SIZE EFFECT IN THE MAGNETORESISTANCE OF SEMICONDUCTORS

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The electron temperature in thin current-carrying semiconducting plates can be changed by a magnetic field perpendicular to the current and lying in the plane of the plate. In the absence of surface heat exchange between the electron gas and lattice, or in the case of equal rates of heat exchange on both surfaces, the mean electron temperature remains equal to the lattice temperature. If the heat exchange rates are different the electron temperature differs from the lattice temperature. In the first case the appearance of a transverse electron thermal emf induces an additional longitudinal current component which appreciably lowers the magnetoresistance in plates that are thin compared with the electron cooling length (it may vanish completely for some scattering mechanisms). In the second case, besides this effect, a resistance change occurs, which is proportional to the product of the electric and magnetic field strengths and which results in rectification of the current passing through the plate.

IN most cases, the time τ_ϵ of scattering of electron energy by lattice vibrations in semiconductors is much larger than the momentum relaxation time τ. In such semiconductors, a size effect exists for the magnetoresistance and arises when the plate thickness is of the order of the electron cooling length $L = \bar{v}\sqrt{\tau\tau_\epsilon}$, which greatly exceeds the mean free path $l = \bar{v}\tau$, where $\bar{v}$ is the average electron velocity. A magnetic field $H_y = H$, lying in the plane of the plate and perpendicular to the electric current j_x, deflects the electrons in the direction of the z axis, towards one of the surfaces of the plate. Owing to the difference in the drift velocities of the high-energy and low-energy electrons, the Lorentz force acting on them is not the same and separates them within the thickness of the plate ($-d \le z \le b$). If the surface electron-energy relaxation rates are not infinite, accumulation of "hot" electrons at one surface and depletion on the other take place. This establishes concentration gradients of electrons having identical energy. The resultant transverse electron currents are deflected by the magnetic field in such a way that they partially compensate the decrease in the current due to the ordinary (thickness-independent) magnetoresistance. When the surface relaxation rates are not equal, a change takes place also in the resistance of the plate, proportional to the product of the electric and magnetic fields $E_x H$, owing to the change in the average energy of the electrons in the plate.

The described effect is similar to the effect in bipolar semiconductors, which was considered in [1,2]. There, however, the characteristic length was the diffusion length determined by the time of the electron-hole recombination and the effect was noticeable only in intrinsic semiconductors. Here, on the other hand, the characteristic length is the cooling length L, and the effect should be observed in all semiconductors for which $L \gg l$, including purely monopolar ones.

We present below a calculation of the effect for a monopolar (electronic) non-degenerate semiconductor with a scalar electron effective mass m. At the sample thicknesses under consideration, the change in the lattice temperature is much smaller than the change in the electron temperature, owing to the smallness of the electronic specific heat compared with the lattice specific heat. Therefore the sample is regarded as isothermal with respect to the lattice temperature. The electron distribution function, which we seek in the form

$$F(\mathbf{v}, z) = F_0(v, z) + \frac{\mathbf{v}}{v}\mathbf{F}_1(v, z), \quad (1)$$

where $\mathbf{v}$ is the velocity, $v = |\mathbf{v}|$, is determined from the equations [3]

$$\frac{v}{3}\frac{\partial F_{1z}}{\partial z} - \frac{e}{3mv^2}\frac{\partial}{\partial v}[v^2(E_x F_{1x} + E_z F_{1z})] + S_0(F_0) = 0, \quad (2)$$

$$v\frac{\partial F_0}{\partial z} - \frac{eE_z}{m}\frac{\partial F_0}{\partial v} + \frac{eH}{mc}F_{1x} + \frac{F_{1z}}{\tau(v)} = 0, \quad (3a)$$

$$-\frac{eE_x}{m}\frac{\partial F_0}{\partial v}-\frac{eH}{mc}F_{1z}+\frac{F_{1x}}{\tau(v)}=0. \tag{3b}$$

Here τ is the relaxation time and $S_0(F_0)$ is the so-called collision integral with respect to energy. Assuming that $S_0(F_0)$ is governed essentially by electron-electron collisions, we solve Eqs. (2) and (3) in the electron-temperature approximation [4], according to which

$$F_0=\left(\frac{T_0}{T}\right)^{3/2}\exp\left(-\frac{mv^2}{2kT}\right), \tag{4}$$

where T_0 is the lattice temperature and T is the electron temperature [1]. Assuming the magnetic and electric fields to be small

$$\omega_c=eH/mc\ll 1/\tau, \tag{5}$$

$$\mathscr{E}=eE_xL/kT\ll 1, \tag{6}$$

and recognizing that $j_z = 0$, we obtain for T the equation (see the appendix)

$$\frac{d^2T}{dz^2}-\frac{T-T_0}{L^2}=0, \tag{7}$$

where the cooling length L can be expressed in terms of $S_0(F_0)$ and $\tau(\epsilon)$:

$$L^2=\lim_{T\to T_0}\left[\frac{2k(T-T_0)}{3m}\frac{\langle\tau\varepsilon^2\rangle\langle\tau\rangle-\langle\tau\varepsilon\rangle^2}{\langle\tau\rangle\langle e^{\varepsilon}S_0(F_0)\rangle}\right], \tag{8}$$

$\epsilon = mv^2/2kT$ is the dimensionless electron energy

$$\langle f\rangle=\frac{4}{3\sqrt{\pi}}\int_0^\infty \varepsilon^{3/2}e^{-\varepsilon}f(\varepsilon)\,d\varepsilon.$$

Solving Eq. (7) with boundary conditions

$$q_z(\pm d)=\pm S^{\pm}k[T(\pm d)-T_0] \tag{9}$$

(here q_z is the energy flux carried by one electron

$$q_z=\int_0^\infty \frac{mv_z^2}{2}vF_{1z}\,dv\Big/\int_0^\infty F_0\,dv,$$

$S_\pm$ are the surface velocities of the energy scattering at $z=\pm d$ respectively), we obtain the distribution of the electron temperature over the cross section of the plates. By determining the total current in the x direction, we calculate the difference between the effective electric conductivity $\sigma(H)$ of the plate in the magnetic field and the conductivity $\sigma(0)$ at $H = 0$:

$$\frac{\sigma(0)-\sigma(H)}{\sigma(0)}=\omega_c^2\left\{\frac{\langle\tau^3\rangle\langle\tau\rangle-\langle\tau^2\rangle^2}{\langle\tau\rangle^2}-\frac{(\langle\tau\rangle\langle\varepsilon\tau^2\rangle-\langle\varepsilon\tau\rangle\langle\tau^2\rangle)^2}{\langle\tau\rangle^2(\langle\tau\rangle\langle\varepsilon^2\tau\rangle-\langle\varepsilon\tau\rangle^2)}\frac{\mathrm{th}\,\delta}{\delta}B\right\}+\omega_c\mathscr{E}\left[\frac{\langle\varepsilon\tau\rangle}{\langle\tau\rangle}-\frac{5}{2}\right]\times\frac{\langle\varepsilon\tau^2\rangle\langle\tau\rangle-\langle\varepsilon\tau\rangle\langle\tau^2\rangle}{\langle\varepsilon^2\tau\rangle\langle\tau\rangle-\langle\varepsilon\tau\rangle^2}\frac{b^+-b^-}{b^++b^-}D. \tag{10*}$$

Here

$$B=\left[1+\mathrm{th}\,\delta\frac{(b^++b^-)\mathrm{th}\,\delta+2b^+b^-}{2\,\mathrm{th}\,\delta+b^++b^-}\right]^{-1},$$

$$D=\frac{\mathrm{th}^2\,\delta}{\delta}\left[1+\mathrm{th}^2\,\delta+2\frac{1+b^+b^-}{b^++b^-}\mathrm{th}\,\delta\right]^{-1},\quad \delta=\frac{d}{L},$$

$$b^\pm=\frac{S^\pm mL}{kT}\frac{\langle\tau\rangle}{\langle\tau\rangle\langle\varepsilon^2\tau\rangle-\langle\varepsilon\tau\rangle^2}.$$

We note that all the differences contained in (10) (with the exception of the difference in the square brackets and $(b^+ - b^-)$) are non-negative for arbitrary $\tau(\epsilon)$.

The first term in the curly brackets of (10) describes ordinary magnetoresistance [5] and the second, "size," term gives the contribution connected with the gradient of the electron temperature. It drops out when $\delta\to\infty$ and is maximal when $\delta\to 0$ and $b^\pm = 0$. The table lists the values of the quantity in the curly brackets (denoted by A) for $\delta = 0$ and $\delta=\infty$ ($b^\pm = 0$), and their ratio for $\tau(\epsilon)\sim\epsilon^s$ as functions of s. We see from the table that for small plate thicknesses ($d \ll L$) the magnetoresistance decreases appreciably; the effect is particularly pronounced when $s > 0$, when the magnetoresistance in thin samples almost vanishes. As is well known [5], when τ is constant the magnetoresistance of the semiconductor is identically equal to zero; in thin plates it is also equal to zero when $s = 1$, and is very small in the entire region of s between 0 and 1.

The term outside the curly brackets in the right side of (10) is connected with the change in the average electron temperature in the sample when $b^-\neq b^+$ (when $b^- = b^+$ the value of T average over the thickness of the sample remains equal to T_0). Since this term is proportional to E_x, rectification of the current takes place in thin plates with unequally "cooled" surfaces. The sign of the rectification depends on the sign of $(b^+ - b^-)$, on the direction of the magnetic field H_y, and also on the scattering mechanism (namely, on the sign of s when $\tau\sim\epsilon^s$, inasmuch as the factor in the square bracket is in this case equal to s). When

[1]It is assumed that d and L greatly exceed the Debye screening radius, and the conductivity in the space-charge layers near the surfaces can be neglected.

*th ≡ tanh.

s	A (∞)	A (0)	A (0)/A (∞)	s	A (∞)	A (0)	A (0)/A (∞)
$-1/2$	0.215	0.116	0.54	1	3.5	0	0
0	0	0	0.18	$3/2$	43.9	1.09	0.025
$1/2$	0.239	0.00873	0.037				

s = 0 and the magnetoresistance effect vanishes, the rectification also vanishes.

In conclusion we note that similar effects (the dependence of the resistance on the thickness when $\delta \sim 1$) can be obtained also in the absence of a magnetic field, if the electric conductivity of the semiconductor is anisotropic. It is merely necessary that the anisotropy exponent $a(\epsilon) = \sigma_{xz}(\epsilon)/\sigma_{zz}(\epsilon)$ in the different energy layers be different, for example, that the hot electrons be more anisotropic than the cold ones.

APPENDIX

To derive Eq. (7) we use a standard procedure (see, for example, [4]): we multiply Eq. (2) by v^4 and integrate with respect to v term by term, taking (4) into account, between the limits zero and infinity; we get

$$\frac{1}{T^3}\frac{d}{dz}(\langle \varepsilon^{1/2}e^{\varepsilon}F_{1z}\rangle T^3) + \frac{eE_x}{kT}\langle \varepsilon^{-1/2}e^{\varepsilon}F_{1x}\rangle + \frac{3}{2}\left(\frac{2m}{kT}\right)^{1/2}\langle e^{\varepsilon}S_0\rangle = 0. \tag{A.1}$$

(The term with E_z in (A.1) is missing, since $\langle \epsilon^{-1/2}e^{\epsilon}F_{1z}\rangle = 0$ if $j_z = 0$). To obtain an equation for the transverse distribution of T, we must determine from (3a) and (3b) the functions F_{1x} and F_{1z}, and substitute them in (A.1), after eliminating the field E_z from the condition $j_z = 0$. This yields

$$\frac{1}{T^2}\frac{d}{dz}\left[T^2\left(\frac{P_1}{T}\frac{dT}{dz} + \omega_c P_2\frac{eE_x}{kT}\right)\right] - \frac{eE_x}{kT} \times \left(\frac{eE_x}{kT}P_3 - \frac{\omega_c P_2}{T}\frac{dT}{dz}\right) - \frac{3}{2}\left(\frac{T}{T_0}\right)^{3/2}\frac{m}{kT}\langle e^{\varepsilon}S_0\rangle = 0,$$

$$P_1 = \langle \varepsilon^2\tau^*\rangle - \frac{\langle \varepsilon\tau^*\rangle^2}{\langle \tau^*\rangle}, \quad P_2 = \langle \varepsilon\tau\tau^*\rangle - \frac{\langle \varepsilon\tau^*\rangle\langle \tau\tau^*\rangle}{\langle \tau^*\rangle},$$

$$P_3 = \langle \tau^*\rangle + \omega_c^2\frac{\langle \tau\tau^*\rangle^2}{\langle \tau^*\rangle}; \quad \tau^* = \frac{\tau}{1+(\omega_c\tau)^2}. \tag{A.2}$$

By virtue of the assumed smallness of the Hall angles (5) and the assumed non-heating electric fields (6), we omit the second terms in (A.1) and (A.2), which gives the heating, we replace τ^* by τ throughout, and retain in the first term of (A.2)[2)] only the term with the second derivative d^2T/dz^2. (Allowance for the discarded terms in (A.2) would add in the resultant formula (10) terms of higher orders in ω_c and $\mathcal{E}$). We then obtain from (A.2)

$$\frac{d^2T}{dz^2} - \left(\frac{T}{T_0}\right)^{3/2}\frac{3m}{2kP_1}\langle e^{\varepsilon}S_0\rangle = 0. \tag{A.3}$$

Assuming that the expansion of $\langle e^{\epsilon}S_0\rangle/P_1$ in powers of $T - T_0$ begins with the linear term, we replace for small values of $|T - T_0|$ the second term in (A.3) by $(T - T_0)/L^2$, where L^2 is given by formula (8).

2)It is convenient here to use the estimate $dT/dz \sim (eE_x/k)\omega_c\tau$.

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