

EXCITATION OF FIRST AND SECOND SOUND IN He³-He⁴ SOLUTIONS

É. Ya. RUDAVSKIĬ and I. A. SERBIN

Physico-technical Institute of Low Temperatures, Academy of Sciences, Ukrainian S.S.R.

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The amplitudes of oscillation of the pressure, temperature and concentration have been computed for waves of first and second sound in superfluid solutions of helium isotopes. It is shown that a surface with periodically changing temperature simultaneously excites first and second sounds in the solution, the intensities of which can have the same order of magnitude.

It is well known that oscillations of a different physical nature can propagate in superfluid He⁴: pressure oscillations ("first sound") and temperature oscillations ("second sound").¹ In the wave of first sound, the normal and superfluid components vibrate so that

$$v_n = v_s, \tag{1}$$

and in the wave of second sound,

$$v_n = -\frac{\rho_s}{\rho_n} v_s. \tag{2}$$

Here v_n , v_s , ρ_n , and ρ_s are the velocity and the density of the normal and superfluid components, respectively. It has therefore turned out that first sound in He⁴ is most suitably excited by mechanical means, and second sound by thermal means.

In superfluid He³-He⁴ solutions, the situation is more complicated, because of the fact that the oscillations of pressure and temperature produce oscillations of the concentration; therefore, special features appear in the propagation and excitation of the sound. Elucidation of these features is the subject of this research.

Starting from a linearized set of hydrodynamic equations for solutions,^[1] one can, by analogy with ^[2] express the amplitudes of the variable parts of the pressure P' , the temperature T' , the concentration c' and v_n in terms of v_s . It is shown that in the wave of first sound,

$$v_n = \left(1 - \frac{\rho_s}{\rho_n} \frac{c}{\rho} \frac{\partial \rho}{\partial c}\right) \left(1 + \frac{c}{\rho} \frac{\partial \rho}{\partial c}\right)^{-1} v_s, \tag{3}$$

$$P' = \rho u_1 \left(1 + \frac{c}{\rho} \frac{\partial \rho}{\partial c}\right)^{-1} v_s, \tag{4}$$

$$T' = -\frac{u_1}{\sigma} \frac{\rho_s}{\rho_n} \frac{\partial T}{\partial \sigma} \frac{\bar{\sigma}^2}{u_1^2} \frac{c}{\rho} \frac{\partial \rho}{\partial c} \left(1 + \frac{c}{\rho} \frac{\partial \rho}{\partial c}\right)^{-1} v_s, \tag{5}$$

$$c' = -\frac{c \rho_s}{u_1 \rho_n} \frac{c}{\rho} \frac{\partial \rho}{\partial c} \left(1 + \frac{c}{\rho} \frac{\partial \rho}{\partial c}\right)^{-1} v_s, \tag{6}$$

and in the wave of second sound,

$$v_n = -\frac{\rho_s}{\rho_n} \left(1 + \frac{c}{\rho} \frac{\partial \rho}{\partial c}\right) \left(1 - \frac{\rho_s}{\rho_n} \frac{c}{\rho} \frac{\partial \rho}{\partial c}\right)^{-1} v_s, \tag{7}$$

$$P' = -\rho u_2 \frac{\rho_s}{\rho_n} \frac{c}{\rho} \frac{\partial \rho}{\partial c} \left(1 - \frac{\rho_s}{\rho_n} \frac{c}{\rho} \frac{\partial \rho}{\partial c}\right)^{-1} v_s, \tag{8}$$

$$T' = -\frac{u_2}{\sigma} \frac{\rho_s}{\rho_n} \frac{\partial T}{\partial \sigma} \frac{\bar{\sigma}^2}{u_2^2} \left(1 - \frac{\rho_s}{\rho_n} \frac{c}{\rho} \frac{\partial \rho}{\partial c}\right)^{-1} v_s, \tag{9}$$

$$c' = -\frac{c \rho_s}{u_2 \rho_n} \left(1 - \frac{\rho_s}{\rho_n} \frac{c}{\rho} \frac{\partial \rho}{\partial c}\right)^{-1} v_s, \tag{10}$$

where u_1 and u_2 are the velocities of first and second sound in a solution with density ρ and weight concentration of He³ equal to c ; σ is the entropy per unit mass, and $\bar{\sigma} = \sigma - c \partial \sigma / \partial c$.

Thus, the conditions (1) and (2) are not satisfied in propagation of sound in He³-He⁴ solutions. For example, in a solution with 20% He³, for which $c \rho^{-1} \partial \rho / \partial c \approx -0.15$, in the wave of first sound, $v_n = (1.2-1.5)v_s$ over a wide range of temperatures, while in second sound, $v_n = -(0.6-0.8) \times \rho_s \rho_n^{-1} v_s$. Moreover, in solutions, oscillations of temperature take place in the wave of first sound, and oscillations of pressure in the wave of second sound, which greatly exceed the corresponding values produced by thermal expansion. These differences are the greater the greater the concentration of the solution (the value of $c \rho^{-1} \partial \rho / \partial c \approx -0.4$ for highly concentrated superfluid solutions).

As a consequence, special conditions of excitation of sound appear. If we periodically change the temperature of a surface that is impenetrable

¹We neglect throughout the thermal expansion coefficient $\alpha = -\rho^{-1} (\partial \rho / \partial T)$ because of its smallness.

for the superfluid helium, then, inasmuch as on the boundary with a solid surface, the z component (perpendicular to the surface) of the total flow of matter is equal to zero ($j_z = 0$). For the ratio of the intensities of the simultaneously excited first and second sounds, we get

$$\frac{I_2}{I_1} = \frac{\rho_n u_2}{\rho_s u_1} \left(\frac{c}{\rho} \frac{\partial \rho}{\partial c} \right)^{-2}. \quad (11)$$

For the same conditions in pure He⁴, if we take into account the coefficient of thermal expansion α , in accord with [2], we get

$$\frac{I_2}{I_1} = \frac{C}{T \alpha^2 u_1 u_2} = \frac{\rho_n u_2}{\rho_s u_1} \left(\frac{\sigma}{\rho} \frac{\partial \rho}{\partial \sigma} \right)^{-2}, \quad (12)$$

where C is the specific heat of the helium.

It is shown that, excluding the vicinity of the λ point, we get $I_2/I_1 \sim 10^3$ for He⁴ (from Eq. (12)), and $I_2/I_1 \sim 10$ for a solution with concentration $c = 0.2$ (from Eq. (11)), that is, in addition to second sound in the solution, first sound of perceptible intensity will be excited, while the intensity of the first sound in He⁴ will be vanishingly small for these same conditions. The temperature dependence of I_2/I_1 for these conditions of excitation for the solutions is shown in the drawing.

If a plane plate vibrates in the solution in a direction perpendicular to its surface, then the boundary condition $j_z = \rho v$ (which, for zero thermal conductivity of the plate, is equivalent to the equality of the velocities v_n and v_s with the velocity of vibration of the plate v : $v_n = v_s = v$) leads for solutions to the relation

$$\frac{I_1}{I_2} = \frac{\rho_n u_1}{\rho_s u_2} \left(\frac{c}{\rho} \frac{\partial \rho}{\partial c} \right)^{-2}, \quad (13)$$

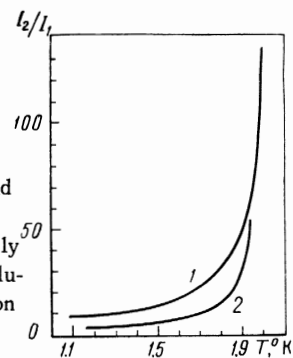
and for pure He⁴

$$\frac{I_1}{I_2} = \frac{\rho_n u_1}{\rho_s u_2} \left(\frac{\sigma}{\rho} \frac{\partial \rho}{\partial \sigma} \right)^{-2}. \quad (14)$$

In a solution with concentration $c = 0.2$ we have $I_1/I_2 \sim 10^4$, while in He⁴, we have $I_1/I_2 \sim 10^6$, i.e., neither in the solution nor in pure He⁴ is any second sound propagated under such conditions.

Equations (11)–(14) admit of an illustrative physical description. They reflect the fact that the coupling between the amplitudes of oscillation of the density (or pressure) and the entropy (or temperature) are brought about in first approximation by the derivative $\partial \rho / \partial c$ (which is proportional to the coefficient of thermal expansion and is therefore small). In He⁴, the coupling of the first and second sound is determined by this fact alone, while the ratio of the intensities is propor-

Ratio of the intensities of second and first sound, excited simultaneously by a surface with a periodically changing temperature. Curve 1 – solution with 10% He³; curve 2 – solution with 20% He³.



tional to a nondimensional quantity $(\sigma \rho^{-1} \partial \rho / \partial \sigma)^{-2}$ is seen to be negligibly small. In superfluid solutions, except for the mentioned factor, which is neglected in this analysis, the coupling between the sounds is more effective because of the coupling in the density and the concentration, while the ratio of the intensities is proportional to the quantity $(c \rho^{-1} \partial \rho / \partial c)^{-2}$ which is several orders of magnitude larger than $(\sigma \rho^{-1} \partial \rho / \partial \sigma)^{-2}$.

Still another feature of the sound propagation in solutions of helium isotopes is connected with the oscillations of the concentration. In the sound propagation, the “attuning” of the concentration to the changing pressure and temperatures comes about. This is shown to be so important that the sound velocity depends on the quantity $(c \rho^{-1} \partial \rho / \partial c)$ (particularly the velocity of fourth sound in solutions^[3-4]).

Let us estimate the amplitude of oscillation of the concentration in the sound wave. If we assume that the amplitude $v_s \approx 10^{-3}$ cm/sec then, for a solution with concentration $c = 0.2$ for $T = 1.5^\circ\text{K}$ we have $c' \sim 10^{-4}$ in the wave of first sound and $c' \sim 10^{-2}$ for the wave of second sound. Such concentration oscillations can be recorded in a sound wave, for example, by the method of nuclear magnetic resonance. The possibility is not excluded that one can also produce a sound wave in the medium by exciting concentration oscillations only.

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