SELF FOCUSING AND DEFOCUSING, INSTABILITY AND STABILIZATION OF LIGHT BEAMS IN WEAKLY ABSORBING MEDIA

Yu. P. RAĬZER

Institute of Problems of Mechanics, Academy of Sciences, U.S.S.R.

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We consider the process of self-focusing and defocusing of light beams in transparent media having weak absorption. The relative roles of electrostriction, the thermal expansion accompanying the absorption, and the Kerr effect in the variation of the dielectric constant of medium, and the extent to which these mechanisms of the nonlinearity of the material properties participate in the effects brought about by the nonlinearity are assessed. The divergence, due to absorption of an inhomogeneous beam is estimated. It is shown that in the case of a beam that is homogeneous over the radius, the absorption during the stage of unsteady motion of the medium exerts, in contrast to the usual case, a focusing action on the medium. The effect of suppression of the self focusing of the beam by a growing defocusing action of thermal expansion is considered. It is shown how absorption stabilizes a plane wave, which in the absence of absorption would be unstable against field perturbations. The possibility of experimentally observing the effects is discussed.

1. INTRODUCTION

THE self focusing (self trapping) of light beams in a nonlinear transparent medium, predicted in general form by Askar'yan,[1] has been attracting persistent attention recently. A large number of papers have been published on this topic within a short time,[2-17] mostly theoretical.[2-15] The first to observe the self-focusing effect experimentally were Pilipetskiĭ and Rustamov.[16] The gist of the phenomenon consists in the following. The electric field E of an electromagnetic wave causes a slight increase in the dielectric constant of the medium ϵ in the light channel,

$$\varepsilon = \varepsilon_0 + \Delta\varepsilon, \ \Delta\varepsilon = \varepsilon_2 E^2 > 0, \qquad (1)$$

and also in the refractive index $n = \sqrt{\epsilon} \approx n_0 + n_2 E^2$ ($n_0 = \sqrt{\epsilon_0}$, $n_2 = \epsilon_2/2\sqrt{\epsilon_0}$; E^2 is averaged over the period). Because of this, transverse gradients of n are produced, causing the beams to be deflected towards the region of larger n, i.e., towards the beam axis. If the radiation power P exceeds a certain critical value P_{cr}, the refractive narrowing offsets completely the diffractive spreading of the beam, and the existence of a self-maintaining waveguide channel becomes possible.[2,3]

A nonlinearity of type (1) is brought about by the Kerr effect (orientation of anisotropically polarizing molecules by the field) and by electrostriction (compression of the dielectric in the electric field). For many substances, the equilibrium value of $\Delta\epsilon_{str}$, due to striction, greatly exceeds $\Delta\epsilon_{Kerr}$, but frequently the principal role is played by the Kerr effect. The reason is that the Kerr increment is established very rapidly, within $\sim 10^{-11}-10^{-12}$ sec (and also locally), whereas the striction compression occurs relatively slowly, being the result of a hydrodynamic process, and actually the values of $\delta\epsilon_{str}$ turn out to be much smaller than the equilibrium values.

In this article we consider the processes of propagation (self focusing and defocusing) of light beams in weakly absorbing media, since even the so-called "transparent" media always have a small absorption.[1)] Absorption of light causes heating and subsequent thermal expansion of the material in the light channel, leading to effects which are in many respects the inverse of striction. The increment $\delta\epsilon_{ther}$ connected with the absorption is also proportional to E^2 (it is proportional to the radiant energy flux density $J = \sqrt{\epsilon_0}\, cE^2/4\pi$), but is negative. If the field in the beam decreases from the axis towards the edge, then the striction and the Kerr effect exert a fo-

[1)]For example, in optical glasses at approximately the ruby-laser wavelength ($\lambda_0 \approx 7000$ Å) the absorption coefficients are $\kappa_\nu = (0.3 - 1) \times 10^{-2}$ cm^{-1}, and the path lengths are $l_\nu = 1/\kappa_\nu = 3 - 1$ m [18]. In water at $\lambda_0 = 7000$ Å we have $\kappa_\nu \approx 5 \times 10^{-3}$ cm^{-1} and $l_\nu \approx 2$ m [19].

cusing action on the beam and absorption causes defocusing.

However, conditions are possible[12] when absorption, paradoxical as it may sound, leads not to defocusing but to focusing of beams. This occurs during the stage of unsteady motion of the material, if the field inside the beam is constant over the radius (see Sec. 5).

Bespalov and Talanov[11] have shown that in a medium with $\Delta\epsilon = \epsilon_2 E^2 > 0$ a plane wave is unstable against perturbations and breaks up spontaneously into individual self-focusing beams. It was noted, however,[20] that in a weakly absorbing medium the self-focusing effect is gradually suppressed and consequently absorption counteracts the instability of the plane wave, exerting a stabilizing action on it. This phenomenon will be treated in Secs. 6 and 7.

We shall assume below that the dielectric constant depends directly only on the density ρ, but not on the temperature T. In fact, $\epsilon = \epsilon(\rho, T)$ and $\delta\epsilon_{ther} = (\partial\epsilon/\partial\rho)_T\delta\rho + (\partial\epsilon/\partial T)_\rho \delta T$. However, for most substances the principal role is played by the change in ϵ due to thermal expansion. Under equilibrium conditions, when $\delta\rho = (\partial\rho/\partial T)_p \delta T$ (p = pressure), the second term in $\delta\epsilon_{ther}$ is smaller by 1–2 orders of magnitude than the first. There are no direct measurements of $(\partial\epsilon/\partial T)_\rho$, since there are likewise no theoretical estimates of this quantity.[21, 22] Even the sign of $(\partial\epsilon/\partial T)_\rho$ has not been fully determined. If we substitute into the thermodynamic formula $(\partial\epsilon/\partial T)_\rho = (\partial\epsilon/\partial T)_\rho + \alpha(\rho\partial\epsilon/\partial\rho)_T$ (α—coefficient of thermal expansion) the experimental values of the terms on the right side (they are given in the book[21]), then $(\partial\epsilon/\partial T)_\rho$ turns out to be a small difference of two relatively large quantities, the sign of the derivative depending on the choice of various experimental values of $(\rho\partial\epsilon/\partial\rho)_T$ and changing from substance to substance. In view of such an uncertainty, we shall assume that $\epsilon = \epsilon(\rho)$, emphasizing that the conclusions of this article do not hold for substances in which $(\partial\epsilon/\partial T)_\rho > 0$ and is sufficiently large. We note that the diffusion of heat due to heat conduction is too slow.

2. EQUATION OF MOTION OF THE MATERIAL. RELATIVE ROLE OF ABSORPTION AND STRICTION

Assume that a directed parallel (or weakly diverging) light beam of radius R enters the medium at an instant t = 0. The coordinate z will be reckoned along the beam axis from the point of its entry into the medium. The changes in the density of the material are usually very small, so that the motion of the medium is described by the linearized equations of hydrodynamics.[2)] Let us write out these equations, bearing in mind that it is practically always possible to neglect the longitudinal motion (along the channel). If $\delta\rho = \rho - \rho_0$ is the change in the density, $p(\rho, S)$ is the pressure as a function of the density and the entropy S, and u is the radial velocity, then

$$\frac{\partial\delta\rho}{\partial t} + \frac{\rho_0}{r}\frac{\partial}{\partial r} ru = 0, \tag{2}$$

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial r} - \frac{\partial p_{str}}{\partial r}, \quad \rho_0 T_0 \frac{\partial S}{\partial t} = J\varkappa_\nu. \tag{3}$$

Here $p_{str} = -(\rho\partial\epsilon/\partial\rho)E^2/8\pi$ is the striction pressure and κ_ν is the coefficient of light absorption (we assume it to be independent of the field). The two equations in (3) combine into a single equation of motion

$$\rho_0 \frac{\partial u}{\partial t} = -a^2 \frac{\partial\delta\rho}{\partial r} - \frac{\partial p_{ther}}{\partial r} - \frac{\partial p_{str}}{\partial r},$$

$$p_{ther} = \Gamma\varkappa_\nu \int_{t_z}^{t} J\, dt. \tag{4}$$

Here $a = (\partial p/\partial\rho)_S^{1/2}$ is the speed of sound, p_{ther} is the pressure rise which would occur at the given instant t were there to be heat release without change in density; $t_z = z/c_1$ is the instant of arrival of the front of the light wave at the considered cross section of the channel z ($c_1 = c/\sqrt{\epsilon_0}$ is the velocity of light in the medium). In the case of solids and liquids the quantity $\Gamma = (\rho_0 T_0)^{-1}(\partial p/\partial S)_\rho$ is the Gruneisen coefficient;[23] for gases $\Gamma = \gamma - 1$, where γ is the adiabatic exponent.

The relative role of absorption and striction is characterized by the ratio of p_{ther} and p_{str}. The equilibrium values of the changes in the density $\Delta\rho_{ther, str} = -p_{ther, str}/a^2$ are related in the same ratio, and also the equilibrium values of the changes in the dielectric constant $\Delta\epsilon = (\partial\epsilon/\partial\rho)\Delta\rho$. Assuming by way of an estimate J(t) = const, we get

$$\frac{|\Delta\varepsilon_{ther}|}{\Delta\varepsilon_{str}} = \frac{p_{ther}}{|p_{str}|} = \frac{2\Gamma\sqrt{\varepsilon_0}}{\rho\,\partial\varepsilon/\partial\rho}\varkappa_\nu c t', \quad t' = t - t_z.$$

The thermal effect under quasiequilibrium conditions becomes larger than the striction effect after a time

2)For simplicity we confine ourselves to liquids and gases, although qualitatively all the conclusions are applicable also to solids.

$$\tau_{c0} = \rho\frac{\partial\varepsilon}{\partial\rho}\Big/2\Gamma\sqrt{\varepsilon_0}\,c\varkappa_\nu. \quad (5)$$

At values $\Gamma = 2$, $\sqrt{\epsilon_0} = 1.5$, and $\rho\partial\epsilon/\partial\rho = 1$, which are characteristic of solids and liquids, and at very weak absorption $\kappa_\nu = 10^{-4}$ cm^{-1} ($l_\nu = 100$ m), we have $\tau_{c0} = 0.5 \times 10^{-7}$ sec = 150 nsec; for $\kappa_\nu = 10^{-3}$ cm^{-1} we get $\tau_{c0} = 5$ nsec. For example, in atmospheric air, $\Gamma = 0.4$ and $\rho\partial\epsilon/\partial p = \epsilon_0 - 1 \approx 6 \times 10^{-4}$. The absorption of light in the visible and infrared regions of the spectrum is connected with the presence of water vapor.3) The absorption spectrum of water is very complicated, and the concentration of the vapor in air varies in a very wide range. If we assume by way of a characteristic value $\kappa_\nu = 3 \times 10^{-7}$ cm^{-1} ($l_\nu \approx 30$ km),[24] we get $\tau_{c0} = 10^{-7}$ sec.

3. ESTIMATE OF THE BEAM DEFLECTION ANGLE

When considering the propagation of the light, we shall disregard diffraction; this can be done if the refractive deflection of the beams greatly exceeds the characteristic diffraction angles. This condition corresponds in principle to the condition that the power is greatly in excess of the critical value, and the beam radius greatly exceeds the radius of the stationary channel corresponding to the given power.[3,8] 4)

For simplicity we solve the problem approximately, in two steps. We find first the density distribution, neglecting the redistribution of the field in the initial beam due to the resultant refraction, after which we consider the beam deflection on the basis of the obtained distribution of $\delta\rho$ and $\delta\epsilon$.

In the case when the beam has axial symmetry, the trajectory of the moving "light particle" (ray) is described by the functions r(t) and z(t). For paraxial rays $z(t) \approx c_1(t - t_0)$, where t_0 is the instant of entry of the beam into the medium. We denote by $\theta = dr/dz$ the angle of deflection of the trajectory from the direction of the z axis. The "substantional" derivative

$$\frac{d\theta}{dz} = \frac{1}{c_1}\frac{d\theta}{dt} = \frac{1}{c_1}\frac{\partial\theta}{\partial t} + \frac{\partial\theta}{\partial z} + \theta\frac{\partial\theta}{\partial r}$$

is determined by the refraction equation

$$\frac{d\theta}{dz} = \frac{1}{2\varepsilon_0}\frac{\partial\delta\varepsilon}{\partial r}, \quad (6)$$

with $\partial\delta\epsilon/\partial r = (\partial\epsilon/\partial\rho)\partial\delta\rho/\partial r + \epsilon_{2k}\partial E^2/\partial r$, where the second term is due to the Kerr effect. The rays are always deflected towards the higher dielectric constant, and therefore the beam is focused if the density decreases from the axis of the light channel towards the periphery, and defocused if the density increases.

Under the assumptions made (almost parallel beam without redistribution of the field), the ray will encounter during its entire path the same radial distribution of the dielectric constant $\delta\epsilon(r, z, t) = \delta\epsilon(r, 0, t_0)$. Consequently we have along the ray trajectory

$$d\theta / dz = {}^1\!/_2 d\theta^2 / dr = ({}^1\!/_{2\varepsilon_0})\, d\delta\varepsilon / dr$$

and the refraction equation can be integrated in general form

$$\theta^2 = \theta_{\text{init}}^2 + \frac{1}{\varepsilon_0}(\delta\varepsilon - \delta\varepsilon_{\text{init}}) = \frac{1}{\varepsilon_0}(\delta\varepsilon - \delta\varepsilon_{\text{init}}) \quad (7)$$

(we assume that $\theta_{\text{init}} = 0$). Here $\delta\epsilon_{\text{init}} = \delta\epsilon(r_0, 0, t_0)$ corresponds to the point at the instant of entry of the beam into the medium, and the value of $\delta\epsilon$ is taken at $z = 0$, t_0, and r equal to the radius of the trajectory in question: $\delta\epsilon = \delta\epsilon(r, 0, t_0)$. Formula (7) describes also the refraction of glancing rays on passing through the surface of the discontinuity $\delta\epsilon$. In this case θ_{init} and $\delta\epsilon_{\text{init}}$ pertain to the regions ahead of the discontinuity, and θ and $\delta\epsilon$ to the region behind the discontinuity. Thus, formula (7) determines the angle of deflection of a beam passing through the medium with arbitrary distribution of weak inhomogeneities.

This formula can be used to estimate the distance at which self focusing of the rays takes place, and the characteristic distance for defocusing of the beam. In the case of self focusing, we have for peripheral rays $\delta\epsilon_{\text{init}} \approx 0$. The rays converge to the axis at distances z_f of the order of $z_f \approx R/\theta_t$, $\theta_t \approx \sqrt{\delta\epsilon/\epsilon_0}$, where $\delta\epsilon$ corresponds to the main part of the light channel, say the midpoint of the radius $r = R/2$. The angle θ_t can be interpreted as being the angle of total internal reflection of the rays incident from outside the beam on the surface of the light channel. The estimate $z_f \approx R/\theta_t$ practically coincides with the result obtained by Kelley.[6] For defocusing, the final value is $\delta\epsilon = 0$ and the final inclination of the rays is $\theta_\infty \approx \sqrt{|\delta\epsilon_{\text{init}}|/\epsilon_0}$ ($\delta\epsilon_{\text{init}} < 0$), where $\delta\epsilon_{\text{init}}$ can also be referred to the midpoint of the

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3)The vibrations excited in water molecules upon absorption of light become de-activated extremely rapidly, unlike nitrogen and oxygen [25]. Therefore the absorbed energy goes over practically instantaneously into heat. The question of the relaxation must be taken into consideration when estimating the thermal effect in other gases.

4)Of course, diffraction cannot be neglected in the region where the beams converge, where the waveguide channel is produced.

radius. The characteristic distance for defocusing is $z_2 = R/\theta_\infty$. We note that a medium is "weakly absorbing" if $l_\nu >> z_f$ or z_2.

4. BEAM IN WHICH THE FIELD DECREASES FROM THE AXIS TOWARDS THE EDGE

For real beams, this case is to some degree typical. We put $J(r, z, t) = J_0(t')f(r/R)$, where $J_0(t')$ is the beam density on the axis and $f(\xi)$ is the profile function ($\xi = r/R$, $f(0) = 1$). We shall henceforth write t in place of $t' = t - t_z$, referring to the section at $z = 0$.

The system (2) and (4) is equivalent to an inhomogeneous cylindrical wave equation. We shall obtain for it an approximate solution sufficient for estimates. Considering the earlier stage of motion, namely times t that are small compared with the characteristic hydrodynamic time $t_s = R/a$, we leave out from (4) the term $-a^2\partial\delta\rho/\partial r$ (such an approximation was made in [15] in the calculation of $\delta\epsilon_{str}$). Integrating the resultant equations, we get

$$\delta\rho(r,t) = \frac{\psi(\xi)}{R^2}\int_0^t dt' \int_0^{t'} (p_{0\,\mathrm{ther}} + p_{0\,\mathrm{str}})\,dt'', \qquad \psi = f'' + f'/\xi, \tag{8}$$

where $p_{0\,\mathrm{ther}}$ and $p_{0\,\mathrm{str}}$ pertain to the beam axis, and the function ψ determines the radial profile of the density. If we approximate the radial distribution of the flux by a cosine function, $f = \cos(\pi\xi/2)$, then

$$\psi = -(\pi/2)^2\left(\cos\frac{\pi}{2}\xi + \sin\frac{\pi}{2}\xi\Big/\frac{\pi}{2}\xi\right).$$

We put $J_0(t) = J_0 = \text{const}$. Then

$$p_{0\,\mathrm{str}} = -\rho\frac{\partial\varepsilon}{\partial\rho}\frac{J_0}{2c\sqrt{\varepsilon_0}}, \qquad p_{0\,\mathrm{ther}} = \Gamma\varkappa_\nu J_0 t$$

and from (8) we get

$$\delta\rho = \frac{\psi}{6}\frac{\Gamma\varkappa_\nu J_0 t^3}{R^2}\left(1 - \frac{\tau_c}{t}\right) = \frac{\psi}{6}\frac{p_{0\,\mathrm{ther}}}{a^2}\left(\frac{t}{t_s}\right)^2\left(1 - \frac{\tau_c}{t}\right), \tag{9}$$

where $\tau_c = 3\tau_{c0}$, and the time τ_{c0} is determined by formula (5). When $t < \tau_c$, the thermal expansion still does not have time to compensate for the effective striction, and the material in the channel is compressed. If $\tau_c = 3\tau_{c0}$ and is at least several times smaller than t_s, then $\delta\rho$ goes through zero before formula (9) becomes invalid, at an instant τ_c (when $t > \tau_c$ we have $\delta\rho < 0$). But if $\tau_c \gg t_s$, then the instant when $\delta\rho$ vanishes can be determined approximately by assuming the density distribution in the channel to be in quasiequilibrium. In this case, in accordance with the definition in Sec. 2, this instant is τ_{c0}. Thus, in all cases a time on the order of τ_{c0} separates the stages in which striction and thermal expansion predominate.

The distribution of the density outside the channel when $t \ll t_s$ and at $\ll R$ can be obtained by assuming the wave traveling from the boundary $r = R$ to be plane. To this end it is necessary to use the expression for the velocity $u(R, t)$ on the boundary, which is obtained by solving the internal problem approximately. Putting $x = r - R$ we find that when $0 < x \le at$

$$\delta\rho = -\frac{f'(1)}{2R}\frac{\Gamma\varkappa_\nu J_0}{a^3}(at - x)\cdot(at - x - 2a\tau_{c0}). \tag{10}$$

We note that formula (9) cannot be applied to the peripheral layer $R > r > R - at$ reached by the perturbations from the outer region. In this layer, the density varies continuously from the values defined by (9) to those given by (10). The radial density distributions are shown in Fig. 1.

In the limit of large $t \gg t_s$, the density in the channel is in quasiequilibrium: $\delta\rho \approx \Delta\rho = -(p_{\mathrm{ther}} + p_{\mathrm{str}})/a^2$. Assuming as before $J_0(t) = \text{const}$, we get

$$\delta\rho \approx -f\frac{\Gamma\varkappa_\nu J_0 t}{a^2}(1 - \tau_{c0}/t), \qquad t \gg t_s. \tag{11}$$

The limiting formulas (9) and (11) can be approximately extrapolated to the instant $t = t_s$, where they join together fairly well.

If $J_0(t) = \text{const}\cdot t = J_m t/t_m$, we get for $t \ll t_s$

$$\delta\rho = \frac{\psi}{24R^2}\frac{\Gamma\varkappa_\nu J_m t^4}{t_m}\left(1 - \frac{\tau_c'}{t}\right), \qquad \tau_c' = 4\tau_{c0}, r < R - at,$$

$$\delta\rho = -\frac{f'(1)}{3R}\frac{\Gamma\varkappa_\nu J_m}{a^4 t_m}(at - x)^2(at - x - 3a\tau_{c0}), 0 < x \le at.$$

These formulas can be used for a description of the first half of a giant laser pulse. At $t_m = 20$ nsec and $a = 1$ km/sec, the condition $t_m < t_s$ is satisfied if $R > 2\times10^{-3}$ cm, i.e., in most cases of interest.

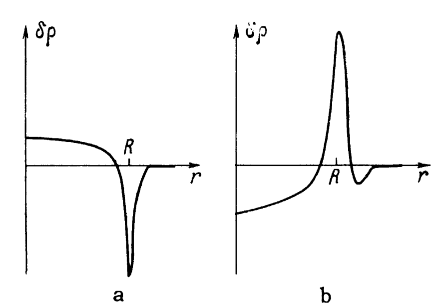

FIG. 1. Radial density distribution for $t \ll t_s = R/a$, when the field drops off from the axis to the edge of the beam: a) $t < \tau_c$, striction compression predominate, b) $t > \tau_c$, thermal expansion predominates.

In liquids at $\kappa_\nu \approx 10^{-3}$ cm^{-1} we have $\tau_{c0} = t$ nsec (see Sec. 2) and $\tau'_c \approx 20$ nsec $= t_m$. Consequently, the focusing action of the striction can come into play only at weaker absorption, $\kappa_\nu < 10^{-3}$ cm^{-1}. It must be stated that in many cases the focusing action of the striction is generally less effective than the analogous action of the Kerr effect. If $\kappa_\nu > 10^{-3}$ cm^{-1}, then the density becomes smaller than normal even before the end of the giant pulse, but the defocusing action of the absorption at such short times still cannot withstand the competition with the focusing influence of the Kerr effect (see Sec. 6).

The defocusing effect can come fully into play in the case of relatively long pulse of solid-state lasers operating in the free generation mode, $t_1 \approx 10^{-3}$ sec. To increase the flux density, the laser beam can be narrowed down with the aid of a telescopic system. Assume, for example, that $R = 0.2$ cm, $J = 10$ MW/cm^2, and $t_1 = 10^{-3}$ sec, corresponding to an energy of 1000 J.[26] At $a = 1$ km/sec we get $t_s = 2 \times 10^{-6}$ sec $\ll t_1$, i.e., the density in the channel is quasiequilibrium. Putting $\rho = 1$ g/cm^3, $\Gamma = 2$, $\rho \partial\epsilon/\partial\rho = 1$, and $\kappa_\nu = 10^{-4}$ cm^{-1}, we get from (11) $\delta\epsilon_{ther} \approx -2 \times 10^{-3} t/t_1$. When $t \sim t_1$ this quantity is larger, by several orders of magnitude, than either $\delta\epsilon_{str}$ or $\delta\epsilon_{Kerr}$. At the end of the pulse the beam-defocusing angle reaches $\theta_\infty \approx 0.03$, and when $\kappa_\nu = 10^{-3}$ cm^{-1} we have $\theta_\infty \approx 0.1$. The characteristic distances $z_2 \approx R/\theta_\infty$ are 6 and 2 cm. Such a defocusing greatly exceeds the initial divergence of the beam (which has increased after passing through the telescopic system), and can be easily measured.

A similar procedure could be used for an experimental determination of weak absorption in media that scatter more strongly than they absorb, and for which the measurement of the beam attenuation yields only the summary coefficient.

5. HOMOGENEOUS BEAM

Let us consider a parallel homogeneous beam, in which the flux density J and the field are constant over the radius and terminate abruptly at the boundary $r = R$. Such a beam can be obtained experimentally by placing at the laser outlet a screen with a small opening in the center.

The concept of a homogeneous beam is, of course, idealized, since diffraction always disturbs the homogeneity. The width of the diffraction-spreading zone of the beam boundary increases on increasing distance from the opening approximately like $x_d \approx \sqrt{\lambda z/2}$, where $\lambda = \lambda_0/\sqrt{\epsilon_0}$ (λ_0 = wavelength in vacuum). The beam spreads out completely over a distance $z_d \approx 2R^2/\lambda$. However, at distances $z \ll z_d$ "quasihomogeneity" is still maintained, so that the idealization is not devoid of meaning.

The homogeneous beam has unusual properties with respect to refraction due to nonlinearity of the medium.[12] These anomalies are connected with the absence of radial gradients of the field inside the light channel. The Kerr effect becomes manifest here only to the extent that diffraction smears out the discontinuity on the boundary and the peripheral rays enter into the region of the field gradient.

The field-dependent external force acting on the material, $-\partial(p_{ther} + p_{str})/\partial r$, is equal to zero inside the channel. It acts only on the lateral boundary of the beam and is equal to $p_{ther} + p_{str}$ per square centimeter of surface.

Starting with instant t_z when the light wave arrives at the given section z, compression and rarefaction waves, due to the action of p_{str} and p_{ther}, propagate from the beam boundary inward with the speed of sound. Propagating outward are respectively rarefaction and compression waves.

We introduce the coordinate $x = r - R$. So long as the perturbation front has not reached the axis, its coordinate x has an absolute value $x_s = at$ (we assume, as before, that $z = 0$ and $t' = t$). When $|x| > at$ we have $\delta\rho = 0$ inside and outside the channel. The density distribution $\delta\rho(x, t)$ in the perturbed region $|x| \le at$ can be readily obtained for the early stage $t \ll t_s = R/a$, $x_s \ll R$, when the motion can be regarded as plane. For the wave traveling inside the channel $\delta\rho = \varphi_1(t + x/a)$, and for the exterior region $\delta\rho = \varphi_2(t - x/a)$, where φ are functions that are as yet arbitrary. At the channel boundary $x = 0$, the velocity u and the summary pressure of the material and of the striction, i.e., $(a^2\delta\rho + p_{ther}) + p_{str}$, are continuous. The first condition yields $\varphi_2(t) = -\varphi_1(t)$, and the second yields $\varphi_2(t) - \varphi_1(t) = p_{ther} + p_{str}$. This determines the form of the functions φ:

$$\varphi_1(t) = -\varphi_2(t) = -\frac{1}{2a^2}[p_{ther}(t) + p_{str}(t)]. \quad (12)$$

Assume that the flux J is constant in time. Then for $|x| \le x_s$

$$\delta\rho(x, t) = \mp\frac{1}{2a^2}\left[p_{ther}\left(1 - \frac{|x|}{x_s}\right) - |p_{str}|\right]$$

$$= \mp\frac{\Gamma\varkappa_\nu Jt}{2a^2}\left[1 - \frac{|x|}{at} - \frac{\tau_{c0}}{t}\right]. \quad (13)$$

The upper sign pertains here to the internal region and the lower to the external one. This distribution is shown in Fig. 2.

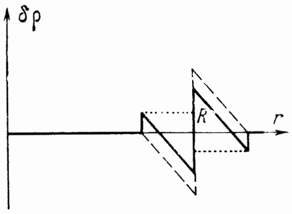

FIG. 2. Radial distribution of the density $\delta\rho = \delta\rho_{str} + \delta\rho_{ther}$ at $t \ll t_s = R/a$, when the field in the beam is radially uniform. The dashed lines show $\delta\rho_{ther}$ and the dotted ones $\delta\rho_{str}$.

In the absence of absorption, the density profile in the channel has the form of a "step" with $\delta\rho_{str}$ equal to half the equilibrium value. The value of $\delta\rho_{ther}$ at the boundary is likewise equal to half the equilibrium value, and the partial jumps in the density on the boundary are equal to their equilibrium values $|\Delta\rho_{ther,\,str}| = |p_{ther,\,str}/a^2|$. (We recall that when $t \gg t_s$ the density in the channel is in quasiequilibrium throughout the channel, $\delta\rho = \Delta\rho = -(p_{ther} + p_{str})/a^2$.)

Let us trace the rays in a medium having a density distribution (13). Those rays which at the given instant t enter the medium in the unperturbed region of the channel, $r < R - at$, are not refracted and retain the initial direction $\theta = 0$. But the rays which enter the perturbed layer, $R > r > R - at$, are deflected toward the axis. The density gradient in this region is constant, and therefore according to (6) $d\theta/dz = \text{const} = \beta$; the constant β is equal to

$$\beta = \rho \frac{\partial \varepsilon}{\partial \rho} \frac{\Gamma \varkappa_\nu J}{4\varepsilon_0 \rho_0 a^3} = \frac{1}{2\varepsilon_0} \frac{\delta\varepsilon(-0, t)}{x_s}.$$

The ray describes a parabolic trajectory $r = r_0 - \beta z^2/2$ and encounters the perturbation front $r_s = R - at$ after covering a distance $z_s = \{(2/\beta)[at - (R - r_0)]\}^{1/2}$ and acquiring a slope $\theta_s = \beta z_s$.

On the perturbation front there is a density discontinuity (see Fig. 2). At $t < \tau_{c0}$, as follows from (7) and (13), the ray experiences here total internal reflection. The same occurs also when $t > \tau_{c0}$ in the case of those rays which enter the medium sufficiently close to the front, in the region where $\delta\rho > 0$. (In the absence of absorption, when $\delta\rho(r) = \text{const} = \delta\rho_{str}$, the rays are not refracted at all.) When $t > \tau_{c0}$, the rays entering the medium in the region $\delta\rho < 0$ are refracted at the discontinuity and enter the unperturbed region, where they propagate linearly towards the axis. At $\tau \gg \tau_{c0}$ we have $\delta\rho_{str} \ll |\delta\rho_{ther}|$, the discontinuity is relatively small, and the refraction hardly changes the angle θ_s. During this stage the rays approach the axis at angles close to θ_s and of order of magnitude of $\theta_s \approx \sqrt{\Delta\epsilon_{ther}/2\epsilon_0}$, where $\Delta\epsilon_{ther}$ is the equilibrium values; the rays reach the axis at distances $z_f \sim R/\theta_s$. The course of the rays is shown in Fig. 3. Thus, in the case of a homogeneous beam, the striction itself does not lead to self focusing, and the thermal expansion during the stage $t < t_s = R/a$, to the contrary, exerts a focusing influence on the peripheral part of the beam. This influence counteracts the diffraction spreading of the boundary. It can contribute to a decrease in the divergence of a homogeneous fan-like diverging beam of rays.

If the field is radially constant in the main part of the beam and drops off gradually at the edges, then the thermal expansion will lead to divergence of only the outermost rays. In the homogeneous part of the beam, on the other hand, everything will proceed approximately in the manner described above. We shall not consider here the action of refraction and diffraction, when their roles are comparable. For estimates it can be assumed that refraction prevails over diffraction if $z_f \ll z_d$, and vice versa. This inequality is equivalent to $\theta_s \gg \theta_d \sim \lambda/R$, which in the theory of self focusing corresponds to the condition that the power greatly exceed the critical value.[3,4,6]

We write out the solution for the case $J = J_m t/t_m$. When $|x| \le at$ we have

$$\delta\rho = \mp \frac{1}{2a^2}\left[p_{ther}\left(1 - \frac{|x|}{x_s}\right)^2 - |p_{str}|\left(1 - \frac{|x|}{x_s}\right)\right] = \mp \frac{\Gamma \varkappa_\nu J_m t^2}{4a^2 t_m}\left[\left(1 - \frac{|x|}{at}\right)^2 - \frac{2\tau_{c0}}{t}\left(1 - \frac{|x|}{at}\right)\right] \quad (14)$$

(the upper and lower signs refer to the inside and outside of the channel).

When $t \gg \tau_{c0}$, the refraction differs little from that occurring in the case $J(t) = \text{const}$. On the

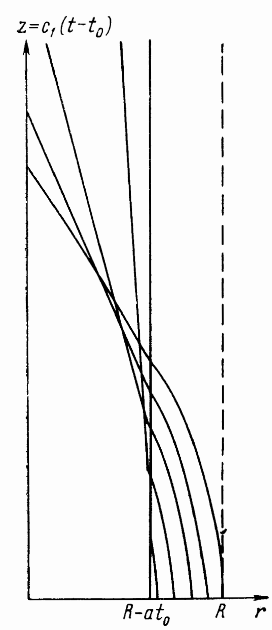

FIG. 3. Course of rays in the case of homogeneous beam.

other hand, if there is no absorption and only striction alone is effective, then the rays entering the perturbed peripheral layer are deflected along parabolas towards the boundary of the channel, but experience there total internal reflection on the density discontinuity and "are returned" back.

We present a numerical example as applied to a giant ruby-laser pulse. Let $R = 2 \times 10^{-2}$ cm, just as in [17], $J = 100$ MW/cm^2, $\Gamma = 2$, $\sqrt{\epsilon_0} = 1.5$, $\rho\partial\epsilon/\partial\rho = 1$, $a = 1$ km/sec, $\rho_0 = 1$ g/cm^3, and $\kappa_\nu = 10^{-2}$ cm^{-1}. With this, $\tau_{c0} = 0.5$ nsec. At the instant $t = 30$ nsec we have $at = 0.15R$ and rays close to the edge converge towards the axis at a distance $z_f \approx 6$ cm; the diffraction distance in this example is considerably larger than z_f, namely $z_d \approx 18$ cm. If the pulse were to continue to have the same power for a duration several times longer, the focusing action could encompass almost the entire beam.

6. SUPPRESSION OF SELF FOCUSING DUE TO THE KERR EFFECT

We return to the case of a field that drops off from the axis toward the edge of the beam, and compare the action of the Kerr effect with that of absorption. We assume, by way of an estimate, that the field is constant in time. We leave out from (9) and (11) the terms that are connected with striction (i.e., we put $\tau_{c0} = 0$). Noting that $\psi/6f \approx 1$, we obtain approximately

$$\frac{|\delta\varepsilon_{\rm ther}|}{\delta\varepsilon_{\rm kerr}} = \frac{t}{\tau_{k0}} \cdot \begin{cases} (t/t_s)^2 & \text{for } t < t_s \\ 1 & \text{for } t > t_s \end{cases}$$

where

$$\tau_{k0} = 4\pi\rho_0 a^2 \varepsilon_{2k}/\rho\frac{\partial\varepsilon}{\partial\rho}\Gamma c\sqrt{\varepsilon_0}\,\varkappa_\nu. \tag{15}$$

The thermal action exceeds the action of the Kerr effect, starting with an instant τ_k equal to

$$\tau_k = \tau_{k0} \cdot \begin{cases} (t_s/\tau_{k0})^{2/3}, & \text{if } t_s > \tau_{k0} \\ 1 & \text{if } t_s < \tau_{k0} \end{cases}. \tag{16}$$

We present numerical estimates. For liquids, in which the Kerr effect is relatively weak, $\epsilon_{2k} \approx 10^{-12}$ absolute units.[15] Putting $\rho_0 = 1$ g/cm^3, $a = 1$ km/sec, $\Gamma = 2$, $\sqrt{\epsilon_0} = 1.5$, $\rho\partial\epsilon/\partial\rho = 1$, and $\kappa_\nu = 10^{-3}$ cm^{-1}, we get $\tau_{k0} = 1.4$ nsec. In all the cases of practical interest, $t_s > \tau_{k0}$ ($R > 1.4 \times 10^{-4}$ cm), so that $|\delta\epsilon_{\rm ther}|$ increases to $\delta\epsilon_{\rm Kerr}$ even before mechanical quasiequilibrium is attained, and $\tau_k = \tau_{k0}^{1/3} t_s^{2/3}$. For example, at $R = 0.1$ cm we have $t_s = 10^{-6}$ sec and $\tau_k = 1.1 \times 10^{-7}$ sec. We recall that the effect of the striction comes into play much earlier: $\tau_c = 15$ nsec. In the most active liquid, carbon disulfide, $\epsilon_{2k} = 6 \times 10^{-11}$ absolute units; at the same value of absorption, $\kappa_\nu = 10^{-3}$ cm^{-1}, we get $\tau_{k0} \approx 60$ nsec; if $R < 0.7 \times 10^{-2}$ cm, then $\tau_k \approx \tau_{k0} \approx 0.6 \times 10^{-7}$ sec; for $R = 0.1$ cm we have $t_s = 0.8 \times 10^{-6}$ sec and $\tau_k \approx 4 \times 10^{-7}$ sec.

Assume that an almost-parallel beam with divergence not exceeding the diffraction value λ/R carries a power $P = \sqrt{\epsilon_0}\, cE^2\pi R^2/4\pi$, which is larger (but by not too many times) than the critical value $P_{cr} = c\lambda_0^2/86n_{2\rm Kerr}$.[3, 17]

The rays converge to the axis at a distance on the order of $z_f \sim R\sqrt{\delta\epsilon_{\rm Kerr}/\epsilon_0} \sim (R^2/\lambda)(P/P_{cr})^{-1/2}$, or more accurately, with diffraction taken into account, at a distance $z_f \sim (R^2/\lambda)(P/P_{cr} - 1)^{-1/2}$.[6] In the presence of absorption, the focusing of the Kerr effect is gradually offset by the defocusing action of the thermal expansion. The total value $\delta\epsilon = \delta\epsilon_{2k} - |\delta\epsilon_{\rm ther}|$ decreases with increasing time. As a result, the critical power increases and the degree of supercriticality of the beam decreases. Accordingly, the distance z_f increases. At some instant, somewhat shorter than τ_k, the critical power becomes equal to the beam power, and the distance z_f becomes infinite. This concludes the self focusing. Further, when $t > \tau_k$, defocusing takes place ($\delta\epsilon < 0$) and the beam divergence increases; an estimate of the divergence was given in Sec. 4. Thus, the time τ_k determines the duration of the self focusing of the light beam in the absorbing medium.

So far we have used for observation of self focusing ruby lasers operating in the giant pulse mode.[16, 17] The durations of such pulses, ≈ 20–40 nsec, are apparently too short to be able to observe the suppression of self focusing (this effect occurs after a time $\tau_k \sim 10^{-7}$ sec). It will be probably necessary to use for this purpose lasers operating in the free-generation mode; the durations of individual spikes, $\sim 10^{-6}$ sec, are already sufficient.

This, of course, raises a difficulty in that the power required for self focusing is quite large. It is necessary that a beam carrying a power, say, double the critical value have a divergence not exceeding the diffraction value (as was the case in the experiments of [17]). On the other hand, if the divergence of the beam exceeds the diffraction value (this was the situation in the experiments of [16]), then the power required is several times larger than critical (this circumstance was explained in our paper[8]). One can, however, hope that modern lasers[26] will make it possible to observe the suppression effect in the most active liquids, such as carbon disulfide, where P_{cr}

≈ 10 kW[17] (at the ruby frequency); the absorption can in this case be varied by adding absorbers. It would be best to operate with a single pulse of $\sim 100-200$ nsec duration.

7. INSTABILITY AND STABILIZATION OF A PLANE WAVE

Let us see how a broad light beam of high power and long duration propagates in a weakly absorbing medium, in the idealized case—a plane wave with amplitude $\sqrt{2}$ E. Bespalov and Talanov[11] have shown mathematically that in a medium with dielectric constant (1) a plane wave is unstable against random field perturbations. This results in a spontaneous decay of the wave into individual beams in which the power exceeds the critical value, and each of such beams becomes self focused independently. The radii R of the beams are of the order of $R_{cr} = \sqrt{4P_{cr}/\sqrt{\epsilon_0}cE^2}$. The self-focusing distances are determined by the same formula $z_f \sim R/\sqrt{\epsilon_2 E^2/\epsilon_0}$ as for a single beam of radius R. The nature of the instability is clear: if the field has for some accidental reason become smaller than normal, then the dielectric constant becomes smaller at this location; the corresponding rays will start to move away from one another and the field along these rays will become even smaller.[5)]

Reasoning in similar fashion, it is easy to conclude that a negative increment $\Delta\epsilon = \epsilon_2 E^2 < 0$ should, to the contrary, exert a stabilizing effect on the wave. Accidentally occurring perturbations will not increase. This can be verified also formally by repeating the calculations of [11] and putting $\epsilon_2 < 0$. The absorption and subsequent thermal expansion of the material play precisely the role of a stabilizer of this type. Here, however, the inertia of the thermal effect comes essentially into play. The process must proceed approximately as follows: The waves in different places will incessantly cause bursts of instability connected with the Kerr effect, as a result of which individual self-focusing beams will appear. In each such beam, by virtue of the very nature of its formation, the field will in general drop off from the axis toward the edge, and the results of the preceding section are fully applicable to this case. The lifetime of the beam will be of the order of τ_k. After a time $\sim \tau_k$, the converging beam becomes defocused to the initial state, and the field in the transverse direction becomes equalized; the density in the region of the beam will also become equalized, and so on until the next burst of the wave in the given region.

It must be assumed that the promptness of appearance of the self-focusing beams will be determined by the ratio of the characteristic time of formation of the beam, the order of which, owing to the instantaneous nature of the Kerr effect, is $\tau_f \sim z_f/c_1$, to the suppression time τ_k. The effect will be clearly pronounced when $\tau_f \ll \tau_k$, and to the contrary, no instability will appear if $\tau_f \gg \tau_k$, since the self focusing simply does not have time to develop. As shown by estimates, the Kerr effect ensures fully the satisfaction of the conditions $\tau_f \ll \tau_k$ or $\tau_f \sim \tau_k$. As regards striction, the situation is apparently reversed here (the time τ_c, which in this case plays the role of τ_k, is small, and the radii R_{cr} and the distances $z_f \sim R_{cr}^2/\lambda$ are, to the contrary, large).

Since the transverse dimensions of the wave are always limited under real conditions, local bursts will apparently be produced against the background of the general evolution of the density in the region of the wave and of the entire light beam as a whole. Here, depending on the radial field distribution, either beam divergence or focusing of the peripheral rays is possible, as described in Secs. 4 and 5.

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5)This is counteracted by diffraction: if $R < R_{cr}$, then the field is restored before the beams converge; but if $R > R_{cr}$, the diffraction is unable to counteract the self focusing. This is precisely why the individual beams have radii $R > R_{cr}$.

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