SELF-SYNCHRONIZATION OF AXIAL MODES IN A LASER WITH SATURABLE FILTERS

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Low-frequency oscillations (with periods 7, 14, and 22 nsec) of the intensity of the giant pulse of a laser with saturable filters were investigated at different filter positions inside of the cavity. A theoretical analysis is presented of the phenomenon, and its results agree well with the observed dependence of the structure of the giant pulse on the filter coordinate.

THE distinguishing features of the interaction between an intense electromagnetic field and matter find a number of characteristic manifestations in processes occurring in lasers with saturable filters. In particular, it was observed recently that a definite coupling can be established in the filter material between several modes of field oscillation, such that the output emission of the laser has a form of a train of equidistant short pulses [1,2] 1). This phenomenon is called self-synchronization of the modes. Great interest in self-synchronization is due also to attempts to obtain in this manner ultrashort radiation pulses.

We wish to call attention to certain new properties of this phenomenon, which make it possible to present more clearly the operating mechanism of the saturable filter, all the more since the theory of the processes occurring in it is far from complete.

1. EXPERIMENTAL DATA

A diagram of the experimental setup is shown in Fig. 1. Here M_1 and M_2 are the cavity mirrors with reflection coefficients 99 and 65% respectively, NR is a neodymium rod with 15 mm diameter and 120 mm length, C is cell with a solution of one of the analogs of pentacarbocyanine in nitrobenzene, used as the saturable filter, ST is a semitransparent mirror, CPC is a coaxial low-inertia photocell connected directly to the plates of an S1-11 oscilloscope (the time constant of this system is ~ 2 nsec), FP is a Fabry-Perot interferometer with distance between plates of 15 cm, L is a lens with f = 1000 mm, P is the photographic plate, and D is a photodiode connected to the input of the S1-16 oscilloscope.

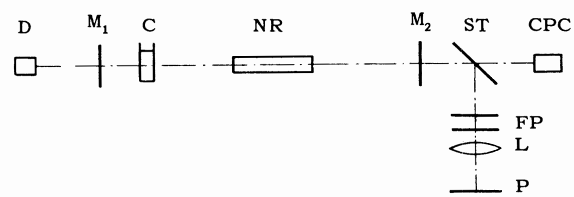

FIG. 1. Diagram of experimental setup.

The experimental setup made it possible to register simultaneously the temporal and spectral characteristics of the output emission of the laser. The photodiode was used to monitor the number of giant pulses per flash. In addition, the emission spectrum was investigated with a diffraction spectrograph with dispersion 1.25 Å/mm. To eliminate the discriminating action of the end of the neodymium rod (see [3]), the axis of the rod was inclined in all experiments to the resonator axis by an angle ~1°. The main part of the experiment was carried out at an optical resonator length L = 324 cm, at a cell transmission coefficient (at wavelength $\lambda = 1.06\ \mu$) of approximately 60%, and at pump energies close to threshold.

The main experimental results consist in the following.

When the solution-containing cell is located near one of the resonator mirrors (at a distance 2–3 cm), the giant pulse (with total duration 100–150 nsec) constituted a train of individual narrow pulses of 3–4 nsec duration, the distance between which was T = 22 ± 1 nsec ≈ 2L/c (Fig. 2a). The output energy of the giant pulse was approximately 0.3 J, so that peak power reached 15–20 MW.

When the cell was at the center of the resonator, the distance between the two nearest pulses was T = 11 ± 1 nsec (Fig. 2b), i.e., smaller by a factor of 2. The width of the individual pulses was also

1) Emission from a neodymium-glass laser, in the form of a train of short pulses with large off-duty cycle, was observed by us earlier [3].

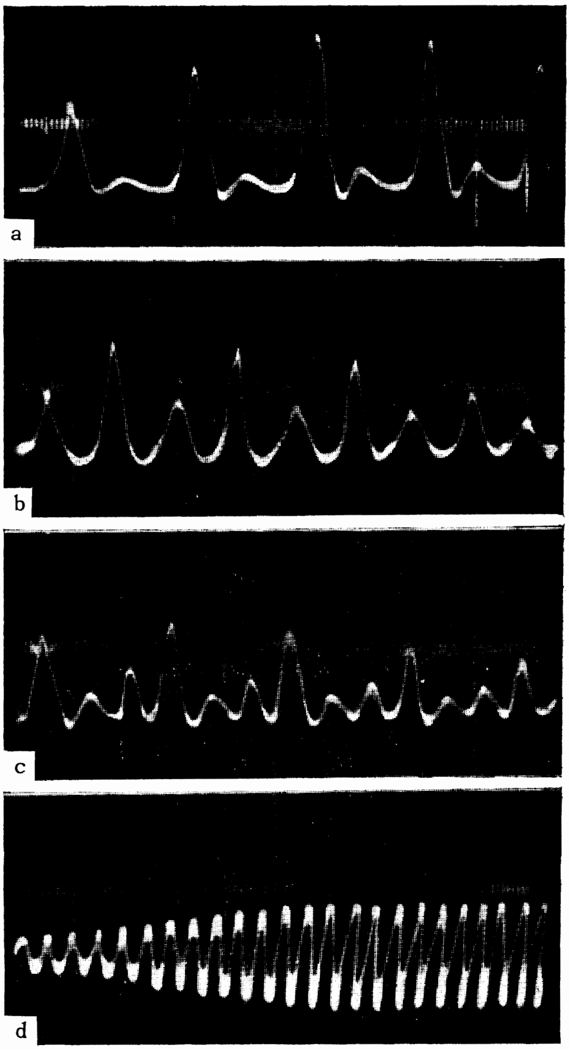

FIG. 2. Time sweeps of giant pulse for different values of X (the distance from the filter to the mirror M_1). The center of the neodymium rod is at a distance 20 cm from M_2. a) X = 2 cm; b) X = 162 cm, c) X = 108 cm; d) calibration sinusoid with period 4 nsec.

3—4 nsec in this case. The relative intensity of the two nearest pulses did not remain constant and varied from flash to flash.

In the case when the cell was located at a distance ~L/3 from one of the mirrors, the output emission consisted of a train of pulses with distances T = 7 ± 1 nsec ≈ (2/3) L/c (Fig. 2c). As in the preceding case, the relative intensity of the three nearest pulses could vary from flash to flash.

No influence of the position of the neodymium rod on the observed phenomenon was noted.

When the transmission coefficient of the cell with the solution was increased from 60 to 85% (in this case the cell was located near one of the mirrors), the width of the individual pulses in the overall train increased, and the form of the modulation of the output signal approached sinusoidal. A decrease in the resonator length led to a decrease in the depth of modulation of the output radiation, which apparently is connected with the excitation of the off-axes modes.

The measurements of the giant pulse, made with the aid of the Fabry-Perot interferometer, have shown that the width of the generation line was $\Delta\nu = 1.1 \times 10^{-2}\ \text{cm}^{-1}$ (the apparatus function of the Fabry-Perot interferometer did not exceed $0.3 \times 10^{-2}\ \text{cm}^{-1}$). This corresponds to seven times the distance between neighboring axial modes. The absolute value of the wavelength of the generation line could vary from flash to flash by ~25 Å.

With increasing pump energy over thresholds, the number of giant pulses (within the time of one pump pulse) increased, but the time evolution of each individual giant pulse did not change. The emission spectrum consisted in this case (Fig. 3) of several individual intense lines, the number of which corresponded to the number of giant pulses, and the width was determined by the apparatus function of the instrument (0.1 Å). It should be noted that the emission spectrum of either the single giant pulse or of several pulses could contain also other lines, but their intensity is smaller by 1—3 orders of magnitude compared with the fundamental lines, so that they could be observed on the spectrogram only by strongly overexposing the fundamental lines.

2. THEORETICAL PART

For a theoretical discussion of the observed phenomena we consider the following model.

We assume that a stationary regime exists, i.e., we do not consider the processes occurring during the bleaching of the filter and the buildup of the field. The amplitudes of the different spectral components of the field will be assumed constant and specified. We determine the absorption of the field energy in the filter for arbitrary phases of the spectral components, and find the phase relations at which the losses in the filter are minimal. We assume that the regime realized in the laser is precisely the one at which these conditions are satisfied.

We assume, to simplify the problem, that the filter material is described by a two-level scheme

$$\frac{d}{dt}(\rho_{11} + \rho_{22}) + \gamma(\rho_{11} - \rho_{22}) = \gamma N - 2i\frac{p}{\hbar}E(\rho_{12} - \rho_{12}^{*}),$$

$$\frac{d}{dt}\rho_{12} + (\Gamma - i\omega_0)\rho_{12} = -i\frac{p}{\hbar}E(\rho_{11} - \rho_{22}). \qquad (1)$$

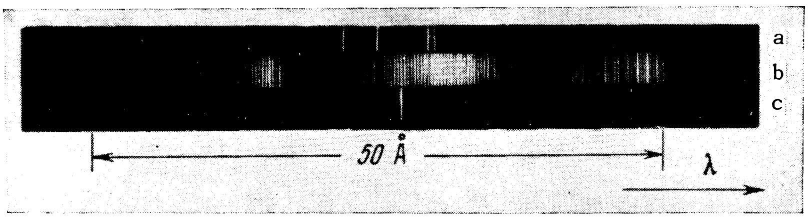

FIG. 3. Spectrograms of emission. a—four giant pulses per pump pulse; b—free generation (no filter); c—single giant pulse.

Here ρ_{mn} are the density matrix elements, γ and Γ are the constants of the population relaxation and of the off-diagonal elements with $\Gamma \gg \gamma$, N is the difference in the populations of the lower and upper levels in the absence of the field, ω_0 is the natural frequency of the transition, p is the matrix element of the dipole moment, and E is the intensity of the electric field. Since we are considering a stationary process, i.e., one independent of the initial conditions, the system (1) must be solved simultaneously with the boundedness condition for ρ_{mn} at $t = \pm\infty$.

We shall assume that the field consists of several discrete spectral components, and the spatial distribution for each component is close to a plane standing wave. We assume that the wave vectors and the corresponding natural frequencies are equidistant and are determined by the distance between the mirrors L, just as in an empty resonator. With this we have

$$E=\sum_m A_m \sin k_m x \cos(\omega_m t+\varphi_m) \equiv \frac{\hbar}{p}\sum_m F_m \cos(\omega_m t+\varphi_m), \quad (2)$$

$$A_m \geqslant 0; \quad k_m = c\omega_m, \quad \omega_m = \omega + m\Omega, \quad \Omega = \pi c/L.$$

We confine ourselves to an examination of the case when the following relation is satisfied

$$\Omega \gg \gamma. \quad (3)$$

In addition, we assume for simplicity that $\omega = \omega_0$, and that for all the spectral components

$$m\Omega \ll \Gamma. \quad (4)$$

If condition (4) is satisfied, then it follows from (1) that

$$\rho_{12}=\frac{-i}{2\Gamma}\sum_m F_m \exp\{i(\omega+m\Omega)t+i\varphi_m\}(\rho_{11}-\rho_{12}), \quad (5)$$

$$\frac{d}{dt}(\rho_{11}-\rho_{22})+\gamma(\rho_{11}-\rho_{22})=\gamma N+\frac{\rho_{11}-\rho_{22}}{\Gamma} \times\sum_l\sum_m F_mF_l\cos[(m-l)\Omega t+\varphi_m-\varphi_l]. \quad (6)$$

The solution of Eq. (6), bounded at $t = \pm\infty$, can be represented in the form

$$\rho_{11}-\rho_{12}=\frac{N}{1+\Gamma^{-1}\gamma^{-1}\sum_s F_s^2}\left\{1-\frac{1}{2\Gamma}\sum_{l\neq m}\sum_m F_lF_m \times\left[\frac{\exp\{i(l-m)\Omega t+i(\varphi_l-\varphi_m)\}}{i(l-m)\Omega+\gamma+\Gamma^{-1}\sum_j F_j^2}+\text{c.c.}\right]\right\}, \quad (7)$$

if the field intensity is not too large

$$\sum_m F_m^2 \ll \Gamma|\gamma+i\Omega|. \quad (8)$$

The first term in the curly brackets of (7) corresponds to the dc component of the population difference, and the second corresponds to quantities that oscillate in time. The oscillating quantities constitute a small correction to the dc component, of the order of $F^2/\Gamma\Omega$. In our approximation, only the oscillating part of the population difference depends on the phases φ_m of the fields.

It should be noted that condition (8) for the applicability of the solution presented here can be satisfied at appreciable saturation. In fact, saturation of the population levels becomes essential at intensities $F^2 \gtrsim \gamma\Gamma$. By virtue of (3) there exist such field intensities

$$\Gamma\gamma \lesssim \sum_m F_m^2 \ll \Gamma|\gamma+i\Omega|, \quad (9)$$

that the saturation is already large yet (8) is still satisfied. For our problem interest attaches precisely to the fields that produce saturation, since the action of the saturating filter is based on the saturation effect.

Let us calculate the electromagnetic energy W absorbed by the filter material per unit time. We assume that the polarization of the filter material P is connected entirely with the considered level pair

$$P = np(\rho_{12}+\rho_{12}^*), \quad (10)$$

where n is the density of the absorbing centers. Then we obtain on the basis of (5), (7), and (10) the following expression for the losses of the electromagnetic energy, averaged over the period $2\pi/\omega$:

$$W_{av} = \left(E\frac{dP}{dt}\right)_{av} = n\hbar\omega\frac{N}{\Gamma}\left\{\left(\sum_r F_r^2\right)\left(1+\frac{1}{\Gamma\gamma}\sum_m F_m^2\right)^{-1}\right.$$

$$-\frac{\gamma}{2\Gamma}\sum_s\sum_{l\neq m}\sum_m F_mF_lF_sF_{m+s-l}$$

$$\left.\times\frac{\cos(\varphi_l-\varphi_m-\varphi_s+\varphi_{m+s-l})}{(l-m)^2\Omega^2+\left(\gamma+\Gamma^{-1}\sum_j F_j^2\right)^2}\right\}. \qquad (11)$$

From (11) we see that the absorption depends on the phases φ_m.

We note that Statz and Tang [4], who explained mode interaction in a laser without a saturable filter, calculated the polarization of the active medium as a function of the phase relations. In that paper, however, all the field components were assumed weak, and therefore the approach used in [4] cannot be used in principle for our problem (see the discussion of the inequality (9)). In addition, the field considered in [4] consisted of only three components. It will be shown later, however, that certain singularities of the output radiation, which depend on the position of the filter inside the resonator, become manifest only in the presence of a large number of components in the generation spectrum.

Attention must be called to the fact that the very dependence of the polarization of the active medium on the phases of the spectral emission components was known before the self-synchronization phenomenon, and was observed in quantum amplifiers [5,6].

Let us see now under what phase relations are the losses minimal. To this end we integrate (11) along the filter. Assuming that the filter is much longer than the wavelength of the radiation, but much shorter than the resonator length L, and also discarding the phase-independent part of the absorption, we proceed to check the minimum of the expression

$$-\sum_s\sum_{l\neq m}\sum_m A_mA_lA_sA_{s+l-m}(l-m)^{-2}$$

$$\times\left[1+\cos 2\pi\frac{X}{L}(m-l)+\cos 2\pi\frac{X}{L}(m-s)\right]$$

$$\times\cos(\varphi_m-\varphi_l-\varphi_s+\varphi_{m+s-l})\equiv\Phi(\varphi). \qquad (12)$$

Here X is the distance from the center of the filter to the mirror.

Let us examine the function $\Phi(\varphi)$ in the case when the filter is at the edge of the resonator $X/L = 0$. It can be verified that in this case Φ is minimal if the phases satisfy the relation

$$\varphi_{m+1}-\varphi_m = \varphi_m-\varphi_{m-1}+2\pi q, \qquad (13)$$

where q is an integer.

If the filter is at the center of the resonator, $X/L = 1/2$, then the minimum of $\Phi(\varphi)$ is insured by the following phase relations

$$\varphi_{m+1}-\varphi_m = \varphi_m-\varphi_{m-1}+\pi(2q+1). \qquad (14)$$

A test of $\Phi(\varphi)$ for a minimum at $X/L = 1/3$ leads to the following conditions on the phases:

$$\varphi_{m+3}-\varphi_m = \varphi_{l+3}-\varphi_l+2\pi q. \qquad (15)$$

We emphasize that the relations (13)—(15) ensure minimum losses for arbitrary amplitudes A_m. On the other hand, if we consider arbitrary values of the filter coordinate, then the phase relations will depend on A_m and will have a rather complicated form. We shall discuss only three filter positions: X = 0, L/2, and L/3.

In the cases when condition (13) (or (14)) is satisfied we can express the phases of all the components in terms of two arbitrary constants (φ_0 and α):

$$\varphi_m = \varphi_0 + m\alpha \qquad \text{for } X/L = 0, \qquad (16)$$

$$\varphi_m = \varphi_0 + m\alpha + {}^1\!/_2 m(m-1)\pi \qquad \text{for } X/L = {}^1\!/_2. \qquad (17)$$

In the case of (15), however, in order to write down the phases of all the components, it is necessary to introduce four coordinates (φ_0, α, β, θ):

$$\left.\begin{aligned}\varphi_{3m} &= \varphi_0+m\theta,\\ \varphi_{3m+1} &= \varphi_0+\alpha+m\theta,\\ \varphi_{3m+2} &= \varphi_0+\beta+m\theta\end{aligned}\right\} \quad \text{for } X/L = {}^1\!/_3. \qquad (18)$$

The dependence of the output radiation on the time in these three cases will, in general, be different. The low-frequency component of the oscillations of the field intensity, which is registered by a quadratic receiver, is proportional to the quantity

$$\sum_{l\neq m}\sum_m A_mA_l\cos[(m-l)\Omega t+\varphi_m-\varphi_l]\equiv B.$$

We write down this quantity, assuming that the phases are given by one of the three relations (16)—(18).

For X/L = 0 we obtain

$$B = \sum_m\sum_l A_lA_{l+m}\cos[m(\Omega t+\alpha)]. \qquad (19)$$

For $X/L = 1/2$

$$B = \sum_m\left\{\sum_l(-1)^l A_lA_{l+2m+1}\cos[(2m+1)(\Omega t+\alpha)]\right.$$

$$\left.-\sum_l A_lA_{l+2m}\cos[2m(\Omega t+\alpha)]\right\}. \qquad (20)$$

For $X/L = 1/3$

$$
\begin{aligned}
B = \sum_m \Big\{ & \sum_l A_l A_{l+3m} \cos\Big[3m\Big(\Omega t + \frac{\theta}{3}\Big)\Big] \\
& + \sum_l A_{3l} A_{3(l+m)+1} \cos[(3m+1)\Omega t + \alpha + m\theta] \\
& + \sum_l A_{3l+1} A_{3(l+m)+2} \cos[(3m+1)\Omega t + \beta - \alpha + m\theta] \\
& + \sum_l A_{3l+2} A_{3(l+m+1)} \cos[(3m+1)\Omega t + (m+1)\theta - \beta] \\
& + \sum_l A_{3l} A_{3(l+m)+2} \cos[(3m+2)\Omega t + \beta + m\theta] \\
& + \sum_l A_{3l+1} A_{3(l+m+1)} \cos[(3m+2)\Omega t + (m+1)\theta - \alpha] \\
& + \sum_l A_{3l+2} A_{3(l+m)+4} \cos[(3m+2)\Omega t + (m+1)\theta + \alpha - \beta] \Big\}.
\end{aligned} \tag{21}
$$

Thus, when $X/L = 0$ all the terms having the same frequency have identical phases. When $X/L = 1/2$ all terms at even frequencies have the same phase and some of those at odd frequencies are in phase opposition to the remainder. In both cases, as seen from (20) and (21), if the amplitudes A_m are known, the time dependence of the signal can be determined completely. The undetermined constant α results only in an inessential phase shift.

In the third case, the form of the output signal depends on the constants α, β, and θ, which are not determined in our model. This is connected with the fact that in this cell position ($X/L = 1/3$) the losses do not depend on α, β, or θ and all these constants remain arbitrary in our analysis. It is possible that in a real system a definite connection between α, β, and θ is ensured by the matter of the active rod, the role of which is not taken into account here. The active rod, however, undoubtedly gives a much weaker effect, since the saturation in its material is much smaller than in the filter material.

From (21) we can see that for arbitrary values of the constants α, β, and θ the components at triple frequency and at frequencies that are multiples of 3Ω are intense in the output signal.

Formulas (19)—(21) were used to plot the intensity of the output signal against the time for the three filter positions. On the basis of an experimental determination of the line width of the generation it was assumed that only the amplitudes of seven components differ from zero, $A_m \neq 0$ for $|m| \leq 3$, and all these amplitudes were assumed to be equal. In the case of $X/L = 1/3$, in addition, additional phase relations were specified: $\beta = 2\alpha + \pi$ and $\theta = 3\alpha$. The obtained time variation of the signal intensity is shown in Fig. 4. When the filter

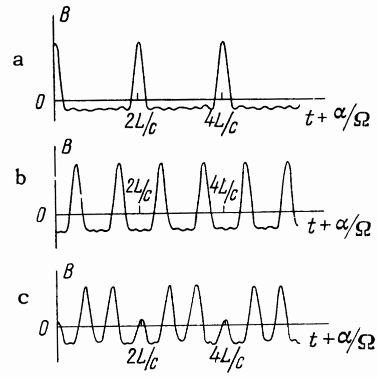

FIG. 4. Dependence of B—intensity of the output signal (in relative units)—on the time. a—X/L = 0, b—X/L = 1/2, c—X/L = 1/3.

is located at the edge of the resonator, the distance between maxima of the intensity on the plot amounts to $2\pi/\Omega = 2L/c$; it is decreased by one-half for the case when the filter is in the center and by a factor of 3 when the filter coordinate is $X = L/3$. The same distances between the pulses were obtained experimentally. The ratio of the intensities of the neighboring pulses differs in the theoretical and experimental curves. For a better reconciliation of the theoretical and experimental curves it is probably necessary to take into account the asymmetry of the resonator, and particularly the difference in the losses in mirrors M_1 and M_2 (when the mirror loss is taken into account, expression (2) for the field changes).

Thus, analysis of model of the mode interaction in a saturable filter makes it possible to describe certain features of the self-synchronization phenomenon, and explains the change occurring in the form of the output signal when the filter position is changed. Therefore such a model can be used in principle as the starting point for further study of a laser with saturable filter. It is of interest, in particular, to investigate the mode interaction under nonstationary conditions, while retaining the basic assumptions concerning the properties of the medium and the spatial distribution of the field. This will make it possible to ascertain which mode amplitudes and phases are established in real cases, and to find a method for synchronizing a large number of modes to obtain very short light pulses.

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