn} + \nu_{ilmn} \partial V_m / \partial x_n. \quad (4.14)$$

Here the tensor η is the ordinary tensor of viscosity coefficients of a solid.^[8] We have discussed the meaning of the tensor γ above. The tensors μ and ν describe the "crossing" viscosity.

To establish their symmetry, we write down the time derivative of the entropy. The thermodynamic identity now has the form

$$dE = T dS + \sigma_{il}^{(0)} du_{il} + \mathbf{V} d\mathbf{P}. \quad (4.15)$$

We now take it into account that

$$\frac{\partial}{\partial t} \left(E + \frac{\rho_L \dot{u}^2}{2} \right) = -\text{div } \mathbf{Q}. \quad (4.16)$$

Then

$$\begin{aligned} \frac{\partial S}{\partial t} + \text{div } \mathbf{S} &= \frac{\sigma_{il}^{(4)} \dot{u}_{il}}{T} + \frac{1}{T} F_{il}^{(4)} \frac{\partial V}{\partial x_i} \\ &+ \frac{1}{T} \chi_{il} \frac{\partial T}{\partial x_i} \frac{\partial T}{\partial x_l} + S^2 \chi_{il}^{-1} V_i V_l. \end{aligned} \quad (4.17)$$

Hence, in addition to the well-known relations for η and the relations for γ and χ obtained above, it follows that

$$\nu_{ilmn} = \mu_{mnil}. \quad (4.18)$$

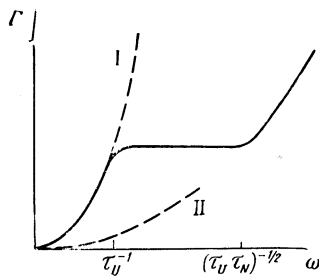
Moreover, it is obviously true that

$$\mu_{ilmn} = \mu_{ilnm}. \quad (4.19)$$

The condition for positive definitiveness of the dissipation function reduces to a certain inequality which must be satisfied by the components of the tensors η , γ , and μ . We shall not investigate this problem any further.

5. ABSORPTION OF ORDINARY SOUND UNDER CONDITIONS OF THERMAL-CONDUCTIVITY DISPERSION

The phenomenological theory of the absorption of a low-frequency sound wave in solids has been



Continuous curve—schematic dependence of the absorption coefficient of ordinary sound on the frequency. Dashed curve I—dependence of Γ_κ on the frequency according to the usual theory. Dashed line II—dependence of Γ_η on the frequency according to the usual theory

developed in the book of Landau and Lifshitz.^[8] The total absorption coefficient Γ is made up of the absorption coefficient due to the viscosity Γ_η and that due to the thermal conductivity Γ_κ . Both these coefficients depend on the frequency as ω^2 ; however, in the situation under consideration, when $\tau_N \ll \tau_U$ and $\Gamma_\eta \ll \Gamma_\kappa$, inasmuch as the viscosity is proportional to τ_N , while the thermal conductivity is proportional to τ_U . This theory is applicable when $\omega \ll \tau_U^{-1}$. The set of phenomenological equations obtained in Sec. 4 permit us to calculate the absorption coefficient in a significantly greater range of frequencies, satisfying the condition $\omega \ll \tau_N^{-1}$.

When ω becomes comparable with τ_U^{-1} , the dispersion of the thermal conductivity begins to play a role and, as will be pointed out below, for $\omega \gg \tau_U^{-1}$ the quantity Γ_κ ceases to depend on the frequency. This result is natural and general for all mechanisms of sound absorption of the Mandel'shtam-Leontovich type when there is a large relaxation time in the system (in the given case, this relaxation time is τ_U). On the other hand, inasmuch as the viscosity coefficient is determined by the time τ_N , it does not have dispersion at these frequencies and Γ_η continues to increase as ω^2 . Therefore, for frequencies $\omega \approx (\tau_N \tau_U)^{-1/2}$, the coefficients Γ_η and Γ_κ are seen to be of the same order, and at higher frequencies Γ_η predominates and the total absorption coefficient Γ again increases as ω^2 . The obtained dependence of Γ on ω is shown in the drawing.

We now proceed to construct the quantitative theory. We consider the propagation of the longitudinal wave along a symmetry axis of the crystal (the x axis). We begin with the case of frequencies satisfying the condition $\omega^2 \tau_N \tau_U \ll 1$. Direct estimates show that in this case all the viscosity terms can be neglected. Then one can use Eqs. (4.6) and (4.8) while one must use (3.3) as relations estab-

lishing the connection between Q and ∇T . For the phase velocity of sound and the absorption coefficient (with respect to amplitude) one can obtain the following expressions:

$$\frac{\omega}{q} = w_I + \Delta w \frac{(1 - w_{II}^2/w_I^2) + (\omega\tau_\kappa)^{-2}}{(1 - w_{II}^2/w_I^2)^2 + (\omega\tau_\kappa)^{-2}}, \quad (5.1)$$

$$\Gamma = \frac{\Delta w}{w_I^2} \frac{w_{II}^2}{w_I^2} \frac{\omega^2 \tau_\kappa}{1 + (1 - w_{II}^2/w_I^2)^2 \omega^2 \tau_\kappa^2}, \quad (5.2)$$

where $w_I = (\lambda/\rho_L)^{1/2}$; $w_{II} = (TS^2/\rho C_V)^{1/2}$ is the velocity of second sound; $\Delta w = (\lambda a)^2 \Gamma / 2\rho_L w_I C_V$ is the difference between the adiabatic and the isothermal velocity of ordinary sound; and $\tau_\kappa = \kappa/w_{II}^2 C_V = 1/D_{XX}$ is a time which is the same as τ_U in order of magnitude. It is seen that at low frequencies Γ increases in proportion to ω^2 and in this region is determined by well known expressions (see the book of Landau and Lifshitz^[8]). At high frequencies, dispersion of the thermal conduction arises and Γ ceases to depend on the frequency.

In the region of frequencies $\tau_U^{-1} \ll \omega \ll \tau_N^{-1}$, the following formula is valid:

$$\Gamma = \frac{\omega^2 \eta}{2\rho_L w_I^3} + \frac{\Delta w_I}{w_I^2 (1 - w_{II}^2/w_I^2)^2} \left[\frac{w_{II}^2}{w_I^2} \left(\frac{\gamma \omega^2}{\rho w_I^2} + \frac{1}{\tau_\kappa} \right) + \frac{\omega^2 \chi}{C_V w_I^2} \right] - \frac{w_{II}^2}{w_I^3 (1 - w_{II}^2/w_I^2)} \frac{\lambda a}{S_{\rho L}} \frac{\mu \omega^2}{w_I^2}. \quad (5.3)$$

It is then seen that for $\omega > (\tau_U \tau_N)^{-1/2}$, the losses due to the viscosity become important and Γ increases in proportion to the square of the frequency. The regions of applicability of Eqs. (5.2) and (5.3) overlap in the interval $\tau_U^{-1} \ll \omega \ll (\tau_U \tau_N)^{-1/2}$, and in this interval they clearly give identical results. As is seen from (5.3), the viscous losses in the region $\omega \gg (\tau_U \tau_N)^{1/2}$ are determined not only by the tensor computed by Akhiezer,^[10] but also by the three tensors γ , χ , and μ , which give a contribution of the same order.

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