DRIFT INSTABILITIES OF A PLASMA SITUATED IN A HIGH-FREQUENCY ELECTRIC FIELD

Ya. B. FAĬNBERG and V. D. SHAPIRO

Physico-technical Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor August 11, 1966

J. Exptl. Theoret. Phys. (U.S.S.R) 52, 293–308 (January, 1967)

The possibility of stabilizing drift instabilities in an inhomogeneous plasma by means of a uniform high-frequency electric field is investigated; the electric field is in the direction of the magnetic field. It is shown that the collisionless drift instability can be stabilized by a longitudinal electric field when $k_z v_{Ti} \ll \omega \ll k_z v_{Te}$ and when $\omega \gg k_z v_{Te}$ (ω and k_z are respectively the frequency and projection of the wave vector associated with the drift instability in the direction of the magnetic field, while v_{Te} and v_{Ti} are the thermal velocities of the electrons and ions). The values of the frequency and amplitude of the applied electric field required for stabilization are determined. The possibility of high-frequency stabilization of the drift-dissipative instability is also considered.

1. INTRODUCTION

It is well known that confinement of a plasma by a magnetic field is hindered by hydromagnetic instabilities as well as a broad class of drift instabilities that arise by virtue of drifts in an inhomogeneous plasma.[1–5] These instabilities lead to enhanced diffusion of the plasma across the magnetic field and for this reason any method of stabilizing these instabilities would be of value in connection with plasma confinement and heating. At the present time, one class of methods for stabilization has been examined in great detail: these methods are based on the use of shear in the magnetic lines of force, "corrugated" magnetic fields (bumpy fields) and so on.

Another possible method of stabilization of the micro-instabilities associated with plasma drifts has been suggested in [6]. This method is based on the modification of the drift by an external high-frequency field. Theoretical and experimental investigations [7–9] have verified the effectiveness of this method for stabilization of two-stream instabilities. Another possibility lies in the use of a uniform high-frequency electric field in which the electrons are constrained to execute forced oscillations. The stabilization of the two-stream instability due to the motion of electrons with respect to the ions by virtue of the existence of a field of this kind has been treated by Aliev and Silin.[10] High-frequency stabilization of hydromagnetic instabilities has been considered by Osovets.[11]

It is of interest to consider the possibility of stabilizing drift instabilities by means of high-frequency fields. In the present paper we investigate the stabilization of drift instabilities in an inhomogeneous plasma by the application of a uniform high-frequency electric field in the same direction as the magnetic field:

$$E_0(t) = \mathscr{E}_0 \cos \Omega t. \tag{1}$$

The possibility of stabilizing drift instabilities by means of high-frequency fields can be understood as follows. In a high-frequency field of sufficient amplitude the oscillations of the electrons in the wave field become important. If a drift instability is excited in the plasma the plasma electrons are subject to an additional force in the direction of the magnetic field due to the high-frequency electric field $\mathbf{E}_0(t) + \mathbf{E}_1(t^*, \mathbf{r})$: this force is given by

$$\frac{\varepsilon_{zz}(\Omega)-1}{4\pi}\frac{\partial}{\partial z}\langle E_0 E_{1z}\rangle$$

(ϵ_{zz} is the longitudinal component of the dielectric tensor and $\mathbf{E}_1$ is the high-frequency part of the wave field while the angle brackets denote averages over the period of the high-frequency oscillations).

When this additional force is in phase with the force due to the gas-kinetic pressure the field in the drift wave $\langle E \rangle$ is increased, as is the frequency ω. As ω increases, the growth rate for the drift instability (here we are considering the collisionless drift instability under the conditions

$\sqrt{T/m_i} \ll \omega/k_z \ll \sqrt{T/m_e}$) contains a stabilizing term which increases, this term being proportional to

$$\frac{\partial f_0^e}{\partial v_z}\left(\frac{\omega}{k_z}\right) \approx -\frac{m_e\omega}{Tk_z} f_0^e(0)$$

(Landau damping) and the instability can then exist only if there are large gradients.

In the second section of the present work we discuss the mechanism associated with high-frequency stabilization of the collisionless drift instability for the simple case in which the Larmor radius of the plasma particles can be neglected. In the third section we carry out a more general analysis of this stabilization effect in which the finite Larmor radius of the ions is taken into account. In the fourth section we investigate the possibility of stabilization of the drift instability in a plasma in which collisions are important (drift-dissipative instability), this stabilization being brought about by the high-frequency electric field.

2. STABILIZATION MECHANISM FOR THE COLLISIONLESS DRIFT INSTABILITY

Consider an inhomogeneous plasma in which the density and temperature depend on the x coordinate; the plasma is located in a magnetic field and an alternating electric field (1) both of which are along the z axis. We consider drift instabilities in this plasma assuming that the frequency of the external field Ω is large compared with the frequency of the drift oscillations ω.

In the equilibrium state the electrons and ions in the plasma oscillate in the external electric field with velocities given by

$$u_\alpha(t) = (e^\alpha \mathscr{E}_0 / m_\alpha\Omega)\sin\Omega t, \quad \alpha = i, e.$$

Small perturbations about the equilibrium state are written in the form $f(t)\exp[i(\int k_x dx + k_y dy + k_z dz)]$. Because of the perturbations the motion of the electrons along the lines of force is governed by the equation

$$\frac{\partial v_z^e}{\partial t} - ik_z u_{0e} v_z^e \sin\Omega t = -\frac{e}{m_e}E_z - ik_z\frac{T}{n_0 m_e}n_e. \quad (2)$$

Here, $u_{0e} = e\mathscr{E}_0/m_e\Omega\ (e > 0)$, T is the plasma temperature and v_z^e and n_e are the deviations of the electron velocity and density from the equilibrium values. We now substitute in (2)

$$v_z^e = W_z^e e^{-ia_e\cos\Omega t}, \qquad n_e = \eta_e e^{-ia_e\cos\Omega t},$$
$$a_\alpha = -\frac{k_z e_\alpha \mathscr{E}_0}{m_\alpha\Omega^2} = \frac{k_z u_{0\alpha}}{\Omega}$$

and average over the high-frequency oscillations. The *inertia* term can be neglected in the averaged equations if $\omega \ll k_z\sqrt{T/m_e}$. Thus we find

$$\langle E_z e^{ia_e\cos\Omega t}\rangle = -ik_z\frac{T}{en_0}\langle\eta_e\rangle. \quad (3)$$

The field E_z, as well as the other quantities which characterize the perturbations from the equilibrium state, can be written in the form of a sum of fields: the first is a slowly varying (in time) field $\langle E_z\rangle$ and the second is a rapidly varying field (frequency Ω) given by E_{1z}. Substituting $E_z = \langle E_z\rangle + E_{1z}$ in Eq. (3), for $a_e \ll 1$ we have

$$\langle E_z\rangle + ia_e\langle\cos\Omega t E_{1z}\rangle + ik_z\frac{T}{en_0}\langle\eta_e\rangle = 0. \quad (4)$$

Using the expression for the longitudinal component of the dielectric tensor

$$\varepsilon_{zz}(\Omega) = 1 - \frac{\omega_{0e}^2}{\Omega^2}, \qquad \omega_{0\alpha}^2 = \frac{4\pi e^2 n_0}{m_\alpha},$$

we can write (4) in the form

$$n_0 e\langle E_z\rangle + ik_z T\langle\eta_e\rangle - ik_z\frac{\varepsilon_{zz}-1}{4\pi}\langle E_0 E_{1z}\rangle = 0. \quad (5)$$

The relation in (5) describes the balance of forces acting on the electrons in the direction of the magnetic field. In the absence of the external high-frequency field this equation contains only the first two terms which, for electrostatic oscillations, lead to the familiar Boltzmann distribution of the density in the field. The last term in (5) is the pressure associated with the high-frequency field $\mathbf{E}_0(t) + \mathbf{E}_1(t, \mathbf{r})$. In the general case this force is given by $\mathbf{F} = (8\pi)^{-1}(\epsilon_{ik} - \delta_{ik})\nabla E_i E_k$ (cf. for example [12]); in the case at hand, because E_0 is uniform the higher order term $\sim E_0^2$ disappears and we are left with the term $\mathbf{E}_0 \cdot \mathbf{E}_1$.

In what follows we limit our analysis to electrostatic perturbations.1) Substituting $n_\alpha = \eta_\alpha e^{-ia_\alpha\cos\Omega t}$ in Poisson's equation, when $a_\alpha \ll 1$ we obtain the following relation for $\mathbf{E}_1$:

$$\mathbf{E}_1 = -4\pi\frac{i\mathbf{k}}{k^2}\sum_\alpha e_\alpha[-ia_\alpha\cos\Omega t\langle\eta_\alpha\rangle + \eta_{1\alpha}]. \quad (6)$$

The subscript 1 denotes the high-frequency part of the density. Making use of (6) we now write (4) in the form

1)If the limitation to electrostatic drift oscillations in the high-frequency electric field is to hold, the following condition must be satisfied:

$$n\left(T + \frac{m_e u_{0e}^2}{2}\frac{k_z^2}{k^2}\frac{\omega_{0e}^2}{\Omega^2}\varphi(\Omega)\right) \ll \frac{H^2}{8\pi},$$

where the function $\varphi(\Omega)$ is given by (11).

$$\langle E_z\rangle + ik_z\frac{T}{en_0}\langle\eta_e\rangle\left(1+\frac{2\pi e^2 n_0}{k^2 T}a_e^2\right) - \frac{4\pi e k_z}{k^2}a_e\langle\cos\Omega t(\eta_{1e}-\eta_{1i})\rangle = 0. \quad (7)$$

Here we have used the fact that $\langle\eta_e\rangle \approx \langle\eta_i\rangle$ (quasi-neutrality of the high-frequency oscillations) and in obtaining (7) we have neglected terms of order m_e/m_i compared with unity.

If the conditions $k_z^2 T/m_e\Omega^2 \ll 1$ and $k_\perp^2 T/m_i\Omega^2 \ll 1$ are satisfied, in finding η_{1e} and η_{1i} we can use the conventional system of equations for two-fluid hydrodynamics for a cold plasma. If $\omega_{Hi} \ll \Omega \ll -\omega_{He}$ ($\omega_{H\alpha} = e_\alpha H_0/m_\alpha c$) this system of equations written for the quantities W_1^α and $\eta_{1\alpha}$ becomes

$$\frac{\partial \mathbf{W}_1{}^i}{\partial t} = \frac{e}{m_i}\mathbf{E}_1,\quad \frac{\partial\eta_{1i}}{\partial t} + i\mathbf{k}\mathbf{W}_1{}^i n_0 = 0,\quad \frac{\partial W_{1z}{}^e}{\partial t} = -\frac{e}{m_e}E_{1z},$$

$$W_{1x}{}^e = \frac{c}{H_0}\left[E_{1y} + \frac{1}{\omega_{He}}\frac{\partial E_{1x}}{\partial t}\right],$$

$$W_{1y}{}^e = -\frac{c}{H_0}\left[E_{1x} - \frac{1}{\omega_{He}}\frac{\partial E_{1y}}{\partial t}\right],$$

$$\frac{\partial\eta_{1e}}{\partial t} + i\mathbf{k}\mathbf{W}_1{}^e n_0 + W_{1x}{}^e\frac{dn_0}{dx} = 0. \quad (8)$$

Invoking the quasi-neutrality of the low-frequency oscillations, on the right side of (8) we have neglected terms of order $a_\alpha \cos\Omega t\langle\mathbf{E}\rangle$ compared with $\mathbf{E}_1 \sim 4\pi e k^{-2}\mathbf{k}a_e\cos\Omega t\langle\eta_e\rangle$. In conjunction with (6), which determines the field $\mathbf{E}_1$, the equations in (8) form a complete system for the description of forced plasma oscillations of frequency Ω in the inhomogeneous plasma. Eliminating W_1^α and $\mathbf{E}_1$ from this system we obtain the following equations for η_{1e} and η_{1i}:

$$\frac{\partial^2\eta_{1i}}{\partial t^2} + \omega_{0i}^2\eta_{1i} = \omega_{0i}^2(\eta_{1e} - ia_e\cos\Omega t\langle\eta_e\rangle),$$

$$\frac{\partial^2\eta_{1e}}{\partial t^2}\left(1+\frac{\omega_{0e}^2}{\omega_{He}^2}\frac{k_\perp^2}{k^2}\right) - i\frac{\omega_{0e}^2}{\omega_{He}}\frac{k_y\varkappa}{k^2}\frac{\partial\eta_{1e}}{\partial t} + \left(\omega_{0e}^2\frac{k_z^2}{k^2}+\omega_{0i}^2\right)\eta_{1e} = -a_e\langle\eta_e\rangle\left[\sin\Omega t\frac{\Omega\omega_{0e}^2}{\omega_{He}}\frac{k_y\varkappa}{k^2} + i\cos\Omega t\left(\Omega^2\frac{\omega_{0e}^2}{\omega_{He}^2}\frac{k_\perp^2}{k^2} - \omega_{0c}^2\frac{k_n^2}{k^2}\right)\right]. \quad (9)$$

Here $\kappa = d\ln n_0/dx$. Solving these equations and substituting the results in (6), we can now write the latter in the form

$$\langle E_z\rangle + ik_z\frac{T}{en_0}\langle\eta_e\rangle\left(1+\frac{2\pi e^2 n_0}{k^2 T}a_e^2\varphi(\Omega)\right) = 0, \quad (10)$$

where we have used the notation

$$\varphi(\Omega) = \frac{\Omega^2[N\Omega^2 - \omega_{0e}^2 k_z^2/k^2 - \omega_{0i}^2]}{[N\Omega^2 - \omega_{0e}^2 k_z^2/k^2 - \omega_{0i}^2]^2 - \Omega^2(\omega_{0e}^4/\omega_{He}^2)(k_y^2\varkappa^2/k^4)},$$

$$N = 1 + \frac{\omega_{0e}^2}{\omega_{He}^2}\frac{k_\perp^2}{k^2}. \quad (11)$$

The averaged equation of continuity for the ions for the condition $\omega \gg k_z\sqrt{T/m_i}$ (which means that the motion along H_0 in the low-frequency oscillations can be neglected) is then written in the form

$$-i\omega\langle\eta_i\rangle + \frac{c}{H_0}\langle E_y\rangle\frac{dn_0}{dx} = 0. \quad (12)$$

Using the neutrality of the low-frequency oscillations $\langle\eta_e\rangle \approx \langle\eta_i\rangle$, which holds when $a_e \ll 1$, and invoking the electrostatic condition, using (10) and (12) we obtain the following relation for the frequency of the drift oscillations in the presence of the high-frequency electric field:[2)]

$$\omega = -k_y\frac{T}{m_i\omega_{Hi}}\varkappa\left(1+\frac{a_e^2}{2k^2\lambda_D^2}\varphi(\Omega)\right). \quad (13)$$

We note that by using (10) and Poisson's equation for the field $\langle E_z\rangle$ we can obtain a relation which gives the deviation from neutrality in the low-frequency oscillations:

$$(1+k^2\lambda_D^2)\langle\eta_e\rangle = \langle\eta_i\rangle(1+{}^1/_2 a_e^2\varphi^*(\Omega)),\quad \lambda_D = \sqrt{T/4\pi e^2 n_0}, \quad (14)$$

$$\varphi^*(\Omega) = \left\{\left[N\Omega^2 - \omega_{0e}^2\frac{k_z^2}{k^2} - \omega_{0i}^2\right]\left[N\Omega^2 - \omega_{0e}^2\frac{k_z^2}{k^2}\right] - \Omega^2\frac{\omega_{0e}^4}{\omega_{He}^2}\frac{k_y^2\varkappa^2}{k^4}\right\}\left\{\left[N\Omega^2 - \omega_{0e}^2\frac{k_z^2}{k^2} - \omega_{0i}^2\right]^2 - \Omega^2\frac{\omega_{0e}^4}{\omega_{He}^2}\frac{k_y^2\varkappa^2}{k^4}\right\}^{-1}. \quad (14')$$

It follows from (14) that the deviation from neutrality in the low-frequency oscillations is important only when $a_e^2\varphi^*(\Omega) \sim 1$.

The possibility of high-frequency stabilization of the drift instability derives from the increase in the frequency of the drift oscillations caused by the application of the high-frequency electric field. Under these conditions the growth rate for the drift instability contains a stabilizing term associated with Landau damping, this stabilization term

2)In the limiting case of high-frequencies for the external field

$$\Omega \gg \frac{1}{N^{1/2}}\left(\omega_{0e}^2\frac{k_z^2}{k^2}+\omega_{0i}^2\right)^{1/2},\quad -\frac{1}{N}\frac{\omega_{0e}^2}{\omega_{He}}\frac{k_y\varkappa}{k^2}$$

we find $\varphi \approx 1/N$. In this case (13) becomes the expression obtained earlier in [13].

being given by

$$\frac{\partial f_0^e}{\partial v_z}\left(\frac{\omega}{k_z}\right)\approx -\frac{m_e\omega}{Tk_z}f_0^e(0)$$

(f_0^e is the equilibrium distribution function for the electrons).

It follows from (11) and (13) that the application of the electric field increases the frequency of the drift oscillations and thus increases the Landau damping when the field frequency Ω satisfies the condition

$$\Omega_- < \Omega < \frac{1}{N^{1/2}}\left(\omega_{0e}^2\frac{k_z^2}{k^2}+\omega_{0i}^2\right)^{1/2} \text{ or } \Omega > \Omega_+, \qquad (15)$$

where $\Omega_\pm$ is given by the expression

$$\Omega_\pm^2 = \frac{1}{N^2}\left[N\left(\omega_{0e}^2\frac{k_z^2}{k^2}+\omega_{0i}^2\right)+\frac{\omega_{0e}^4}{2\omega_{He}^2}\frac{k_y^2\varkappa^2}{k^4}\right.$$
$$\pm\left\{\left[N\left(\omega_{0e}^2\frac{k_z^2}{k^2}+\omega_{0i}^2\right)+\frac{\omega_{0e}^4}{2\omega_{He}^2}\frac{k_y^2\varkappa^2}{k^4}\right]^2\right.$$
$$\left.\left.-N^2\left(\omega_{0e}^2\frac{k_z^2}{k^2}+\omega_{0i}^2\right)^2\right\}^{1/2}\right]. \qquad (15')$$

When (15) is satisfied the frequency increases because the pressure associated with the high-frequency field is in phase with the gas kinetic pressure and this increases the electric field associated with the drift wave $\langle E\rangle$. For all other values of Ω the frequency of the drift wave is reduced and the alternating field becomes a destabilizing factor.

The increase of the Landau damping for the drift oscillations means that the plasma can become unstable only if the temperature gradients become high enough. In order to be convinced of this result we consider the work done by the drift wave on the resonance electrons in the plasma:

$$Q = -e\overline{\langle E_z\rangle\left\langle\int v_z f_1^e\,d\mathbf{v}\right\rangle}.$$

Here, in the usual way the brackets denote averages over the high-frequency oscillations while the bar denotes an average over the wavelength of the drift wave; f_1^e is the deviation from equilibrium of the electron distribution function.

In order to find this deviation we use the kinetic equation in the drift approximation. Introducing the quantity $w_z = v_z + (e\mathscr{E}_0/m_e\Omega)\sin\Omega t$ as the independent variable in this equation we write it in the form

$$\frac{\partial f_1^e}{\partial t}+ik_z\left(w_z-\frac{e\mathscr{E}_0}{m_e\Omega}\sin\Omega t\right)f_1^e-\frac{e}{m_e}\left(E_z\frac{\partial f_0^e}{\partial w_z}\right.$$
$$\left.+\frac{1}{\omega_{He}}E_y\frac{\partial f_0^e}{\partial x}\right)=0. \qquad (16)$$

In solving (16) it will be found convenient to introduce the function $\psi_e(t, x, w_z) = f_1^e e^{ia_e\cos\Omega t}$ which can be easily shown to satisfy the equation

$$\frac{\partial\psi_e}{\partial t}+ik_zw_z\psi_e-\frac{e}{m_e}E_ze^{ia_e\cos\Omega t}\left(\frac{\partial f_0^e}{\partial w_z}+\frac{k_y}{k_z}\frac{1}{\omega_{He}}\frac{\partial f_0^e}{\partial x}\right)=0. \qquad (17)$$

When $\Omega \gg k_z\sqrt{T/m_e}$, the quantity $\langle\psi_e\rangle \gg \psi_{1e}$ where the subscript 1 denotes the high-frequency part of the function ψ_e. Thus

$$\langle f_1^e\rangle = \langle\psi_e\rangle\langle e^{ia_e\cos\Omega t}\rangle = J_0(a_e)\langle\psi_e\rangle.$$

Determining $\langle\psi_e\rangle$ from (17)

$$\langle\psi_e\rangle = -\frac{e}{m_e}\frac{\langle E_ze^{ia_e\cos\Omega t}\rangle}{i(\omega-k_zw_z)}\left(\frac{\partial f_0^e}{\partial w_z}+\frac{k_y}{k_z}\frac{1}{\omega_{He}}\frac{\partial f_0^e}{\partial x}\right), \qquad (18)$$

and substituting the result in the expression for Q, when $a_e \ll 1$ we have

$$Q = -\frac{\pi e^2}{m_e}\frac{\omega}{k_z|k_z|}\left(\frac{\partial f_0^e}{\partial w_z}+\frac{k_y}{k_z}\frac{1}{\omega_{He}}\frac{\partial f_0^e}{\partial x}\right)\Bigg|_{w_z=\omega/k_z}$$
$$\times\overline{\langle E_z\rangle\langle E_z\exp\{ia_e\cos\Omega t\}\rangle}$$
$$=\frac{1}{16}\sqrt{\frac{T}{2\pi m_e}}\frac{\omega_{0e}^2}{\omega_{He}^2}\frac{k_y^2\varkappa^2}{k_z^2|k_z|}\left(\frac{a_e^2}{k^2\lambda_D^2}\varphi(\Omega)+\frac{d\ln T}{d\ln n}\right)\langle E_z\rangle^2, \qquad (19)$$

where the equilibrium electron distribution function has been written in the form

$$f_0^e(x, w_z) = n_0(x)\sqrt{\frac{m_e}{2\pi T(x)}}\exp\left\{-\frac{m_ew_z^2}{2T(x)}\right\}$$

and we have used (13) for the frequency of the drift oscillations.

In the absence of the high-frequency field for the case being considered here $\sqrt{T/m_i} \ll \omega/k_z \ll \sqrt{T/m_e}$ the drift oscillations are unstable in the zero-Larmor-radius approximation when $d\ln T/d\ln n < 0$. In accordance with (19), the high-frequency field brings about stabilization in the range given by

$$-\frac{a_e^2}{k^2\lambda_D^2}\varphi(\Omega) < \frac{d\ln T}{d\ln n} < 0. \qquad (19')$$

The applied field amplitudes for which stabilization becomes important are determined from the condition

$$\frac{a_e^2}{k^2\lambda_D^2}\varphi(\Omega) = \frac{m_eu_{0e}^2}{T}$$
$$\times\frac{\omega_{0e}^2k_z^2k^{-2}[N\Omega^2-\omega_{0e}^2k_z^2/k^2-\omega_{0i}^2]}{[N\Omega^2-\omega_{0e}^2k_z^2/k^2-\omega_{0i}^2]^2-\Omega^2(\omega_{0e}^4/\omega_{He}^2)(k_y^2\varkappa^2/k^4)} \gtrsim 1. \qquad (20)$$

For the high-frequencies

$$\Omega \gg \frac{1}{N^{1/2}}\left(\omega_{0e}^2\frac{k_z^2}{k^2}+\omega_{0i}^2\right)^{1/2}, \qquad -\frac{1}{N}\frac{\omega_{0e}^2}{\omega_{He}}\frac{k_y\varkappa}{k^2},$$

considered in [13] this condition can only be satisfied in strong electric fields in which the velocity associated with the electron excursions is appreciably greater than the electron thermal velocity $m_e u_{0e}^2/T \gg 1$.[3)] However, it follows from (19) that stabilization obtains for all external field frequencies that satisfy one of the conditions in (15) (for these frequencies $\varphi(\Omega) > 0$). Then (20) can be satisfied for lower field amplitudes when $m_e u_{0e}^2/T \lesssim 1$. As resonance is approached $\Omega = \Omega_\pm$ there is a further reduction in the field strength required for stabilization. However, the formulas obtained here do not apply at exact resonance.

At high field amplitudes, for which $m_e u_{0e}^2/T > 1$, the presence of the field can lead to a new instability associated with the relative oscillations of the electrons and ions in the electric field.[14,15] The plasma diffusion coefficients associated with these instabilities have not been calculated; however, it will be evident that these instabilities, which cause oscillations at rather high frequencies, will not be as important in terms of plasma confinement as the slow drift instabilities. In the experiments described in [9] the high-frequency modulation of a beam traversing a plasma led to the suppression of the drift instabilities and to a considerable reduction of plasma diffusion across the magnetic field. Nevertheless in this case the plasma exhibited high-frequency oscillations at harmonics of the modulating frequency.

3. KINETIC THEORY FOR DRIFT INSTABILITIES IN THE PRESENCE OF A HIGH-FREQUENCY ELECTRIC FIELD

In this section we consider high-frequency stabilization of drift instabilities in the kinetic approximation, taking account of the finite Larmor radius of the ions.

The equilibrium distribution function for an inhomogeneous plasma subject to an alternating electric field (1) is given by

$$f^\alpha = f_0{}^\alpha\left[x+\frac{v_y}{\omega_{H\alpha}},\ v_\perp{}^2, v_z-\frac{e^\alpha\mathscr{E}_0}{m_\alpha\Omega}\sin\Omega t\right]. \qquad (21)$$

[3)]Under these conditions the energy of the stabilizing fields can be somewhat smaller than the thermal energy of the plasma since

$$E_0{}^2/nT = 4\pi\frac{\Omega^2}{\omega_{0e}^2}\frac{m_e u_{0e}^2}{T},$$

and Ω can be small compared with ω_{0e}.

We consider small perturbations about the equilibrium state. It is assumed that the spatial gradients in the equilibrium state are small and that the WKB approximation can be used in considering perturbations. Writing these perturbations of the distribution function in the form

$$\delta f^\alpha = f_1{}^\alpha(t, x, \mathbf{v})\exp\left[i\left(\int k_x dx + k_y y + k_z z\right)\right],$$
$$|k_x{}^{-1}\partial\ln f_1{}^\alpha/\partial x| \ll 1$$

and linearizing the kinetic equation, we have

$$\frac{\partial f_1{}^\alpha}{\partial t} + i(k_\perp v_\perp\sin\vartheta + k_z v_z)f_1{}^\alpha + \frac{e^\alpha E_0(t)}{m_\alpha}\frac{\partial f_1{}^\alpha}{\partial v_z} - \omega_{H\alpha}\frac{\partial f_1{}^\alpha}{\partial\vartheta}$$
$$+\frac{e^\alpha}{m_\alpha}\left\{E_\perp\sin\vartheta\frac{\partial f_0{}^\alpha}{\partial v_\perp} + E_z\frac{\partial f_0{}^\alpha}{\partial v_z} + E_y\frac{1}{\omega_{H\alpha}}\frac{\partial f_0{}^\alpha}{\partial x}\right\} = 0. \quad (22)$$

In this equation we have introduced cylindrical coordinates in velocity space $v_\perp$, v_z and θ, $\vartheta = \pi/2 + \theta - \varphi$; $k_\perp$, k_z and φ are the cylindrical coordinates of the vector **k**. In writing (22) we have made use of the fact, as follows from (21), that

$$\frac{\partial f_0{}^\alpha}{\partial v_y} = \sin\theta\frac{\partial f_0{}^\alpha}{\partial v_\perp} + \frac{1}{\omega_{H\alpha}}\frac{\partial f_0{}^\alpha}{\partial x},$$

where the derivative with respect to $v_\perp$ is computed for $x + v_y/\omega_{H\alpha} = \text{const}$; we have also neglected terms which contain magnetic field perturbations since we shall be interested in electrostatic perturbations since we shall be interested in electrostatic perturbations only.

The solution of (22) can be obtained by means of a procedure similar to that used by Aliev and Silin.[10] In (22) we transform to the independent variables t, $w_z = v_z - (e^\alpha\mathscr{E}_0/m_\alpha\Omega)\sin\Omega t$ and $w_\perp = v_\perp$. In terms of these variables the function $\psi_\alpha(t, x, w)$, which is defined by the relation $\psi_\alpha = f_1^\alpha\exp(ia_\alpha\cos\Omega t)$, can then be obtained from (22):

$$\frac{\partial\psi_\alpha}{\partial t} + i(k_\perp w_\perp\sin\vartheta + k_z w_z)\psi_\alpha - \omega_{H\alpha}\frac{\partial\psi_\alpha}{\partial\vartheta} + \frac{e^\alpha}{m_\alpha}E_z e^{ia_\alpha\cos\Omega t}$$
$$\times\left(\frac{\partial f_0{}^\alpha}{\partial w_z} + \frac{k_y}{k_z}\frac{1}{\omega_{H\alpha}}\frac{\partial f_0{}^\alpha}{\partial x} + \frac{k_\perp}{k_z}\sin\vartheta\frac{\partial f_0{}^\alpha}{\partial w_\perp}\right) = 0. \quad (23)$$

Determining the field E_z from Poisson's equation and averaging (23) over the high-frequency oscillations we have

$$i(k_\perp w_\perp\sin\vartheta + k_z w_z - \omega)\langle\psi_\alpha\rangle - \omega_{H\alpha}\frac{\partial\langle\psi_\alpha\rangle}{\partial\vartheta}$$
$$-\frac{4\pi i k_z}{k^2}\sum_\beta\frac{e^\alpha e^\beta}{m_\alpha}\left[\frac{\partial f_0{}^\alpha}{\partial w_z} + \frac{k_y}{k_z}\frac{1}{\omega_{H\alpha}}\frac{\partial f_0{}^\alpha}{\partial x}\right.$$
$$\left.+\frac{k_\perp}{k_z}\sin\vartheta\frac{\partial f_0{}^\alpha}{\partial w_\perp}\right]$$

$$\times\Big[J_0(\gamma_{\alpha\beta})\int\langle\psi_\beta\rangle d\mathbf{w}+\sum_{s\neq0}J_s(\gamma_{\alpha\beta})\langle e^{is(\Omega t+\pi/2)}\eta_{1\beta}\rangle\Big]=0. \tag{24}$$

We now substitute $\langle\psi_\alpha\rangle\sim e^{-i\omega t}$ and $\gamma_{\alpha\beta}=a_\alpha-a_\beta$ in this equation. As in the preceding section $\eta_{1\alpha}=\int\psi_{1\alpha}d\mathbf{w}$ denotes the high-frequency part of the density.

The solution of (24) is

$$\langle\psi_\alpha\rangle=\frac{4\pi k_z}{k^2}e^{-ib_\alpha\cos\vartheta}\sum_l\frac{e^{il\pi/2}}{k_zw_z-\omega-l\omega_{H\alpha}}\sum_\beta\frac{e^\alpha e^\beta}{m_\alpha}J_l(b_\alpha)$$
$$\times\left[\frac{\partial f_0^\alpha}{\partial w_z}+\frac{k_y}{k_z}\frac{1}{\omega_{H\alpha}}\frac{\partial f_0^\alpha}{\partial x}-\frac{l}{b_\alpha}\frac{k_\perp}{k_z}\frac{\partial f_0^\alpha}{\partial w_\perp}\right]$$
$$\times\Big[J_0(\gamma_{\alpha\beta})\int\langle\psi_\beta\rangle d\mathbf{w}+\sum_{s\neq0}J_s(\gamma_{\alpha\beta})\langle\eta_{1\beta}e^{is(\Omega t+\pi/2)}\rangle\Big], \tag{25}$$

where we have used the notation $b_\alpha=k_\perp w_\perp/\omega_{H\alpha}$. Integrating with respect to w we then have

$$\int\langle\psi_\alpha\rangle d\mathbf{w}-D_\alpha\sum_\beta\frac{e^\beta}{e^\alpha}\Big[J_0(\gamma_{\alpha\beta})\int\langle\psi_\beta\rangle d\mathbf{w}$$
$$+\sum_{s\neq0}J_s(\gamma_{\alpha\beta})\langle\eta_{1\beta}e^{is(\Omega t+\pi/2)}\rangle\Big]=0. \tag{26}$$

When the equilibrium distribution function is

$$f_0^\alpha=n_0\left(\frac{m_\alpha}{2\pi T}\right)^{3/2}\exp\left\{-\frac{m_\alpha(w_z^2+w_\perp^2)}{2T}\right\},$$

the quantity D_α is determined from the relation

$$D_\alpha=-\frac{1}{k^2\lambda_D^2}+\frac{4\pi e^2}{m_\alpha}\frac{k_z}{k^2}\left(\frac{k_y}{k_z}\frac{1}{\omega_{H\alpha}}\frac{\partial}{\partial x}-\frac{m_\alpha}{T}\frac{\omega}{k_z}\right)$$
$$\times\frac{n_0}{\sqrt{2\pi}}\left(\frac{m_\alpha}{T}\right)^{3/2}\sum_l\int_{-\infty}^{+\infty}dw_z\frac{\exp(-m_\alpha w_z^2/2T)}{k_zw_z-\omega-l\omega_{H\alpha}}$$
$$\int_0^\infty dw_\perp w_\perp J_l^2(b_\alpha)\exp\left(-\frac{m_\alpha w_\perp^2}{2T}\right). \tag{27}$$

For the case $\omega\ll\omega_{Hi}$, which is the one being treated here, in the summation over l that appears in D_i we need only consider the $l=0$ term.[3]

In this case D_i is given by

$$D_i=-\frac{1}{k^2\lambda_D^2}+\frac{4\pi e^2}{m_ik^2}\left(\frac{k_y}{\omega_{Hi}}\frac{\partial}{\partial x}-\frac{m_i}{T}\omega\right)n_0A(\rho_i)$$
$$\times\left(\frac{m_i}{2\pi T}\right)^{1/2}\int_{-\infty}^{+\infty}dw_z\exp\left(-\frac{m_iw_z^2}{2T}\right)\frac{1}{k_zw_z-\omega}, \tag{28}$$

where

$$A(\rho_i)=\frac{m_i}{T}\int_0^\infty dw_\perp w_\perp J_0^2(b_i)\exp\left(-\frac{m_iw_\perp^2}{2T}\right)=e^{-\rho_i^2/2}I_0\left(\frac{\rho_i^2}{2}\right)$$
$$\rho_i=\frac{k_\perp}{\omega_{Hi}}\sqrt{\frac{2T}{m_i}}.$$

In similar fashion, in obtaining D_e, in which we need not consider the finite Larmor radius of the electrons, from (27) we have

$$D_e=-\frac{1}{k^2\lambda_D^2}+\frac{4\pi e^2}{m_ek^2}\left(\frac{k_y}{\omega_{He}}\frac{\partial}{\partial x}-\frac{m_e}{T}\omega\right)n_0\left(\frac{m_e}{2\pi T}\right)^{1/2}$$
$$\times\int_{-\infty}^{\infty}\frac{dw_z\exp(-m_ew_z^2/2T)}{k_zw_z-\omega}. \tag{28'}$$

Using (23) and assuming that $\Omega\gg k_z\sqrt{T/m_e}$, ω, we can show that the following inequality holds:

$$\int\psi_{1\alpha}d\mathbf{w}\ll\int\langle\psi_\alpha\rangle d\mathbf{w}.$$

However, by virtue of the neutrality of the low-frequency oscillations, when $a_e\ll1$

$$\int\langle\psi_e\rangle d\mathbf{w}\approx\int\langle\psi_i\rangle d\mathbf{w},$$

and terms containing the quantity $\eta_{1\alpha}=\int\psi_{1\alpha}d\mathbf{w}$ must be retained in (26).[4)] Using (9) to express $\eta_{1\alpha}$ in terms of $\langle\eta_\alpha\rangle$ and substituting the result in (26), we obtain the dispersion relation for the low-frequency oscillations for the case

$$D_e+D_i-\frac{a_e^2}{2}\varphi(\Omega)D_eD_i=0, \tag{29}$$

where the function $\varphi(\Omega)$ is given by (11). In the case at hand $|D_\alpha|\gg1$ and the effect of the high-frequency field is important even when $a_e\ll1$.

At large amplitudes of the external field, in which case $a_e\sim1$, if the condition $\Omega\gg\omega$ is satisfied[5)] the relation in (29) is replaced by the following dispersion equation:

$$D_e+D_i-(1-J_0^2(a_e))D_eD_i=0. \tag{30}$$

A. We first consider the low-frequency case

$$\omega\ll k_z\sqrt{T/m_i}. \tag{31}$$

In this case the quantity D_α that appears in the dispersion equation is given by the following formulas:

$$D_e=-1/k^2\lambda_D^2,$$
$$D_i=-\frac{1}{k^2\lambda_D^2}\left\{1+\frac{k_y}{k_z}\frac{A(\rho_i)}{\omega_{Hi}}\left[\frac{\omega}{k_z}\Gamma+i\left(\frac{\pi T}{2m_i}\right)^{1/2}\Gamma_1\right]\right\}, \tag{32}$$

where we have used the notation

4)The authors are indebted to B. B. Kadomtsev for this observation.

5)When $a_e\sim1$ the deviation from quasi-neutrality becomes important and the frequency ω for this case is of the order of the characteristic frequency of the plasma oscillations for a plasma with "magnetized" electrons.

$$\Gamma = \varkappa\left(1 - \frac{d\ln T}{d\ln n}\left(1+\frac{\delta}{2}\right)\right),\ \Gamma_1 = \varkappa\left(1 - \frac{d\ln T}{d\ln n}\frac{1+\delta}{2}\right),$$

$$\delta = \rho_i^2\left(1 - I_1\left(\frac{\rho_i^2}{2}\right)I_0^{-1}\left(\frac{\rho_i^2}{2}\right)\right).$$

Substituting D_α in (29) we have

$$\frac{\omega}{k_z} = -\left[\frac{2+\beta}{1+\beta}\frac{1}{A} - i\left(\frac{\pi T}{2m_i}\right)^{1/2}\frac{k_y}{k_z}\frac{\Gamma_1}{\omega_{Hi}}\right]$$

$$\times\left[\frac{k_y}{k_z}\frac{\Gamma}{\omega_{Hi}} + i\left(\frac{\pi m_i}{2T}\right)^{1/2}\right]^{-1}. \qquad (33)$$

In this equation we have used the notation

$$\beta = \frac{a_e^2}{2k^2\lambda_D^2}\varphi(\Omega). \qquad (33')$$

In the case at hand the instability condition ($\mathrm{Im}\,\omega > 0$) assumes the form

$$\left[1 - \frac{d\ln T}{d\ln n}\left(1+\frac{\delta}{2}\right)\right]\left[1 - \frac{d\ln T}{d\ln n}\frac{1+\delta}{2}\right]$$

$$+\frac{2+\beta}{1+\beta}\frac{k_z^2}{k_y^2}\frac{1}{A}\frac{m_i\omega_{Hi}^2}{\varkappa^2 T} > 0. \qquad (34)$$

For values of Ω that satisfy one of the conditions in (15), we find $\varphi(\Omega) > 0$ and as the amplitude of the high-frequency field increases the second term in the inequality in (34) is reduced. However, under the conditions for which (33) applies this term is small and the instability region actually is the same as in the absence of the high-frequency field:

$$d\ln T / d\ln n > 2/(1+\delta). \qquad (34')$$

B. We now consider the intermediate-frequency case, for which

$$k_z\sqrt{T/m_i} \ll \omega \ll k_z\sqrt{T/m_e}. \qquad (35)$$

This case has been considered in the second section in the limit of zero ion-Larmor radius $\rho_i \to 0$. In the present section we shall take account of the finite ion Larmor radius.

Computing the integrals that appear in D_α in the approximation given by (35) we have

$$D_e = -\frac{1}{k^2\lambda_D^2}$$

$$-\pi i\frac{4\pi e^2}{m_e k^2}\left(\frac{m_e}{T}\frac{\omega}{|k_z|} - \frac{k_y}{|k_z|}\frac{1}{\omega_{He}}\frac{\partial}{\partial x}\right)n_0\left(\frac{m_e}{2\pi T}\right)^{1/2},$$

$$D_i = -\frac{1-A(\rho_i)}{k^2\lambda_D^2} - \frac{\omega_{0i}^2}{\omega\omega_{Hi}}\frac{k_y\varkappa}{k^2}\left(1 - \frac{\delta}{2}\frac{d\ln T}{d\ln n}\right). \qquad (36)$$

Substituting the result in (29) and solving the resulting dispersion equation we obtain at the following formulas for the frequency and growth rate:

$$\omega = -k_y\frac{T}{m_i\omega_{Hi}}\varkappa A(1+\beta)\left[1 - \frac{\delta}{2}\frac{d\ln T}{d\ln n}\right]$$

$$\times[2-A+\beta(1-A)]^{-1}, \qquad (37)$$

$$\gamma = \sqrt{\frac{\pi}{2}\left(\frac{T}{m_e}\right)^3}\frac{k_y^2\varkappa^2}{|k_z|}\frac{A}{\omega_{He}^2}\left[1 - \frac{\delta}{2}\frac{d\ln T}{d\ln n}\right]$$

$$\times[2-A+\beta(1-A)]^{-3}\Big\{2(1-A)+\beta(1-2A)$$

$$-\frac{1}{2}\frac{d\ln T}{d\ln n}[2-A(1+\delta)+\beta(1-A+\delta A)]\Big\}. \qquad (37')$$

It is evident from the expression for the growth rate that in the case at hand, as in the absence of the high-frequency field,[3] there are two instability regions. The first

$$d\ln T / d\ln n > 2/\delta, \qquad (38)$$

arises only when the finite ion Larmor radius is taken into account ($\delta \to 0$ when $\rho_i \to 0$) and is not changed by the high-frequency field. The second is determined by the relation

$$\frac{d\ln T}{d\ln n} < 2\frac{2(1-A)+\beta(1-2A)}{2-A(1+\delta)+\beta(1-A+\delta A)}. \qquad (39)$$

Thus, when $\rho_i \to 0$, in which case $A \approx 1$ and $\delta \approx 0$, we obtain the same boundary for the instability region $d\ln T/d\ln n = -2\beta$ as is obtained from (19). As ρ_i increases this boundary is displaced toward larger values of $d\ln T/d\ln n$ and when $\rho_i \to \infty$ ($A \approx 0$, $\delta \approx 1$) it approaches $d\ln T/d\ln n = 2$. Thus, the range of values of the parameter $d\ln T/d\ln n$ in which high-frequency stabilization occurs is a maximum for $\rho_i \to 0$ and approaches zero when $\rho_i \to \infty$ (as in the case above, we limit our analysis to the case $\beta > 0$).

It should also be noted that when $\rho_i \neq 0$ (in which case $A < 1$) and for sufficiently large values of β, the growth rate as given by (37′) is reduced with increasing β ($\gamma \sim \beta^{-2}$ when $\beta \gg 1$). Thus, the stabilizing effect of the high-frequency field appears in an expansion of the stability region which is a maximum for small ρ_i and in a reduction of the growth rate for oscillations characteristic of finite ρ_i.

C. Finally we consider the high-frequency region

$$\omega \gg k_z\sqrt{T/m_e}. \qquad (40)$$

We assume that the following condition is satisfied:

$$k_0\sqrt{\frac{T}{m_i}}\rho_i \gg \omega, \qquad k_0 = \frac{1}{n_0 T}\frac{d}{dx}(n_0 T), \qquad (41)$$

and obtain the following relations for D_α:

$$D_e = -\frac{\omega_{0e}^2}{\omega_{He}\omega}\frac{k_y\varkappa}{k^2} - \frac{\omega_{0e}^2}{\omega_{He}\omega^3}\frac{k_z^2 k_y}{k^2}\frac{1}{n_0}\frac{d}{dx}\left(\frac{n_0 T}{m_e}\right);$$

$$D_i = -\frac{\omega_{0i}^2}{\omega_{Hi}\omega}\frac{k_y\varkappa}{k^2}A(\rho_i)\left(1-\frac{\delta}{2}\frac{d\ln T}{d\ln n}\right). \quad (42)$$

For small values of ρ_i (the instability being considered corresponds to precisely this case[3]) in the sum $D_i + D_e$, which appears in the dispersion equation (29), the term in D_e is almost completely balanced by D_i and the second term must be taken into account. For this reason the last term in (29) is important even when a_e is small. Using (42) we write the dispersion equation in the form

$$\omega^2\left(1-A+A\frac{\delta}{2}\frac{d\ln T}{d\ln n}\right)+\omega\frac{\beta}{\sqrt{2}}A\varkappa\rho_i\left(\frac{T}{m_i}\right)^{1/2}\left(1-\frac{\delta}{2}\frac{d\ln T}{d\ln n}\right)$$
$$+k_z^2\frac{T}{m_e}\left(1+\frac{d\ln T}{d\ln n}\right)=0. \quad (43)$$

It follows from (43) that the condition in (40) is satisfied only when $\rho_i \ll 1$. In this case ω is given by

$$\omega = -\frac{\beta}{\rho_i}\sqrt{\frac{T}{2m_i}}\frac{\varkappa}{1+d\ln T/d\ln n}$$
$$\pm\left[\frac{\beta^2}{2\rho_i^2}\frac{T}{m_i}\frac{\varkappa^2}{(1+d\ln T/d\ln n)^2}-\frac{2}{\rho_i^2}k_z^2\frac{T}{m_e}\right]^{1/2}. \quad (44)$$

When $\beta = 0$ the instability arises for all values of the parameter $d\ln T/d\ln n$ (the validity of (44) requires only that the parameter not be close to -1).[3] In accordance with (44), the high-frequency field stabilizes this oscillation when

$$\beta > 2\sqrt{\frac{m_i}{m_e}}\frac{k_z}{\varkappa}\left(1+\frac{d\ln T}{d\ln n}\right).$$

It follows from a comparison of (41) and (40) that $k_z \ll k_0\rho_i\sqrt{m_e/m_i}$; thus, in the case being considered stabilization can be achieved with modest field amplitudes for which the condition $\beta \ll \rho_i(1 + d\ln T/d\ln n)^2 \ll 1$ is satisfied.

4. DRIFT-DISSIPATIVE INSTABILITY IN THE PRESENCE OF A HIGH-FREQUENCY ELECTRIC FIELD

It is well known that electron-ion collisions in an inhomogeneous plasma can lead to the excitation of drift oscillations (drift-dissipative instability).[4,5] In the present section we consider the effect of a uniform high-frequency electric field on the drift-dissipative instability.

The dispersion relation is derived on the basis of the usual hydrodynamic equations. The friction term due to electron-ion collisions must be taken into account in the equation for the electron motion along $\mathbf{H}_0$. This term is given by $m_e\nu_e v_z^e$ where ν_e is the effective electron-ion collision frequency. In this equation we then write the nonequilibrium deviations for the velocity and density of the electrons in the form

$$v_z^e = W_z^e e^{-ia_e\cos\Omega t}, \qquad n_e = \eta_e e^{-ia_e\cos\Omega t}$$

and average over the period of the high-frequency oscillation. Then, as before, neglecting the inertia term in the averaged equations, when $a_e \ll 1$ we have

$$\langle W_z^e\rangle = -\frac{e}{m_e\nu_e}\left\{\langle E_z\rangle + ik_z\frac{T}{en_0}\langle\eta_e\rangle + ia_e\langle\cos\Omega t E_{1z}\rangle\right\}. \quad (45)$$

In similar fashion, substituting in the equation of electron motion across the magnetic field $v_\perp^e = W_\perp^e e^{ia_e\cos\Omega t}$ and averaging over the high-frequency oscillations, to higher order in the parameter ω/ω_{He} we find

$$\langle W_x^e\rangle = \frac{c}{H_0}[\langle E_y\rangle + ia_e\langle\cos\Omega t E_{1y}\rangle],$$

$$\langle W_y^e\rangle = -\frac{c}{H_0}[\langle E_x\rangle + ia_e\langle\cos\Omega t E_{1x}\rangle]. \quad (46)$$

In (45) and (46) the term containing the high-frequency part of the field oscillation $\mathbf{E}_1$ can be written

$$ia_e e\langle\cos\Omega t\mathbf{E}_1\rangle = \frac{\varepsilon_{zz}-1}{4\pi n_0}i\mathbf{k}\langle E_0 E_{1z}\rangle, \quad (47)$$

where we have substituted a_e and taken account of the electrostatic nature of the oscillations. The right side of this relation contains the pressure due to the high-frequency field computed per plasma electron. Thus, in contrast with the analysis in,[4,5] in the present work the equations for electron motion along the magnetic field and across the field take account of the pressure term due to the high-frequency field.

If $\Omega \gg \nu_e$, the field $\mathbf{E}_1$ can be found from (6) and (8) in the second section of this paper. Using these equations to express $\mathbf{E}_1$ in terms of $\langle\eta_\alpha\rangle$ and neglecting terms of order m_e/m_i compared with unity, we have

$$\langle W_x^e\rangle = \frac{c}{H_0}\left[\langle E_y\rangle + ik_y\frac{2\pi e}{k^2}a_e^2\varphi(\Omega)\langle\eta_e\rangle\right],$$

$$\langle W_y^e\rangle = -\frac{c}{H_0}\left[\langle E_x\rangle + ik_x\frac{2\pi e}{k^2}a_e^2\varphi(\Omega)\langle\eta_e\rangle\right],$$

$$\langle W_z^e\rangle = -\frac{e}{m_e\nu_e}\left[\langle E_z\rangle + ik_z\frac{T}{en_0}\langle\eta_e\rangle\right.$$
$$\left.+ik_z\frac{2\pi e}{k^2}a_e^2\varphi(\Omega)\langle\eta_e\rangle\right]. \quad (48)$$

Substituting in the averaged equation of continuity for the electrons

$$-i\omega\langle\eta_e\rangle+\langle W_x^e\rangle\frac{dn_0}{dx}+i\mathbf{k}\langle\mathbf{W}^e\rangle n_0=0 \qquad (49)$$

where we have taken $\langle W^e\rangle$ from (48), we have

$$\left[-i\omega+\frac{k_z^2T}{m_e\nu_e}\left(1+\frac{a_e^2}{2k^2\lambda_D^2}\varphi(\Omega)\right)-i\frac{k_y\varkappa}{2k^2}a_e^2\varphi(\Omega)\frac{\omega_{0e}^2}{\omega_{He}}\right]\langle\eta_e\rangle$$

$$-i\frac{k_zen_0}{m_e\nu_e}\langle E_z\rangle+\frac{c}{H_0}\langle E_y\rangle\frac{dn_0}{dx}=0. \qquad (50)$$

In considering the drift-dissipative instability the inertia term must be retained in the averaged equations of motion for the ions across the magnetic field. The averaged equation of continuity for the ions, as averaged over the high-frequency field, is then given by

$$-i\omega\langle\eta_i\rangle+\frac{c}{H_0}\langle E_y\rangle\frac{dn_0}{dx}+\frac{c}{H_0}n_0\frac{\omega}{\omega_{Hi}}(k_y\langle E_y\rangle$$

$$+k_x\langle E_x\rangle)=0 \qquad (51)$$

(the ion motion along $\mathbf{H}_0$ can be neglected for the low-frequency oscillations characterized by $\omega \gg k_z\sqrt{T/m_i}$).

Making use of the electrostatic nature of the low-frequency oscillations and the fact that these oscillations are quasi-neutral[6)] $\langle\eta_e\rangle \approx \langle\eta_i\rangle$, from the condition that must be satisfied in order that (50) and (51) yield a non-trivial solution, we obtain the following dispersion equation:

$$\omega^2+\omega\left[i\omega_s-\omega_e\frac{\beta}{1+\beta}\left(1+\frac{\beta}{2}\rho_i^2\right)\right]+i\omega_e\omega_s$$

$$-\frac{2\beta}{\rho_i^2}\frac{\omega_e^2}{(1+\beta)^2}\left(1+\frac{\beta}{2}\rho_i^2\right)^2=0. \qquad (52)$$

In this equation we have used the following notation

$$\omega_s=-\frac{k_z^2}{k_\perp^2}\frac{\omega_{He}\omega_{Hi}}{\nu_e}\left(1+\frac{\beta}{2}\rho_i^2\right),$$

$$\omega_e=k_y\frac{T}{m_i\omega_{Hi}}\varkappa\frac{1+\beta}{1+{}^1/_2\beta\rho_i^2},$$

$$\frac{\rho_i^2}{2}=-\frac{\omega_{0i}^2}{\omega_{Hi}^2}k_\perp^2\lambda_D^2,$$

and the quantity β is determined by (33′). When $\beta \to 0$ (52) becomes the dispersion equation for the drift-dissipative instability in the absence of a high-frequency field which has been obtained in [4]. When $\nu_e \to 0$, we find from (52) $\omega = -\omega_e$ which, in turn, coincides with (37) for the drift frequency in the case $\rho_i \ll 1$.

We shall limit our analysis to the solution of (52) for large values of β lying in the range

$$1 \ll \beta\rho_i^2 \ll \rho_i^{-2}. \qquad (53)$$

For these values of β

$$a_e^2\varphi(\Omega) \gg \omega_{Hi}^2/\omega_{0i}^2 \qquad (53')$$

(the condition in (53′) can also be satisfied when $a_e \ll 1$ since ω_{Hi}/ω_{0i} is a small parameter). In this case, expanding the solution in powers of the parameter $1/\beta\rho_i^2$ we obtain the following expression for the imaginary root of the dispersion equation

$$\operatorname{Im}\omega_-=-i\omega_s+2\frac{\omega_e^2}{\beta\rho_i^2}\frac{i\omega_s}{\omega_s^2+\omega_e^2\beta^2\rho_i^4/4}, \qquad (54)$$

$$\operatorname{Im}\omega_+=-2\frac{\omega_e^2}{\beta\rho_i^2}\frac{i\omega_s}{\omega_s^2+\omega_e^2\beta^2\rho_i^4/4}. \qquad (54')$$

When $\beta\rho_i^2 \gg 1$ the second term in (54) is small and if β is positive (that is to say, for values of Ω that satisfy one of the conditions in (15)) we find that both roots yield $\operatorname{Im}\omega < 0$, corresponding to stability.

Thus, the drift-dissipative instability can also be stabilized by a high-frequency field; however, the field amplitude required for this case is higher than for the collisionless drift instability.

The authors are indebted to B. B. Kadomstev, V. P. Silin and A. B. Mikhaĭlovskiĭ for valuable comments and to V. I. Shevchenko for assistance.

6)In the presence of a high-frequency field quasi-neutrality obtains when $a_e^2\varphi^* \ll 1$, where $\varphi^*(\Omega)$ is given by (14′).

[1] L. I. Rudakov and R. Z. Sagdeev, DAN SSSR 138, 581 (1961), Soviet Phys. Doklady 6, 415 (1961). Nuclear Fusion, Supplement 2, 1962, p. 481.

[2] B. B. Kadomtsev and A. V. Timofeev, DAN SSSR 146, 581 (1962), Soviet Phys. Doklady 7, 826 (1963).

[3] A. A. Galeev, V. N. Oraevskiĭ and R. Z. Sagdeev, JETP 44, 903 (1963), Soviet Phys. JETP 17, 615 (1963).

[4] A. A. Galeev, S. S. Moiseev and R. Z. Sagdeev, Atomnaya énergiya (Atomic Energy) 16, 451 (1963).

[5] B. B. Kadomtsev, Reviews of Plasma Physics Consultants Bureau, New York, 1966, Vol. 4.

[6] Ya. B. Faĭnberg, Atomnaya energiya (Atomic Energy) 11, 313 (1961).

[7] Ya. B. Faĭnberg and V. D. Shapiro, Atomnaya énergiya (Atomic Energy) 19, 336 (1965); V. D. Shapiro, ZhTF 37, #3 (1967), Soviet Phys. Tech. Phys. 12, in press.

[8] A. K. Berezin et al, Atomnaya énergiya (Atomic Energy) 18, 315 (1965).

[9] E. A. Kornilov, Ya. B. Faĭnberg and O. Kovpik JETP Letters 3, 354 (1966), transl p. 229.

[10] Yu. M. Aliev and V. P. Silin, JETP 48, 901 (1965), Soviet Phys. JETP 21, 601 (1965).

[11] S. M. Osovets, JETP 39, 311 (1960), Soviet Phys. JETP 12, 221 (1961).

[12] L. D. Landau and E. M. Lifshitz, Electrodynamics of Continuous Media, Addison-Wesley, Reading, Mass., 1960.

[13] Ya. B. Faĭnberg and V. D. Shapiro, JETP Letters 4, 32 (1966), transl. p. 20.

[14] V. P. Silin, JETP 48, 1679 (1965), Soviet Phys. JETP 21, 1127 (1965).

[15] L. M. Gorbunov and V. P. Silin, JETP 49, 1973 (1965), Soviet Phys. JETP 22, 1347 (1966).

Translated by H. Lashinsky
36