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GENERATION OF HEAVY PARTICLE PAIRS

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An important result obtained in recent years is the discovered existence of high-energy hadron interaction processes that proceed via the compound state ("head-on collisions"). Such processes determine not only the binary scattering $A + B \to C + D$, with large four-momentum transfer $\sqrt{-t} > 1$ GeV, but also, as shown in the present paper, heavy-pair $p\bar{p}$ and $d\bar{d}$ production. It is possible in this case to differentiate between pair production processes in NN collisions, accompanied by production of other particles such as pions (Sec. 2), and processes of the type $A + \bar{A} \to B + \bar{B}$, which are not so accompanied (Sec. 3). The latter include also the production of heavy pairs by a γ quantum, when the form factor F of the $\gamma p\bar{p}$ vertex is determined by the formation of the compound state (Sec. 5). As applied to the cross section σ_q for the production of the quark pair $\bar{q}q$ in NN and $N\bar{N}$ collisions, it is shown that if the quarks interact with the hadrons at distances from m_N^{-1} to μ^{-1} just as strongly as the hadrons interact with one another, then σ_q is exceedingly small and decreases exponentially with the mass m_q (formulas (7) and (8)). But if they interact more weakly (Sec. 4), then σ_q can be large and approach the geometric cross section. This result is valid practically for any generation mechanism. On the basis of the same considerations, the form factor is estimated in the time-like region for the $\gamma p\bar{p}$ vertex and the result is compared with experiments on the $p\bar{p} \to e^- e^+ (\mu^- \mu^+)$ annihilation.

1. INTRODUCTION

EXPERIMENT has shown[1,2] that a high-energy ($E_{lab} \sim 5$–10 GeV) hadron interaction of the type $AB \to CD$ in the region of large angles ($\theta_{cms} \sim \pi/2$) leads to a universal distribution of the final particles with respect to the transverse momentum $p_\perp$, namely $n(p_\perp)\,dp_\perp \sim \exp(-ap_\perp)$, where $a \approx 160\ (\text{MeV}/c)^{-1}$. This was observed for the reactions $pp \to pp$, $np \to np$, $\pi^+ d \rightleftarrows pp$, $\pi^\pm p \to \pi^\pm p$, and even for multiple pion production at $E_{lab} > 100$ GeV. The weak dynamic dependence of the final state on the input channel (a dependence which nonetheless exists, inasmuch as the distribution depends on $p_\perp$, i.e., the direction of the initial momentum is remembered) is similar to that observed in low-energy nuclear reactions, in the region of energies above resonance. This makes it possible to advance the following hypothesis: At high energies, when many channels are open, the interaction of strongly-interacting particles proceeds in the extreme cases either as a direct d-process or as a compound c-process (intermediate cases are of course possible, but are apparently of secondary significance). Starting from this, we can write for $T_{ab,l}$ (the l-component of the amplitude of the process $a \to b$, where l—orbital angular momentum)[1)]

$$T_{ab,l} = T^d_{ab,l} + T^c_{ab,l} = |T^d_{ab,l}|e^{i\delta^d_{ab,l}} + |T^c_{ab,l}|e^{i\delta^c_{ab,l}}. \quad (1)$$

A criterion which makes it possible to distinguish between the two types of processes is the dependence of the phases $\delta^d_{ab,l}$ and $\delta^c_{ab,l}$ on the energy E and on l, and also their absolute magnitude. For d-processes the phase depends relatively little on E and l, and always remains of the order of π or less. For c-processes, it varies strongly with the initial conditions, and it is reasonable to consider it as a quantity that builds up with increasing E, i.e., to regard it as large quantity

$$\delta^d \lesssim 1, \qquad \delta^c \gg 1. \quad (1a)$$

If the initial state a is specified in the form of a packet, then T^d leads to a final state b which is likewise in the form of a packet, while T^c leads to a state b′ which does not overlap b spatially

1)For simplicity we disregard spin variables or other possible supplementary variables.

(owing to the large phase shift) and is therefore already practically orthogonal to b. Moreover, owing to the mutual incoherence of the parts of the packet that are scattered with different phase shifts, the state b′ is no longer essentially a packet but constitutes a mixture.

For strictly defined E and l (the index l will henceforth be omitted), the following unitarity condition is valid

$$\operatorname{Im} T_{ab} = \operatorname{Im}(T_{ab}{}^{d} + T_{ab}{}^{c}) \\ = \sum_{n} (T_{an}{}^{d} + T_{an}{}^{c})(T_{nb}{}^{d} + T_{nb}{}^{c}). \qquad (2)$$

Let us average (2) over a small energy interval (this corresponds to the assumption that the initial state is specified in the form of a packet). Then the terms which contain in linear fashion the rapidly oscillating function T^c drop out. On the other hand, the term $T^c_{an}T^c_{nb}$ may not drop out if b = a, i.e.,

$$\operatorname{Im} \overline{T_{ab}{}^{d}} = \sum_{n} (\overline{T_{an}{}^{d} T_{nb}{}^{d}} + \overline{T_{an}{}^{c} T_{nb}{}^{c}} \delta_{ab}). \qquad (2a)$$

For elastic scattering, subtracting (2a) from (2), we can obtain separately, under certain assumptions regarding the width of the packet, the unitarity condition for the rapidly oscillating amplitude T^c_{aa}. The cross section, averaged over a small interval of initial momenta, is made up of the incoherent cross sections of d- and c-processes:

$$d\sigma_{ab}/d\Omega = d\sigma_{ab}{}^{d}/d\Omega + d\sigma_{ab}{}^{c}/d\Omega. \qquad (2b)$$

The c-process, inasmuch as the phase is large, has a quasiclassical character. Together with the circumstance that the final state represents approximately not a pure state but a mixture, this indeed justifies the application of statistical, thermodynamic, and hydrodynamic methods to this state.

The c-process corresponds to what is usually called head-on collisions. The d-process contains a contribution from diagrams with exchange of a small number of particles or Regge trajectories, and also a contribution from diffraction as the result of the absorption causing the c-process (the last term in (2a)). These contributions, in general, interfere with one another. Only the first of them contains the peripheral interaction, defined as exchange of one (when a ≠ b) or two (when a = b) pions (see, for example,[3]). (All the foregoing applies also to particle-nucleus interaction in high-energy physics, but then the d-process contains additional contributions of isolated Breit-Wigner levels).

Thus, the classification into d- and c-processes presupposes that their characteristic energy scales of phase variation, Γ^d and Γ^c, are greatly different ($\Gamma^d \gg \Gamma^c$). Consequently, inasmuch as these scales have the meaning of the reciprocal lifetimes of the states ($\Gamma^d = 1/T^d$, $\Gamma^c = 1/T^c$), it is assumed that the lifetimes are also different, $T^d \ll T^c$. It is obvious that T^d is of the same order as the time of flight of the particles past one another: $T^d \sim 1/\mu$, where μ is the pion mass, and therefore the separation is possible if $\Gamma^c \ll \Gamma^d \sim \mu$.

The requirement that the phases be large can hardly be proved theoretically in convincing fashion at present. The liklihood of this assumption is supported by different considerations (the Levinson theorem generalized to the relativistic case, etc.), all of which start from the fact that at high energies the number of open channels is large (this evidences that the number of degrees of freedom of the system is also very large).

It can be thought that d-processes play a decisive role when θ_{cms} is close to 0 and to 180°, and in general at small values of $|t|$ and $|u|$ in the s-channel, when one trajectory (or a small number of them) can exist, exchange of which ensures the required final state (s, t, and u are the usual Mandelstam variables). The c-process predominates when θ_{cms} is close to 90° (and to 180° if the required trajectory cannot be indicated),

A confirmation of the foregoing point of view may be, first of all: a) the universality of Orear's formula [2] (see above); b) the good agreement between the statistical-thermodynamics estimates [4-7] and experiment on large-angle pp scattering [4]; c) the good agreement of similar estimates with experiments on heavy pair production[8] (see below).

It should be noted that although the system "remembers" partly the initial direction of motion (inasmuch as $d\sigma/d\Omega$ depends on $p_\perp$), this does not contradict the compound-state hypothesis and its statistical-thermodynamical treatment. When $E \gg m_N^{-1}$, where m_N is the nucleon mass, the nucleons are oblate along the axis of motion and the initial state of the c-system is anisotropic. In Landau's hydrodynamic theory of multiple production, which is the most consistent form of the statistical-thermodynamic treatment, this leads to a sharp anisotropy of the front-back scattering, at which an invariant characteristic, which is exceedingly weakly dependent on the energy, is precisely the distribution with respect to $p_\perp$ [9] (unfortunately, by statistical theory is frequently meant only its simplest and not fully consistent version - the Fermi theory with isotropic particle scattering).

2. HEAVY-PARTICLE PAIR PRODUCTION

Appreciable experimental material has by now been accumulated on the production of heavy interacting antiparticles - antiprotons $\bar{p}$ and antideuterons $\bar{d}$ - so that certain essential regularities can be traced. Moreover, it is possible to draw on this basis several conclusions on the possible value of the quark production cross section. It turns out [9] that the cross section for the production of a pair of strongly interacting particles upon collision of two nucleons at accelerator energies depends nearly exponentially on the particle mass and decreases by approximately five orders of magnitude when the mass is increased by a single nucleon mass m_N. We shall show that this experimental result agrees well with the thermodynamic theory of the compound process and is explained by competition of the many-meson production process. As applied to the quark problem, it signifies that if the quark mass m_q exceeds $2.5m_N$, then the cross section for quark production is exceedingly small (less than 10^{-12} of the geometric cross section).

a) Experimental foundations. The experiments usually consist of registering the ratio of the number of antiprotons and antideuterons, $n_{\bar{p}}$ and $n_{\bar{d}}$, to the number of π^- mesons n_{π^-}. Thus, for example, in a collision between a proton and a Be nucleus, if we take the data for an initial energy $E_{lab} = 30$ GeV at an emission angle $\theta_{lab} = 4.5°$ and a secondary-particle momentum $p = 5$ GeV/c, then [10,11]

$$n_{\bar{p}}/n_{\pi^-} = (1 \pm 0{,}1)\cdot 10^{-2}, \quad n_{\bar{d}}/n_{\pi^-} = (5{,}5 \pm 1{,}5)\cdot 10^{-8}, \quad (3)$$

$$(d^2\sigma/d\Omega dp)_{\bar{d}} = 7\cdot 10^{-33}\ \mathrm{cm^2 sr^{-1} (GeV/c)^{-1}} \quad (3a)$$

When the values of p, θ_{lab}, and E_{lab} change, these data change little, and are practically the same also for the pN collision. At any rate, they are characteristic, and the uncertainty in the indicated parameters can change the result of interest to us by not more than one order of magnitude; we are dealing, however, with more significant effects. The greatest effect is produced by the closeness to the production threshold. However, say for $\sigma_{\bar{d}}$, when E_{lab} is reduced to 19.2 GeV/c, so that the excess over the threshold of the $d\bar{d}$ pair production decreases in the c.m.s. to 0.5 GeV, the cross section (3a) is decreased by only a factor of 4—5 [12].

In order to exclude the influence of the spin and isospin factors, we shall henceforth refer all the cross sections to a certain standard particle of the same mass, by dividing by certain weight factors. When $\bar{p}$ is generated paired with some N, the weight factor is $g_{\bar{p}N} = 2 \times 4 = 8$; for π^-, $g_{\pi^-} = 1$; for $\bar{d}$ in conjunction with any two nucleons, $g_{\bar{d}NN} = 3 \times 4 \times 4 = 48$; for the $q\bar{q}$ pair $g_{\bar{q}q} = (g_q)^2 = 6 \times 6 = 36$, etc. Starting from the data of (3) and (3a), for example by multiplying $n_{\bar{p}}/n_{\pi^-}$, where σ is the cross section for inelastic nucleon collision, $\sigma \sim \sigma_0 \sim$ mb, and n_{π^-} is the average number of π^- mesons per interaction act, $n_{\pi^-} \sim 2$—3, and also approximately estimating σ_d from (3a) for control purposes (both results coincide), we obtain the experimental values of the production cross sections σ_p and σ_d per pN collision act.

We further take into account the fact that, in spite of intuitive notions, $\sigma_{\bar{d}}$ differs only by a factor $1/6$ - 1 from the cross section for the production of a point like particle of the same mass [13]. This is connected with the fact that in the c.m.s. the $\bar{N}$ (like other particles) are produced with momenta on the order of several hundred MeV, and readily produce $\bar{d}$ (cf. the large cross section of the reaction $p + p \to d + \pi$ at a kinetic energy of the same order). As a result we obtain experimental values for the ratio of the number of pairs to the number of π^- mesons and for the production cross sections, which depend only on the mass of the generated particle m_q:

m_q/m_N	1	2
$\dfrac{n_{q\bar{q}}}{n_{\pi^-}}\dfrac{g_{\pi^-}}{g_{q\bar{q}}}$	$\dfrac{n_{\bar{p}}}{n_{\pi^-}}\dfrac{g_{\pi^-}}{g_{\bar{p}N}} = (1 \pm 0{,}1)\cdot 10^{-3}$	$(1-6)\dfrac{n_{\bar{d}}}{n_{\pi^-}}\dfrac{g_{\pi^-}}{g_{\bar{d}NN}} = (0{,}1-1)\cdot 10^{-8}$
$\dfrac{\sigma_q}{g_{q\bar{q}}}$	$(6-2)\cdot 10^{-29}$ cm^2	$(0{,}5-5)\cdot 10^{-34}$ cm^2

These conclusions can also be applied to quarks, if it is assumed that the quarks interact with nucleons and pions at distances from about m_N^{-1} to μ^{-1}, the same as any other strongly-interacting particles. In fact, a virtual decay $q \to q + (q + \bar{q}) \equiv q + \pi \to q$ is possible. Consequently, the quark should have the usual pion shell (and also other usual shells of smaller radius).

In Fig. 1 we have drawn through the pairs of experimental points for m_q/m_N, equal to 1 and 2, the following interpolation curves

$$1{:}\quad \frac{n_q}{g_{q\bar{q}}\, n_{\pi^-}} = A\left(\frac{m_q}{T_c}\right)^3 \exp\left(-\frac{2m_q}{T_c}\right), A = 6,\ T_c = 0{,}93\mu, \quad (4a)$$

$$2{:}\quad \frac{\sigma_q}{g_{q\bar{q}}} = a\left(\frac{m_q}{T_c}\right)^3 \exp\left(-\frac{2m_q}{T_c}\right),$$

$$a = 4\cdot 10^{-25}\ \mathrm{cm}^2,\ T_c = 0{,}94\mu. \quad (4b)$$

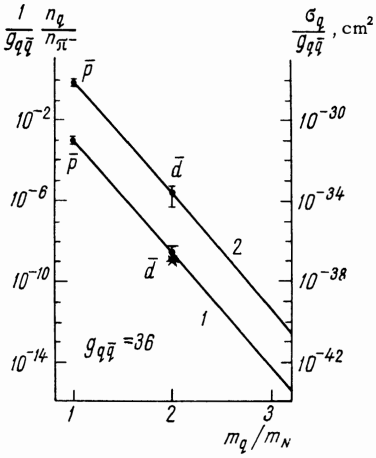

FIG. 1

The choice of the type of interpolation formula is governed by the theoretical considerations presented below.

b) Theoretical considerations. The experimental fact that the cross section decreases by 5—6 orders of magnitude when the particle mass changes from m_N to $2m_N$ can be understood as being the result of statistical competition between many-pion final states: when a pair of energy $2m_q$ is replaced by a number of pions of equivalent energy $(2m_N/\mu)(m_q/m_N) \sim 13m_q/m_N$, the phase volume increases sharply (in the thermodynamic analysis like $\exp(\Delta S)$, where ΔS is the change of the entropy). Since S is proportional to the particle number n, a factor of the type $\exp(k\Delta n) \sim \exp(2km_q/\mu)$, distinguishing between the probabilities of the two final states ($k \sim 1$), can appear when the $q\bar{q}$ pair is replaced by pions.

We obtain a theoretical formula more consistently by taking into account the fact that the generated particles move apart, interacting and experiencing transformations so long as their mutual distances do not exceed the force radius μ^{-1} (strictly speaking, it is necessary to choose distances in the rest system of each subgroup of neighboring particles; at accelerator energies this is still not very important). In such a state, the system consists of many weakly-interacting particles, i.e., is a gas whose temperature (which is of the order of the relative energy of the neighboring particles in the rest system of the given subgroup) is determined by those momenta at which the particles (principally pions) still interact strongly, $T \sim p \sim \mu$. This gives the order of magnitude of the critical temperature for the disintegration of the system into individual particles, $T_c \sim \mu$. However, knowledge of the order of magnitude of this quantity is, of course, not sufficient. The value of T_c cannot be determined more accurately theoretically. We shall obtain it later by comparison with experiment.

The distribution by particle kinds (masses) and momenta is indicated by the formulas of the Bose and Fermi statistics. For example, the number of pions is [14,15]

$$n_\pi = \frac{g_\pi}{(2\pi)^3}\int \frac{Vd^3p}{e^{\varepsilon_\pi/T_c} - 1} = \frac{g_\pi T_c^3}{2\pi^2} F_-\left(\frac{\mu}{T_c}\right) V;$$

$$\varepsilon_\pi = \sqrt{\mu^2 + p^2}, \quad F_-(z) = z^3 \int_0^\infty \left(e^{z\sqrt{1+x^2}} - 1\right)^{-1} x^2\, dx; \quad (5a)$$

the function $F_-(z)$ has been tabulated [13], $F_-(1) \approx 2$, and V is the summary volume of all the elements of the system (in their rest systems) at the instant of decay, when $T = T_c$.

We thus use here the fundamental idea of the Heisenberg-Landau hydrodynamic theory, which is valid for the disintegration of any strongly-interacting aggregate of meson-nucleon matter, for example in head-on NN interactions or for a center strongly excited by peripheral interaction. But we need no other details of this theory, neither the multiplicity nor the dynamics of the scattering, nor the equation of state. From this we can obtain n_q/n_{π^-}. However, a distinction must be made between the case $n_q \gg 1$, which was analyzed earlier [14,15] (in which case

$$n_q = \frac{1}{(2\pi)^3}\int \frac{g_q V d^3p}{e^{\varepsilon_q/T_c} + 1}, \quad \varepsilon_q = \sqrt{p_q^2 + m_q^2}\Big),$$

and the case $n_q \lesssim 1$, which corresponds to our situation and to which the earlier formulas are not applicable, since we are dealing with a rare fluctuation, whose probability must be calculated differently. It is determined, for example, from the Gibbs distribution by

$$n_{q\bar{q}} = g_{q\bar{q}} K \iint e^{-(\varepsilon_q + \varepsilon_{\bar{q}})/T_c} \frac{V^2 d^3p_q d^3p_{\bar{q}}}{(2\pi)^6}, \quad \varepsilon_{\bar{q}} = \sqrt{p_{\bar{q}}^2 + m_{\bar{q}}^2}, \quad (5b)$$

where K is the product of the integrals with respect to the variables describing the simultaneously produced mesons. If the fluctuation is small, then $K \approx 1$.

Expressing V from (5a) in terms of n_π, we find that for $T_c = 0.93\mu$ (see below) $g_\pi = 3$ ($n_\pi = n_{\pi^+} + n_{\pi^-} + n_{\pi^0}$)

$$V = V_c \approx \frac{1{,}24\pi}{4} n_\pi V_0 \approx n_\pi V_0, \quad V_0 = \frac{4}{3}\frac{\pi}{\mu^3}. \quad (5c)$$

Thus we have

$$n_q = \frac{g_q}{g_\pi}\frac{\sqrt{\pi/2}\, n_\pi}{F_-(\mu/T_c)}\left(\frac{m_q}{T_c}\right)^{3/2} e^{-m_q/T_c}, \quad n_q \gg 1, \quad (6)$$

$$n_q = \left(\frac{g_q}{g_\pi}\right)^2 \frac{{}^1\!/_2 \pi n_\pi^2}{[F_-(\mu/T_c)]^2}\left(\frac{m_q}{T_c}\right)^3 e^{-2m_q/T_c}, \quad n_q \lesssim 1 \quad (7)$$

(in summing the number of particles q and $\bar{q}$ we must multiply both formulas by 2)[2)].

The experimental data for $m_q = m_N$ and $2m_N$ correspond to $n_q < 1$ (see Fig. 1). It is obvious that the agreement with experiment is very good[3)]. Indeed, $T_{c.exp} \sim \mu$ and for $n_\pi \sim 6-9$ and $g_\pi = 3$ we have $A_{theor} \sim 1$. All this corresponds to a head-on collision, for the cross section of which we can assume $\sigma^c \sim 0.1\,\sigma_0$.

Even at very large n_π, for example upon collision of a Ca nucleus (with energy $E_{lab} > 10^{12}$ eV per nucleon) in emulsion [17], we obtain, using (6) when $n_q \gg 1$ and (7) when $n_q \sim 1$, for $m_q/m_N = 1$, 2, and 3 the respective values $n_q \approx 12$, 0.6×10^{-3}, and 1×10^{-9}.

3. SPECIAL CASE: THE PROCESS $N\bar{N} \to q\bar{q}$

A special analysis is required for the case of "inelastic scattering" $N + \bar{N} \to q + \bar{q}$ without a pion accompaniment, when a calculation by means of formulas (5a)—(7), which presuppose equilibrium with a large number of mesons, is not valid.

By $q\bar{q}$ we mean a pair of any strongly interacting particles: $N\bar{N}$, $\pi^+\pi^-$, etc., and particularly a quark and an antiquark. A system produced in a head-on $N\bar{N}$ or NN collision can decay into only two particles only in the initial stage of the expansion of the cluster, so long as $V \lesssim V_0$; during the stage when $V \gg V_0$, if one component of the pair is emitted, then it is not probable that the remaining particles of the cluster, which scatter apart to distances larger than the interaction radius, will again produce a single nucleon without accompaniment. Accordingly, a statistical calculation [5] as well as thermodynamic estimates [6,7] afford a splendid description of the statistical pp → pp scattering through 90° in the c.m.s., when it is assumed that $V = V_0$. They lead to the formula

$$d\sigma / d\Omega \sim \exp\{-3{,}3(W - 2m_N)\} = \exp\{-(W - 2m_N) / T'\},$$

where $W = \sqrt{2m_N(E_{lab} + m_N)}$ is the total c.m.s. energy of the system, expressed in GeV. This formula corresponds to a decay temperature $T' \approx 2.2\mu$ [5,6]. At the same time, for a decay into many particles, occurring from a volume $V \approx n_\pi V_0$, comparison with experiment gave us $T_c \approx 0.93\mu$. This means that upon expansion from $\sim V_0$ to $V_c \approx n_\pi V_0$, where $n_\pi \sim 10$, the temperature drops by a factor ~2. This is what we should get from the Stefan-Boltzman formula

$$W/V \sim T^4, \quad T_c/T' \sim (V_0/V_c)^{1/4} \sim n_\pi^{-1/4} \sim 1/2.$$

Let us apply the same reasoning to the $N\bar{N} \to q\bar{q}$ process. The entire difference from the pp → pp process, from the point of view of the thermodynamic concept (according to which the probability of emission of only two particles is determined by the competition of the pion generation process), should reduce: a) to a replacement of $W - 2m_N$ in the exponential by W, since the energy going to pion production includes in the case of annihilation also the rest masses of $\bar{N}$ and N; b) to a multiplication by the ratio of the isotopic and spin weights g_{NN}/g_{pp} (for quarks it is equal to 36/9 = 4); and c) to a multiplication by the ratio of the momentum volumes.

Thus, for the transformation of $\bar{N}N$ into any pair of strongly interacting particles that are scattered (without accompaniment) at an angle $\theta_{cms} \sim \pi/2$, we obtain

$$\left(\frac{d\sigma^{(c)}}{d\Omega}\right)_{N\bar{N}\to q\bar{q}} = Q\left(\frac{d\sigma^{(c)}}{d\Omega}\right)_{pp\to pp},$$

$$Q = \frac{g_{q\bar{q}}}{g_{pp}}\frac{\sqrt{W^2 - 4m_q^2}}{\sqrt{W^2 - 4m_N^2}}e^{-2m_N/T'}$$

$$(T' \approx 2{,}2\mu,\ W = \sqrt{2m_N(E_{lab} + m_N)}). \qquad (8)$$

At sufficiently high energy, we can use for $(d\sigma^{(c)}/d\Omega)_{pp}$ the experimental value. For $W \lesssim 4m_N$, when diffraction can make a contribution to the experimental value, it is better to use the theoretical formula of [13] or else (which is the same) extrapolate the experimental curve from the region of large W. Corresponding examples are given in the table. According to this table, the cross section for quark production turns out to be

2)The fact that formula (7) is the square of (6) has a simple physical meaning. If $n_q \gg 1$, then (6) can be regarded not as the relative number of heavy particles, but as the probability of appearance of a single q-particle. If they can appear only in pairs, then the probability of such a process is equal to the square of (6), which is just what we have in (7).

3)Domokos and Fulton [16] calculated σ_q in accord with the statistical model in purely theoretical fashion, without resorting to the experimental data [11,12], and specified V arbitrarily as $V \sim V_0$. They subsequently used the formula (6), which does not apply here. As a result they obtained $T_c \approx 2\mu$ and the exponent in the exponential relation $\sigma_q \approx \exp(-m_q)$ turned out to be underestimated by a factor ~4.

Values of Q

Energy $\begin{cases} W \\ E_{lab} \end{cases}$	$3{,}74m_N$ $6m_N = 5{,}6$GeV	$4{,}70m_N$ $10m_N = 9{,}4$GeV	$7{,}88m_N$ $30m_N = 28{,}2$GeV	$\frac{g_{\bar{q}q}}{g_{pp}}$
$\pi^+\pi^-$ or $2\pi^0$	$6.5\cdot 10^{-4}$	$6.05\cdot 10^{-4}$	$5.66\cdot 10^{-4}$	$1/4$
$N\bar{N}$	$2{,}2\cdot 10^{-3}$	$2{,}2\cdot 10^{-3}$	$2{,}2\cdot 10^{-3}$	1
$\Sigma^-\bar{\Sigma}^-$	$1.9\cdot 10^{-3}$	$2\cdot 10^{-3}$	$2.15\cdot 10^{-3}$	1
Quark-antiquark $m_q = 2m_N$	0	$1.1\cdot 10^{-2}$	$1.8\cdot 10^{-2}$	9
Quark-antiquark $m_q = 3m_N$	0	0	$1.3\cdot 10^{-2}$	9
$\left(\frac{d\sigma}{d\Omega}\right)_{pp\to pp}$, cm^2 при $\theta_{cms} \sim \pi/2$	$1.5\cdot 10^{-30}$	10^{-31}	$2\cdot 10^{-36}$	

much larger in such a process then in the process $NN \to q\bar{q} + n\pi$ (Sec. 2).

For $E_{lab} = 30$ GeV and $m_q = 3m_N$ we get

$$\sigma_q \sim 4\pi (d\sigma/d\Omega)_{N\bar{N}\to q\bar{q}} \sim 0{,}3\cdot 10^{-36}\text{cm}^2,$$

whereas Fig. 1 yields $\sigma_q \sim 10^{-38}$ cm^2.

From this analysis we can draw a general conclusion: Heavy pairs of strongly interacting particles should be produced with a cross section greatly exceeding (7), in the case of $N\bar{N}$ collisions at an energy close to the threshold of production of these pairs.

Unfortunately, this has no practical value at the contemporary methods of obtaining fast $\bar{p}$; the high-energy $\bar{p}$ themselves are obtained from pN collisions at exponentially small intensity (if $\epsilon_p = W/2 - m_N > T_c$, then m_N in (7) must be replaced by $\epsilon_{\bar{p}}$). However, if beams of fast $\bar{p}$ become attainable by acceleration of slow $\bar{p}$, which in accord with (7) are produced more effectively, then the indicated property of the $NN \to qq$ reaction may turn out to be useful.

4. ROLE OF DEVIATIONS FROM EQUILIBRIUM

In the calculations of the thermodynamic probabilities in Sec. 2 we started from the assumption that an equilibrium compound state is established upon collision and that ideal equilibrium is maintained at each instant during the course of the expansion. Actually, the system has small dimensions and expands rapidly. Only the success of such a crude theory when it comes to describing $\bar{p}$ and $\bar{d}$ production allows us to depend on it. Actually, however, the collision may turn out to be direct, and in particular peripheral. On the other hand, even in the compound state, the produced heavy pairs may escape during early stages of the expansion, when $T \sim m_N$, and accordingly the equilibrium number of pairs is large.

Even if the deviations from the considered equilibrium scheme are relatively small, on going to larger masses - at $m_q \gtrsim 3m_N$, when σ_q is very small, they can still turn out to be appreciable and lead to values of σ_q exceeding those corresponding to (7). We have no experimental data on the production of pairs with $m_q > 2m_N$. However, formula (8) reflects precisely the fact of escape during the early stage of the expansion, when $V \lesssim V_0$. When $m_q/m_N \sim 3$ it gives a value which is larger by two orders of magnitude than shown in Fig. 1 (if we apply formulas (4) and (7) to $N\bar{N}$ collisions).

We can therefore conclude that escape during this and later stages can lead to a σ_q which shifts the results from formula (7) to a formula such as (8). In applying these considerations to the NN collision, it is necessary to leave out from (8) the factor $\exp(-3.3\times 2) \approx 2\times 10^{-3}$. Therefore, the extreme limits of the true value of σ_q in NN collisions are

$$A \lesssim \sigma_q \lesssim B, \qquad (9)$$

where A follows from (7) and B from formula (8) multiplied by $\sim$500.

Let us consider separately certain effects that play an important role here.

a) Peripheral character of the collisions. If the collision is not head-on but occurs via exchange of a particle (say a pion), then two or more excited centers are produced, to each of which we can apply everything said in Secs. 2 and 3 con-

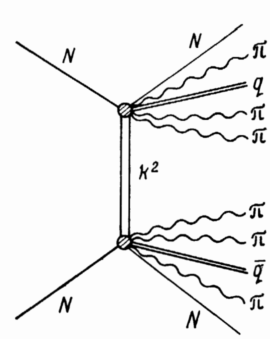

FIG. 2

cerning head-on collisions[4)]. In order for a peripheral collision to take place, the transferred 4-momentum squared k^2 should be $\lesssim \mu^2$ (if $k^2 \gg \mu^2$, then multiparticle exchanges are equally probable, and the collision becomes close to head-on). We can be interested only in the special case when quark exchange takes place in a peripheral collision ($k^2 \lesssim \mu^2$), so that the q and $\bar{q}$ of the same pair turn out to be in different excited aggregates after the collision and cannot annihilate (Fig. 2).

Of course, dependable calculations for such a diagram are impossible. But we can attempt to estimate the matrix element by means of the formula

$$M_q \sim F \frac{1}{m_q^2 + k^2} F, \tag{10}$$

where F are the form factors for qN collision, F = 1 if the virtuality of the transferred q is equal to zero, i.e., if $k^2 = -m_q^2$, and is a decreasing function of the virtuality $k^2 + m_q^2$ of the transferred particle. If we assume (of course, quite arbitrarily) that this function is the same algebraic function as for the Hofstadter electron-proton collision, then $F \sim (k^2 + m_q^2)^{-1}$. Recognizing that μ^2 and $k^2 \ll m_q^2$ in our case, we get

$$d\sigma / d\Omega \sim |M_q|^2 \sim (m_q^2)^{-6}. \tag{11}$$

Thus, the dependence of the cross section on m_q is not exponential, as in central collisions, but algebraic, but the degree of m_q is very high. From dimensional considerations we can expect

$$\frac{d\sigma_q}{d\Omega} \sim \sigma_0 \frac{1}{4\pi} \cdot \frac{\mu^\lambda m_N^{12-\lambda}}{m_q^{12}} = \frac{\sigma_0}{4\pi} \left(\frac{\mu}{m_N}\right)^\lambda \left(\frac{m_N}{m_q}\right)^{12}, \tag{12}$$

where λ is a certain number. Backward $\pi^- p$ scattering also corresponds to a diagram with nucleon exchange, reminiscent of Fig. 2. Experiment yields for this case $(d\sigma/d\Omega)_{\theta_{cms} \sim \pi} \sim 10^{-2} (d\sigma/d\Omega)_{\theta_{cms} \sim 0}$, which prompts us to assume that $\lambda \sim 2$.

Thus, introducing the g-factors and the solid angle Ω_0 of the rear cone, we have

$$\sigma_q \sim \sigma_0 \cdot 10^{-2} \frac{g_{\bar{q}q}}{g_{\pi^- p}} \left(\frac{m_N}{m_q}\right)^{12} \frac{\Omega_0}{4\pi}. \tag{12a}$$

From $m_q = 2m_N$ and $g_{\pi^- p} = 2$ this yields $\sigma_q/g_{qq} \sim 10^{-6} \sigma_0 \Omega_0/4\pi$. In Fig. 1 we have $\sigma_q/g_{q\bar{q}} \sim 10^{-8} \sigma_0$, which coincides with the preceding value when $\Omega_0/4\pi \sim 10^{-2}$; but even when $m_q = 3m_N$ formula (12a) yields $\sigma_q/g_{q\bar{q}} \sim 10^{-8} \Omega_0 \sigma_0/4\pi$ while Fig. 1 yields $\sim 10^{-14} \sigma_0$. This indicates that when $m_q \gtrsim 2m_N$ the cross section can lie above the curve of Fig. 1. There is no need to emphasize how unreliable these estimates are.

b) Leakage from the cluster. To maintain equilibrium in the system, the quark free path time τ with respect to scattering should be much shorter than the time of flight of the particle through the system. Even a very crude estimate of this condition is possible only at the later stage, when one can speak to some degree of collision in a gas of particles. If the cross section of the $g\pi$ scattering is $\sigma^{(q\pi)}$, then we should have

$$\tau = \frac{V}{v n_\pi \sigma^{(q\pi)}} \ll \frac{1}{v} V^{1/3} \tag{13}$$

Here v is the velocity of the quark, n_π/V is the density of the scattering pions. Since $V \sim n$ $V \sim n_\pi (4/3) \pi \mu^{-3}$, this yields

$$\sigma^{(q\pi)} \gg n_\pi^{-1/3} \sigma_0. \tag{14}$$

Inasmuch as $n_\pi \sim 10$, this is a very stringent condition. Therefore, if in spite of the initial premise the cross section for the interaction between the quarks and the pions is smaller than the geometric cross section, then the possibility of leakage may turn out to be considerable.

This results in a rather paradoxical situation: if the quarks interact with hadrons, at an approximate distance from $\sim m_N^{-1}$ to μ^{-1}, with the same intensity as the hadrons themselves, then the cross section for quark production is exceedingly small. On the other hand, if they interact more weakly (perhaps it would be sufficient to reduce the interaction by one or two orders of magnitude), then, being produced at high temperature, they can

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4)There are several published calculations of the cross section for the production of a quark-antiquark pair in πN and NN collisions, based on a peripheral diagram in which the initial particles, as a result of pion exchange or diffraction scattering by each other, are excited to a mass exceeding $2m_q$, and then disintegrate [18,19]. Sometimes it is concretely assumed that the intermediate decaying particle is a ρ meson. However, since this particle is actually very far from the mass shell of the ρ meson, it has little in common with the decay properties of either the ρ meson or of other stable particles of small mass. We therefore assume that the "ρ" vertex of $q\bar{q}$ cannot be taken from any data on the decay of such particles (by "ρ" we mean any intermediate particle which decays into the final particles $q\bar{q}$). More to the point seems to be calculation of the "ρ" vertex of $q\bar{q}$ in accordance with the same statistical theory which we have employed above. Here again, owing to the competition of the "ρ" $\to n\pi$ decay, $n \gtrsim 2m_q/\mu$, an exponentially small factor arises, of the type determined in the text, Therefore the relatively large cross sections obtained in the cited papers (on the order of microbarns and higher) should be decreased by introducing exponentially small time-like form factors in the decay vertices. As a result we return again to formulas of the type (7).

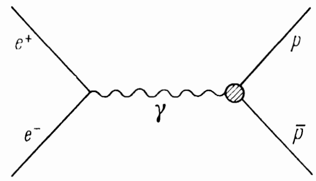

FIG. 3

escape subsequent annihilation and their production cross section will be large. Obviously, any quantitative estimates would be unreliable here.

5. ELECTROMAGNETIC PRODUCTION AND ANNIHILATION OF HEAVY PAIRS

If we assume that the production of pairs of heavy particles proceeds via a compound state (via a head-on collision) and is controlled by processes of a statistical type, then the results obtained are essentially valid for any process of heavy-pair production, particularly for the electromagnetic process.

Let us consider $p\bar{p}$ production, for example, in annihilation of an electron pair (Fig. 3). Then the 4-momentum k transferred by the γ quantum is timelike. In the c.m.s. we have $k = (W, 0, 0, 0)$, where W is the c.m.s. pair energy. We do not know the form factor of the $\gamma p\bar{p}$ vertex. If, however, we consider this vertex in accordance with the $\gamma \to \pi^+\pi^- \to p\bar{p}$ scheme or in accordance with some similar scheme, then it becomes obvious that it contains in the $\pi^+\pi^- \to p\bar{p}$ section precisely that process which we considered in Sec. 3. It proceeds via a compound state, for the decay of which the nature of the primary particles is immaterial.

For the square of the matrix element of the process we can write

$$|M|^2 = |M_0|^2 e^{-3.3W} = |M_0|^2 e^{-\sqrt{k^2}/T_c'}, \quad T_c' \approx 2.2\mu, \quad (15)$$

where M_0 is the matrix element of the process for a pointlike proton, and W is in GeV. We can also state it differently: In calculating the pair production under the assumption that the proton is pointlike, it is necessary to take into account the fact that the components of the pair enter into an interaction during the course of which competition arises with the $p\bar{p} \to n\pi$ process. The probability that only p and $\bar{p}$ will be emitted is determined by the factor exp (−3.3W) (see Sec. 3).

Starting from this, we can find the form factor for the inverse process $p\bar{p} \to \gamma \to e^+e^-$ (or $p\bar{p} \to \gamma \to \mu^+\mu^-$). Since $|M^2|$ are the same for the direct inverse processes, we have

$$d\sigma_{p\bar{p}\to e^+e^-} = (d\sigma^0)_{p\bar{p}\to e^+e^-}|F|^2, \quad |F|^2 = e^{-\sqrt{k^2}/T'_K} \quad (16)$$

(the same holds for $p\bar{p} \to \mu^+\mu^-$), where $d\sigma^0$ is the cross section calculated for pointlike nucleons. Upon collision between an antiproton with momentum $p_{lab} = 2.5$ GeV/c with a proton at rest we get $\sqrt{k^2} = W \approx 2.6$ GeV and $\exp(-3.3W) \sim 2 \times 10^{-4}$. The latest experimental data [20] on this process lead to $|F|^2 < 2.2 \times 10^{-3}$ (at such small values of W the statistical formula may be very inaccurate).

Direct experiments of photogeneration of antiprotons were carried out [21] for γ quanta using H and N nuclei. This process deserves a separate consideration. We note for the time being that in any case the cross section for the $p\bar{p}$ pair production in the $\gamma p\bar{p}$ vertex is lower than calculated for pointlike p and $\bar{p}$ by a factor on the order of $(m_N/T_c')^3 \exp(-2m_N/T_c')$, i.e., by a factor of approximately 10^2 (this factor, and not formula (8), should be taken because the number of pions was not fixed in these experiments).

In conclusion we wish to express sincere gratitude to Ya. B. Zel'dovich, who called our attention to the quark problem, for interesting discussions.

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