EXPERIMENTS IN SEARCH OF FRACTIONAL CHARGES

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A method for measuring the electric charge of small bodies is developed. The accuracy of the method is of the order 0.1e for a mass of the order of $(0.3\text{—}2.0) \times 10^{-8}$ gram. For this purpose, use is made of the Braunbeck method proposed by Becchi, Gallinaro and Morpurgo, whereby a diamagnetic body can be suspended by means of a stationary magnetic field. The results of two series of measurements are described. Effects which could have been caused by fractional charges have been observed in some experiments. Possible background effects which could immitate the presence of fractional charges are discussed. Under the conditions of the experiments, the possible background effects do not allow one to assert that fractional charges exist. Necessary improvements of the method are discussed.

1. DESCRIPTION OF THE METHOD, MEASUREMENT RESULTS

As already indicated [1-3], one can utilize a modification of the Millikan method to search for stable quarks [4]. It is desirable to determine a minimal charge (smaller than the electron charge) of a test body of mass several orders of magnitude larger than the mass of the oil-drops in Millikan's experiments. In this case it becomes necessary to suspend the test body either by means of a servomechanism or by means of a Braunbeck suspension [1,3,5]. The latter method was proposed for searching for quarks by Becchi, Gallinaro, and Morpurgo. A test body placed in this manner in a magnetic potential well will be displaced from its equilibrium position if the potential well is inside an electric field and the body is charged. If the absolute values of the displacements are small, they are proportional to the charge, and they will change by discrete amounts if the charge of the body is changed. The presence of a quark in the test body will have the effect that instead of having the possible charge values $(\ldots, -2, -1, 0, +1, +2, \ldots)e$, one should observe the charges $(\ldots, -2\tfrac{1}{3}, -1\tfrac{1}{3}, -\tfrac{1}{3}, +\tfrac{2}{3}, +1\tfrac{2}{3}, \ldots)e$ or $(\ldots, -2\tfrac{2}{3}, -1\tfrac{2}{3}, -\tfrac{2}{3}, +\tfrac{1}{3}, +1\tfrac{1}{3}, +2\tfrac{1}{3}, \ldots)e$.

Thus, instead of measuring the time of motion of drops in the electric field of a capacitor, as was done by Millikan, it is necessary to make use of the distribution function of the displacements of the test body with respect to its equilibrium position in the potential well. It is necessary to have sufficiently small displacements in order that they be linearly dependent on the magnitude of the electric charge.

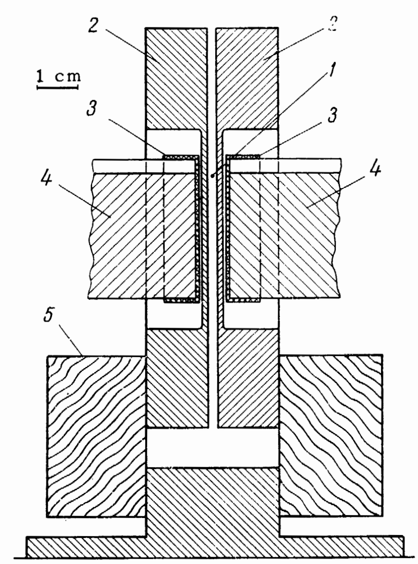

FIG. 1

Figure 1 represents the central part of the installation which was used in the measurements. The test body 1—a graphite particle—is situated near the gap between the plane pole pieces of the electromagnet 4. The upper part of each pole piece is hollowed out. The sharply decreasing field creates near the gap an upward directed force on the diamagnetic graphite, of magnitude $(1-\mu)(8\pi)^{-1}\partial H^2/\partial z$, which balances the weight of the particle. The electric field between the plates of the capacitor 2, which is placed inside the gap of the magnet, is almost parallel to the lines of force of the magnetic field. The equili-

Table I

Particle No.	$\Delta q = \bar{x}/x_e$	$m, 10^{-9}$ g	x_e
1	−0.021±0.100	3.7	14.4
2	−0.059±0.140	6.7	8.0
3*	−0.256±0.207	6.5	8.3
4	+0.033±0.058	3.1	17.3
5	−0.036±0.079	3.2	17.1
6*	+0.334±0.339	6.9	7.8
7	−0.071±0.160	7.0	7.7

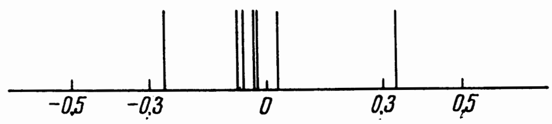

Table II

Particle No.	$\Delta q = \bar{x}/x_e$	$m, 10^{-9}$ g	x_e
1	−0.021±0.100	9.9	9.5
2	−0.048±0.078	13.0	7.2
3	−0.022±0.033	4.2	22.4
4*	+0.260±0.090	13.0	6.5
5	−0.000±0.115	8.8	10.7
6	+0.012±0.150	14.0	6.9
7	−0.013±0.110	7.0	12.6
8*	+0.375±0.185	16.0	4.8
9*	−0.296±0.244	16.0	4.9
10	+0.040±0.060	10.0	10.0
11	−0.061±0.072	7.0	13.0
12	+0.023±0.170	7.0	12.8
13	−0.023±0.180	14.0	7.0
14	−0.006±0.030	6.5	12.0
15	+0.054±0.080	7.0	13.0
16	−0.020±0.060	10.0	10.2
17	−0.030±0.100	6.6	15.5
18	−0.060±0.100	8.9	10.4
19*	+0.385±0.182	20.0	5.2

brium position of the particle under the simultaneous action of both fields is, for small charges, a linear function of the charge q, so long as the electric field E is constant.

Figure 1 also shows the insulators 3 and the glass blocks 5 to which the plates are rigidly fixed. The dimensions of the graphite particles are enlarged 10 times compared to the other parts in the picture. The position of the particle is registered on a photographic film by means of an optical system (magnification 90) with horizontal axis passing close to the particle 1.

The measurements were carried out in the following sequence: the condenser plates 2 were connected alternately to voltages of opposite signs coming from the same source, and the image of the position of the particle relative to a fixed reference system is registered on film. These positions are denoted by Δ_n^+; Δ_{n+1}^-; Δ_{n+2}^+; Δ_{n+3}^- (the upper sign corresponds to the sign of the field, the subscript labels the measurement). The differences $x_{n+1} = \Delta_n^+ - \Delta_{n+1}^-$; $x_{n+2} = \Delta_{n+2}^+ - \Delta_{n+3}^-$ are proportional to q for constant absolute value of E. However, the individual values of x_n are subject to strong fluctuations for the same value of q (these fluctuations have been observed experimentally and their magnitude ranges on the average from 1e to 0.2e in different runs). (For more details cf. Sec. 3.) For this reason 50 to 100 measurements have been performed for the same q and the sums of the x_n obtained in this manner were used for a statistical estimate of $\bar{x}$. The space between the plates was then exposed to weak x-rays and a new series of measurements were made. On the whole, with one given probe, 8 to 12 series of measurements were carried out, i.e. about 800 to 1200 individual measurements of Δ_n. The differences between the individual $\bar{x}$ either turned out to be statistically indistinguishable, or were integral multiples of the same number x_e, which was adopted as the magnitude corresponding to the charge of the electron. In the experiments under discussion x_e varied for various test bodies between 12 and 4 microns. Knowing the characteristics of the potential well one can estimate from the magnitude of x_e the mass of the particle, which varied between 0.3×10^{-8} gram (cf. Sec. 3 for details).

For each particle one or several series of measurements refer to the minimal absolute value of the charge of the test body. In the majority of these experiments the average $\bar{x}_m$ for such a series was statistically indistinguishable from zero. However in six cases out of 26 (cf. Tables I and II, where these cases are marked by asterisks), deviations from $\bar{x}_m$ were observed, in the form of fractions of x_e, i.e. in units of e correspond to the fractional charges: $-0.256e \pm 0.207e$, $+0.334e \pm 0.339e$, $+0.260e \pm 0.09e$, $+0.375e \pm 0.185e$, $-0.296e \pm 0.244e$, and $+0.385e \pm 0.182e$ (the confidence limits are indicated at a confidence level of 0.99). The listed confidence intervals, which are approximately three times larger than the probable deviation, correspond only to the statistical fluctuation of $\bar{x}_m$. The total number of experiments was 40, however only 26 were subjected to a full statistical treatment (cf. Sec. 3 for details). The likelihood that the observed effect is a statistical fluctuation is negligible. For example for the case $\Delta q = 0.260e$ an increase of the confidence level from 0.990 to $(1 - 1 \times 10^{-7})$ leads to a doubling of the confidence interval, i.e. to $q = +(0.260 \pm 0.183)e$. Thus, for this particle the confidence level of the statement that an effect simulating a fractional charge was observed, rather than a rare deviation, is larger than $(1 - 1 \times 10^{-7})$. In Sec. 2 are given more detailed examples of two statistical analyses of the displacements of two particles.

A possible source of systematic error is the presence of a static dipole moment of the test body, interacting with the inhomogeneities of the electric field[1)] possibly due to unevenness of the surface of the condenser plates, which are partially covered with a nonuniform layer of graphite dust. Such a field inhomogeneity will change sign together with the field itself, $\partial E_x/\partial x = \alpha E$. Assuming that the magnetic field rigidly orients the electric dipole due to the magnetic anisotropy of the graphite, we find that the force is $F = \alpha D_x E$, where D_x is the dipole moment. For $D_x \sim 5 \times 10^{-9}$ cgs esu (cooresponding to a potential difference of 1—2 V on the surface of the particle) and for the values of αE corresponding to the conditions of the experiment, the force $F = \alpha D_x E$ could be as large as 0.3eE (cf. Sec. 3 for details).

Thus the deviations corresponding to fractional charges could in reality be caused by the joint action of the dipole and the inhomogeneity of the electric field. For just this reason one cannot conclude that fractional charges have been observed, in spite of the statistical confidence in the observed deviations.

On the other hand from the observation of 20 particles with zero minimal charge it follows that even if quarks exist, their concentration is of the order of 10^{-16} per nucleon, or less.

We plan a direct measurement of the dipole moment of the particles and a reduction of the inhomogeneity of the electric field related to the method of introducing the graphite particles into the potential well. Another possibility consists in reducing the differences among the test bodies and increasing their number. Should it turn out that the minimal charges computed without taking into account the possible dipole moments group themselves around 0, $+1/3$ e and $-1/3$ e, one could assert that fractional charges have actually been observed. However now, after three months of measurements, the number of experiments with apparent fractional charges is not sufficiently large in order to be able to make such a statement with statistical confidence.

Table I summarizes the data from a group of measurements with E = 0.9 kV/cm; Table II is for E = 1.5 kv/cm. For each group of measurements the averages $\bar{x}$ in units x_e are indicated for the minimal charge and the confidence limits also in units x_e, as well as the masses of the particles of graphite and the values of x_e in divisions (1 div $= 4.8 \times 10^{-5}$ cm). The histograms for $\bar{x}$ are constructed without indication of the confidence limits from the data in Tables I and II.

2. DESCRIPTION OF TWO SERIES OF EXPERIMENTS

As an example we consider in more detail the results of one of the experiments with "zero" charge (i.e., an experiment with a particle for which the minimal absolute value of the charge is statistically indistinguishable from zero), and one of the experiments with "fractional" charge.

In the upper part of Fig. 2 the distribution functions for all x_n are constructed for particle No. 5 from the second group (cf. Table II). The distribution function has six clearly expressed maxima, one of which corresponds approximately to zero displacement. The height of each maximum is not a characteristic quantity, since it depends only on the time interval during which the given graphite particle had the given charge. The different markings of the points in the maxima correspond to different series of measurements. The lower part of the figure illustrates the time dependence of the quantity x_n (the time axis points downward). The letter R denotes the instants at which the x-rays have been switched on. In order to follow more clearly the discrete character of the variation of the displacement, the lower part of the figure contains not individual values of the displacement x_n, but averages over eight successive values $\bar{x}_8$. It is not difficult to estimate from the listed data that the possible spontaneous jumps of the charge (which are relatively rare: about one case in $\sim 10^3$ values of x) can be statistically distinguished by using 4—8 successive values of x_n. Therefore in computing $\bar{x}$ for one maximum we are allowed to group together all x_n for which $\bar{x}_8$ refers to the same charge state, obtained in different series, even when the separate x_n differ by a distance which is larger than the distance between the maxima. It is clearly seen in the figure how the same states are repeated twice (i.e., states where the x_n are statistically indistinguishable).

Grouping the x_n for the various states, we obtain the following data for $\bar{x}_n$ and their differences:

$\bar{x}_n$	−32.6		−21.1		−9.3		0.0		+10.2		+21.6
$(\bar{x}_n)_i - (\bar{x}_n)_j$		11.5		11.8		9.3		10.2		11.4	

We find from here that $\overline{(\bar{x}_n)_i - (\bar{x}_n)_j} = 10.7$. Assuming that the states under investigation correspond to the charges (−3e, −2e, −1e, 0, +1e, 2e) we obtain $x_e = 10.7$ $(10 = 4.8 \times 10^{-4}$ cm). We have

[1)]This circumstance was indicated to us by E. L. Feĭnberg.

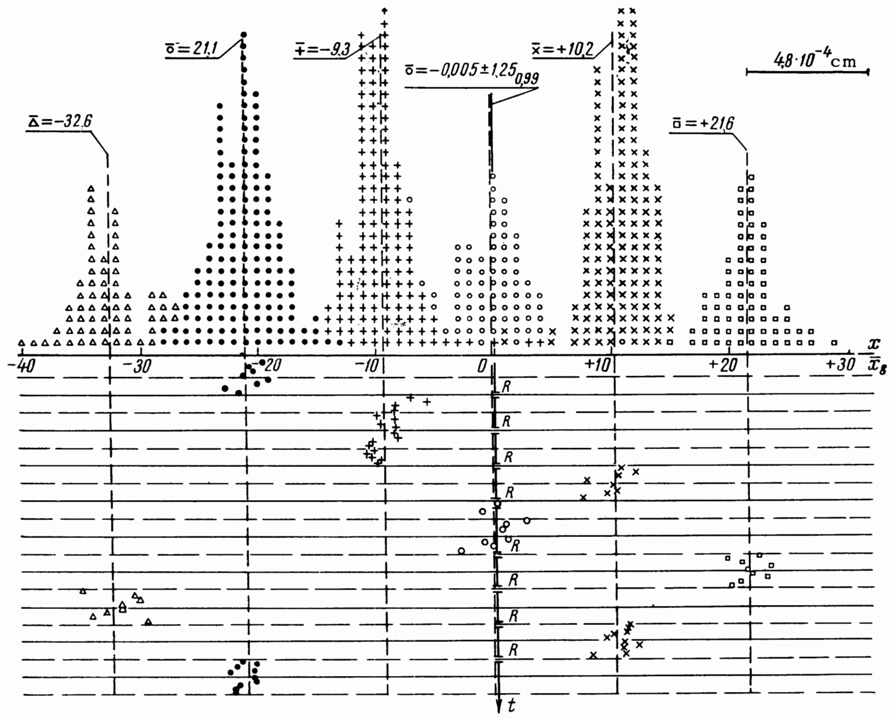

FIG. 2

quoted the most favorable case, when all states between −3e and +2e have been observed, without omissions, and such that the charge jumps for individual irradiations were 5e, 3e, 2e, 1e, and 0e. Usually the charge of the particle varies from −5e to +5e with several omissions (jumps of 2—3 electron charges). The likelihood that in the experiment under consideration we actually deal with a series of states −6e, −4e, −2e, 0, +2e, +4e is very small, since there are no reasons that the change should change under irradiation by a special multiple of the elementary charge. The values of $\overline{x}_8$ illustrated in Fig. 2 for states with charge −2e and +1e (repeated twice) can be statistically distinguished with a significance level of 0.90 (and are indistinguishable at a level of 0.990). This circumstance is apparently caused by a change in the geometric dimensions of the installation as a result of temperature drift, which in turn, influences the coefficient of optical magnification and consequently produces a drift in the magnitude of x_e. Therefore, in order to decide whether or not a given graphite particle contains a fractional charge it is more convenient to analyze the statistical distinguishibility of $\overline{x}$ from zero for a series corresponding to a charge smaller than the electron charge. The other maxima are useful only for determination of the scale of x_e. It is clear that a drift of this scale will have the smallest possible influence on the "zero" maximum.

Figure 3 illustrates the distribution of x_n for particle No. 4 from the second series (cf. Table II) and the time variations of $\overline{x}_8$, in the same manner as was done before for particle No. 5 (Fig. 2). The scale of x_e for this particle turned out to be $x_e = 6.5$ and its "zero" maximum is displaced from zero by 1.67 ± 0.6 (confidence level 0.990), corresponding to $q = (+0.26 \pm 0.09)e$. As can be seen from the figure and the values of $\overline{x}$ on it, one observes jump-like variations of the charge by 4e, 2e, 6e, 1e, and 0e, for the various maxima observed for this graphite particle. The states with charge 15e—16e (the eighth interval in the figures) are not shown, since these states have not been subjected to statistical treatment. This has been the rule for all graphite particles, in order not to have to take into account the deviations of the form of the potential well from a parabolic one. Thus, for this particle one charge jump with odd electron number has been observed, against six jumps with even numbers.

In conclusion we note that the confidence intervals for $\overline{x}_n$ have been computed by means of the quantiles of Student's t-distribution, without using the sums of all x_n corresponding to one maximum, but only the sum of the averaged $\overline{x}_4$. The normality of the distribution of $\overline{x}_4$ was tested by means of the K(λ)-criterion of Kolmogorov [6]. The complete data on observed displacements listed in Figs. 2 and 3 for the particles No. 4 and 5 could be subjected to any other statistical treatment.

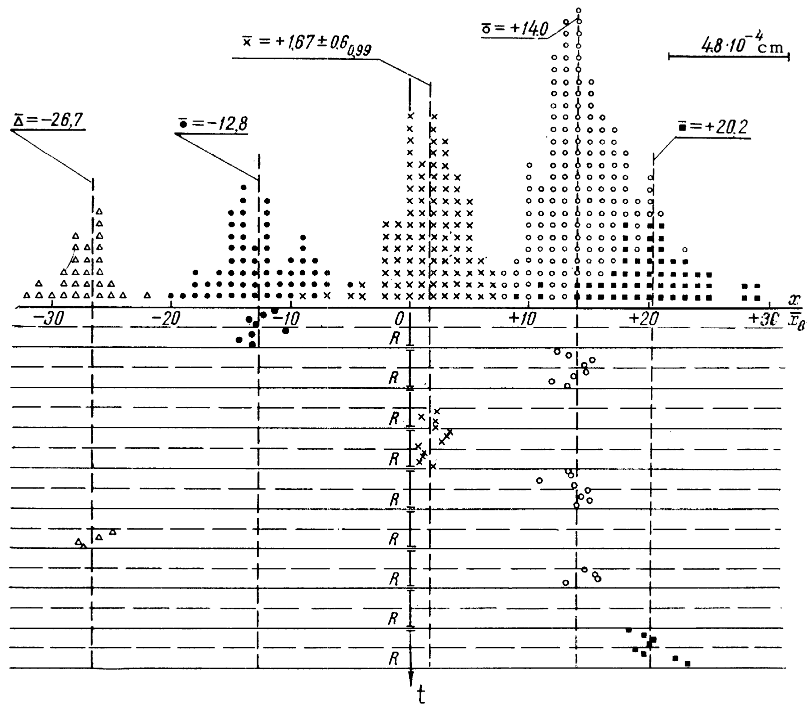

FIG. 3

3. INCOHERENT NOISE. PECULIARITIES OF THE METHOD

Under the conditions of the experiment the graphite particles (their dimensions are 10—25 microns) are overdamped, owing to air viscosity, and their relaxation time in the potential well is approximately 1 sec. The exposure time of the particle on the photographic film for a given direction of the electric field was of the order of 5 seconds. The same amount of time was spent as a pause after reversing the field. Therefore, for a total number of $n \sim 10^3$ the total time necessary for obtaining the distribution function with 4—6 maxima was approximately 10^4 seconds.

The error in the measurement of x_8 and of the mass of the particle is small (mean square deviation of the order of 5%) if there was no error of an integral number of times: 2, 3, . . . times due to a change in charge under x-ray exposure by a number of electrons which is a multiple of 2, 3, ... Such a possibility was discounted in all all statistical treatments. Then the probable error in the determination of the charge and the corresponding confidence intervals are completely determined by the error in the measurement of $\bar{x}$ for the state of minimal charge. For this state we have made on the average 80 measurements for each particle with a total duration of 80 × 5 sec = 400 sec.

In the total case, when the scatter of the data is determined only by brownian motion, we have, for the utilized method (which is analogous to the method of synchronous detection in keyed regime [7]), taking into account that x is the difference of two Δ,

$$\overline{(x - \bar{x})^2} = 2\sqrt{\varkappa T\tau / m\omega_0^2 t} = 5\cdot 10^{-6} \,\text{cm},$$

where τ is the relaxation time of the particle, t is the observation time.

For an average $x_e = 5 \times 10^{-4}$ cm we find the mean square error in the determination of the minimal charge to be 0.01e, and the confidence interval 0.03e, in the ideal case, which is in fact 5—10 times smaller than the errors which were actually found.

A possible cause of the relatively large noise level in the conditions of our experiment could be the presence of nonstationary air currents. In order to displace the graphite particle in the direction of the electric field by an amount equal to the mean square deviation of a unit x_n a velocity of the air current of 3×10^{-4} cm/sec is necessary. Estimates of the possible influence of seismic noises on the scatter of x have not explained the observed excess of the scatter as compared to the Brownian scatter.

We stop briefly to discuss the peculiarities of the method. It is sufficiently difficult to "place" a particle of mass $\sim 10^{-8}$ gram in a potential well. We have used the following approach. Approximately 20—30 mg graphite are introduced on a metallic support into the region close to the potential well. The graphite is expelled by the action of the magnetic field and partially deposits itself on the capacitor plates. Then the magnetic field is decreased by a factor of 3—4 (and the suspended particles drop, whereas part of the graphite that was deposited on the condenser will fall off, leaving only a thin layer, of the order of 100—150 microns). After that the experimenter applies to the capacitor plates a strong alternating electric field (of amplitude 10 kV/cm and frequency 5 Hz), which causes the graphite particles to oscillate and to fall at the same time between the plates. When the "needed" particle of correct size passes near the stable position of equilibrium, the alternating field is switched off, and the magnetic field is simultaneously increased and captures the falling particle.

Such a suspended particle carries an excess charge, usually of ± (1000–300)e. This charge is removed by using Millikan's method. For this purpose the capacitor is connected to a dc voltage and a weak source of x-rays is switched on and ionizes the air between the plates. The small voltage applied to the capacitor displaces the particle from the equilibrium position. If the sign of the field is correctly selected, the particle will return to its equilibrium position in a few minutes. In case the particle continues to move away from its equilibrium position with the x-ray source on, it is necessary to reverse the polarity of the electric field. Owing to a small asymmetry of the x-ray irradiation with respect to the particle, the sign of the derivative of the charge will change when the polarity is reversed. Making use of this method, a sufficiently experienced operator will "reduce" the excess charge to 0—5e in about 3—5 minutes. However in several instances the particle would accumulate an excessive charge and would "fall" out of the well. In this case there was no possibility of continuing the measurement on the given particle, which undoubtedly is a disadvantage of this method of introducing the particle into the potential well. Another disadvantage of this method is the impossibility to repeat measurements even if the particle is conserved.

After suspending the particle and reducing its charge to several electron charges, the operator determines its relaxation time in the potential well, which leads to a rough estimate of its mass (cf. infra). The image of the particle, magnified 90 times by means of a "Gelios-40" objective and a short-focus lens ($f = 2$ cm, $D = 0.8$ cm) was focused on the film. After exposing 1000 frames at small particle charges, the film was developed and the displacements were measured. The operator would measure the distance on the film from fixed fiducial marks, which were also projected onto the film, to the center of the optical image of the particle. The measurement was carried out visually on a projection of the film frame magnified 24 times.

Control measurements were carried out in order to determine the magnitude of the error committed by the operator in the individual reading of the position of the image of the graphite particle. Out of several dozen randomly selected frames handed to the operator, he measured forty times the distance to the center of the image of the particle on the same frame, without knowing it. The estimate of the mean square deviation of the operator error was $S_{op} = 5.2 \times 10^{-5}$ cm, corresponding to an error in the determination of the difference of two readings of $S = \sqrt{2}\, S_{op} = 7.3 \times 10^{-5}$ cm.

In addition, two independent operators were supposed to obtain the distribution of displacements for the same particle (1000 frames), and from the differences in displacements in the same frames another estimate for the operator error was derived: $S_{op} = 6.7 \times 10^{-5}$ cm, for each operator, under the assumption that their errors are the same. Thus the error introduced by the operator in measuring the position of the center of the image was $(6.7—5.2) \times 10^{-5}$ cm. We notice that such a difference between the two values of S_{op} for our small sample is admissible according to Fisher's criterion [6]. Since the mean square deviation observed in the measurements is larger (in the first series $S_{meas} = 1.9 \times 10^{-4}$ cm and in the second series $S_{meas} = 1.4 \times 10^{-4}$ cm), the main source of the dispersion is not the operator error in determining the distances.

Although the operators did not know in advance what the distribution function was—it was obtained only after reading about 1000 displacement values —in the most interesting cases, i.e., those where "fractional charges" were observed, the distribution function was determined independently several times by two or three operators. The operator who photographed the particle would not be involved in obtaining the distribution function of displacements for a given film.

In principle it would be possible to use a photometric operation in order to determine the center of the spot on the film more precisely, thus excluding the operator error. However such a complication of the method did not seem justified, taking into account the large statistics. After obtaining the distribution function and the maxima corresponding to discrete charges on the particle, those distribution functions for which some arbitrariness in the determination of x_e was possible were not subjected to any further treatment (for example, in the case when only three maxima were obtained). The same was true for those distribution functions for which the operator did not succeed in obtaining a state with $q > 1e$ (a "zero" maximum). These rules eliminated 14 of the 40 particles on which measurements were carried out, and the remaining 26 are listed in Tables I and II. We note that for those of the 14 eliminated particles for which it was possible to determine x_e one could also estimate the possible value of the excess Δq from the common displacement of the maxima (in the other 26 cases this was determined from the state $q < 1e$). Only for one of the 14 particles was a displacement of $0.11e \pm 0.20e$ (confidence level 0.99) observed. This value is statistically indistinguishable from zero.

In the second series of measurements (Table II), in distinction from the first (Table I), the interaction volume, which was previously rid of all dust, was sealed more thoroughly, and the electromagnet was force-cooled. In addition, the statistics were increased 1.5 times in the second series and a larger electric field was used ($E_2 = 1.5$ kV/cm compared to $E_1 = 0.9$ kV/cm). The difference in the values of the confidence intervals for each of the observed cases of minimal charges in each of the series (cf. Tables I and II) is due to the following causes: a) different statistics of the displacements corresponding to the states with $q < 1e$, b) differences in the masses of the particles, and consequent differences in the contribution of the operator error to x and x_e, since lighter particles correspond to larger x_e.

As can be seen, the method does not necessitate knowledge of the mass of the particle. Only the equidistance of the maxima of the distribution function, observed in all experiments, is necessary. It is also essential that in all cases with somewhat heavier particles as well as in those cases included in Tables I and II, where the particles had a sufficiently elongated shape, no rotation of the particles took place, and only translational displacements along the electric-field lines were observed.

The mass of the particles can be roughly estimated from the dimensions of its geometric shadow, assuming that its shape does not deviate too much from a spherical one. In Tables I and II mass values determined by another method are listed. Heavier graphite particles, of masses from 5×10^{-7} to 1×10^{-2} gram, suspended in the same magnetic field, turned out to be resonating oscillators with the eigenfrequency varying between 4.8 and 5.1 Hz (on the average, the larger frequency corresponds to lighter particles). This is easily checked by means of a simple photoelectric transducer, which converts displacement of the particle into an electric signal, and determining the resonance curves of the oscillators, by varying the frequency of the voltage applied to the capacitor plates. It is clear that the average value of this frequency characterizes the potential well shape in the direction of the electric field lines, and depends but weakly on the mass of the particle. Extrapolating this rule to lighter particles, one can estimate their mass to be $m = eE(\omega_0^2 x_e)^{-1}$, where x_e is determined as described above. Both tables contain estimates of the masses, computed in this manner. It was assumed that $\omega_0 = 2\pi \times 5\ \mathrm{sec}^{-1}$. We note that during the measurements all particles were "suspended" at the same height, the magnetic field varying for different particles (from 14 to 18 kOe) which seems to be caused by the strong magnetic anisotropy of graphite and the inhomogeneity of the particles.

The eigenfrequencies for vibrations along a direction perpendicular to the lines of force is determined by the shape of the hollowed part of the pole piece of the electromagnet. In our case it was $2\pi \times 0.9\ \mathrm{sec}^{-1}$.

The quantity $\partial E/\partial x$ was estimated from the magnitude of the force $F_{inhom} = v\ \mathrm{grad}\ E^2$, which can be measured by comparing the coordinates of the particle with the field off and the arithmetic mean of the coordinates for two field directions. The force F_{inhom} displaced the particle by an amount ranging from $0.1\ x_e$ to $1\ x_e$. The latter value depended on the quantity of graphite dust deposited on the condenser plates. Knowing m, and consequently v, one can determine $\partial E/\partial x$ from a measurement of F_{inhom}. It is important that the quantity F_{inhom} may not be taken into account in the determination of x_e and of the possible value of Δq, since in the normal measuring procedure only the polarity of the electric field is changed and consequently F_{inhom} is always directed in one direction, and has no influence on the magnitude of the differences of displacements of the particle. Only the dipole moment influences the magnitude of Δq.

As can be seen from what was said, the experiment which was described is not a metrological one to determine the value of e. However, if suitable control experiments for the determination of $\partial E/\partial x$ are made, or the homogeneity of the field is significantly improved, this method can also be used for the determination of the magnitude of the fractional charges.

In conclusion, the authors express their gratitude for valuable discussions to A. S. Borovik-Romanov, N. V. Zavaritskiĭ, V. I. Kogan, L. B. Okun', B. M. Pontecorvo, I. M. Samoĭlov, A. D. Sakharov, E. L. Feĭnberg, Yu. B. Khariton, and M. C. Khaĭkin.

Note added in proof (December 6, 1966). In discussing the results of this paper the question arose whether or not one can consider that the data contained in Tables I and II and in the histograms under the tables contain a statistically significant grouping of the observed deviations from zero around $\pm(1/3)$ e. It is necessary to stress that the apparent value of the charge of the fractional charge is obtained by dividing the average minimal deviation $\bar{x}_n$ by the value for the electron, x_e: $q = \bar{x}_n/x_e$. The confidence interval listed in Tables I and II corresponds to the probable errors in the determination of the numerator $\bar{x}_n$, but does not take into account possible errors in the denominator x_e. In the determination of the denominator, in addition to the regular errors (of the order of 10–20%), there is also the possibility of an error by an integral factor, if the charge jumps are attributed to an incorrect number of electrons. A change of scale does not modify qualitatively the result for the zero experiment (e.g. 0.06 ± 0.10–0.03 ± 0.05, and as before, zero is inside the confidence interval). However the grouping around $\pm(1/3)$ e is strongly affected (e.g. 0.26 ± 0.09–0.13 ± 0.05). Therefore, as already stated in the body of the paper, the data which were obtained do not allow to answer the question about the froupings around $\pm(1/3)$ e.

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