ELASTIC ed SCATTERING AND CP-INVARIANCE VIOLATION. II.

V. M. DUBOVIK, E. P. LIKHTMAN[1)], and A. A. CHESHKOV[1)]

Joint Institute for Nuclear Research

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We analyze in detail the possibility of checking CP-invariance violation in electromagnetic hadron interaction by determining correlation effects in elastic ed scattering. We show in the single-photon approximation that when T-invariance is violated in scattering of electrons by a target polarized perpendicular to the scattering plane, the angular asymmetry of the recoil deuterons can reach ~ 5% at a squared momentum transfer $q^2 = 12\ F^{-2}$ and $\theta_d \approx 20°$. It is estimated that the T-invariant asymmetry, which results from the interference of one- and two-photon amplitudes, is too small (0.02% at the indicated values of q^2 and θ_d) to interfere with the observation of the main effect.

1. INTRODUCTION

As noted in a number of papers[1-3], T-noninvariant effects in electromagnetic interactions of hadrons can be observed by scattering electrons from nuclei.

In our preceding paper[3] (henceforth cited as I) we considered the elastic scattering of relativistic electrons by deuterons and used the experimental data of Buchanan and Yearian[4] to present a preliminary estimate of the expected angular asymmetry of scattering by a polarized target.

We consider in this paper in detail the feasibility of the proposed experiment. We make use of the principal papers on the calculation of the electromagnetic structure of the deuteron[5,6]. We estimate the effects that hinder the observation of CP-invariance violation.

2. CORRELATION EFFECTS IN ed SCATTERING WITH VIOLATION OF T-INVARIANCE

In I we presented in the single-photon approximation the cross sections for scattering of unpolarized electrons by 1) unpolarized and 2) polarized deuteron targets. In case 1) the cross section is determined by the usual formula

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{unpol}}_{\text{lab}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left\{E + \frac{M}{2\eta} + M\,\mathrm{tg}^2\frac{\theta}{2}\right\}; \quad (1)$$

$$E = E_{10} + E_{12} = f_{10}^2(q^2) + {}^{32}/_9\xi^2 f_{12}^2(q^2),$$
$$M = M_{20} + M_{31} = {}^4/_3\xi\eta f_{20}^2(q^2) + {}^{64}/_3\xi^3 f_{31}^2(q^2),$$
$$\xi = q^2/4\varkappa^2, \quad \eta = 1 + \xi, \quad (2)$$

$\varkappa$ is the deuteron mass; f_{10}, f_{12}, f_{20}, and f_{21} are the charge, electric-quadrupole, magnetic-dipole, and second-kind magnetic quadrupole (T-noninvariant) deuteron form factors; θ is the l.s. scattering angle.

If the T-noninvariant form factor f_{31} differs from zero, then the recoil deuterons are polarized in a direction normal to the scattering plane and the degree of polarization P_0 turns out to be

$$P_0 = \frac{1}{2}\left[3E_{12}M_{31}\left(\frac{1}{2\eta} + \frac{1}{2}\,\mathrm{tg}^2\frac{\theta}{2}\right)\right]^{1/2} \Big/ \left[E + \frac{M}{2\eta} + M\,\mathrm{tg}^2\frac{\theta}{2}\right]. \quad (3)$$

The polarization is maximal at an electron-scattering angle given by

$$\mathrm{tg}^2(\theta_{max}/2) = E/M - 3/2\eta, \quad (4)$$

and is equal to

$$P_0^{max} = \frac{\sqrt{3}}{4}\left(\frac{E_{12}M_{31}}{2M(E - M/2\eta)}\right)^{1/2}. \quad (5)$$

In case (2) the polarization of the target leads to an angular scattering asymmetry a equal to

$$a(\theta) = \left|\frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(-\theta)\right| \Big/ \left[\frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma}{d\Omega}(-\theta)\right]. \quad (6)$$

The spin-state density of the target deuterons has in the general case the form

$$\rho = {}^1/_3\{I + {}^3/_2(P_l j_l) + Q_{ik}([j_i j_k]_+ - {}^4/_3\delta_{ik})\}, \quad (7)$$

[1)]Moscow Aviation Institute.

where P_l is the deuteron polarization vector, which varies in the range $0 \leq |\mathbf{P}| \leq 1$, and Q_{ik} is the deuteron alignment tensor.

In the purely polarized state (when $Q_{ik} = 0$) the polarization cannot exceed $|\mathbf{P}| = 2/3$. The maximum polarization $|\mathbf{P}| = 1$ is attained for the pure quantum state with spin projection on the **P** direction equal to +1. The alignment is then different from zero and is equal to

$$Q_{ik} = {}^3/_4 (P_i P_k - {}^1/_3 \delta_{ik}). \tag{8}$$

In the case of a purely polarized beam ($Q_{ik} = 0$) the asymmetry in the angular distribution of the scattered electrons has been calculated in I and found to be

$$a = \frac{3\sqrt{3}}{4}\left[E_{12}\frac{M_{31}}{2\eta}\left(1+\eta\,\mathrm{tg}^2\frac{\theta}{2}\right)\right]^{1/2}\left[E+\frac{M}{2\eta}+M\,\mathrm{tg}^2\frac{\theta}{2}\right]^{-1} \times |(\mathbf{P}_l\mathbf{N}_\perp)|, \tag{9}$$

where $\mathbf{N}_\perp$ is a unit vector perpendicular to the scattering plane. Thus the maximum asymmetry in case 2) is equal to the degree of polarization of the recoil deuterons (3) in case 1).

In the case of a maximally polarized beam ($|\mathbf{P}| = 1$) we have

$$a = \frac{3\sqrt{3}}{4}\left[E_{12}\frac{M_{31}}{2\eta}\left(1+\eta\,\mathrm{tg}^2\frac{\theta}{2}\right)\right]^{1/2}\left[E+\frac{M}{2\eta}+M\,\mathrm{tg}^2\frac{\theta}{2} + C_{\text{align}}\right]^{-1}. \tag{10}$$

The increase of the denominator in (10) is the result of a contribution from the alignment (8). The cross section for electron scattering from an aligned deuteron target in the single-photon approximation, with allowance for the terms violating T-invariance, is given in the Appendix.

3. ESTIMATE OF THE EFFECT

The contributions contained in (7) can be estimated by using a number of considerations given in[1-3]. Since non-T-invariant factors are generally forbidden for free particles with spin 1/2, their presence in nuclei can be due to two causes[1]: a) the nucleons in the nuclei are not on the mass shell (effect $\sim O(V/\kappa_N)$, where V is the potential energy), and b) exchange currents ($\sim O(q^2/\kappa_\pi^2)$. These effects of nucleon virtuality and the presence of exchange pion currents cause in general the nucleus to be non-elementary and it is natural to assume that the T-invariant and T-noninvariant form factors have the same order of smallness.[2]

In order to obtain an upper limit for the effect on a nucleus with spin j, we assume that for sufficiently large q^2 we have $E \approx E_{12}$, $M \approx M_{31}$, and $E \gg M$; then $a = \sqrt{3/2}\,[j(j+1)]^{-2}$. For the deuteron $a = {}^1/_4\sqrt{3/2}$.

In accordance with the experimental data for the deuteron[4], we can propose that $M_{31} \approx M/2$, which agrees with the general considerations. We then have

$$a = \alpha\sqrt{3}/8, \quad \alpha = [E_{12}/(E - M/2\eta)]^{1/2}. \tag{11}$$

The coefficient $\alpha(q^2)$ can be estimated from the results of Glendening and Kramer[5], where the electromagnetic form factors of the deuterons are calculated for different phenomenological nucleon-nucleon potentials.

The comparison of the results of that paper with the experimental data of Buchanan and Yearian[4] shows that the value of $\alpha(q^2)$ at $q^2 = 12\ \mathrm{F}^{-2}$ lies in the range 0.4—0.7. With this, the polarization (5) in case 1) and of the asymmetry at $q^2 = 12\ \mathrm{F}^{-2}$ and $\theta = 127°$ in case 2) lie in the range 3—6%; for a maximally polarized beam it follows from (10) that a = 2—5% for $\theta = 150°$.

4. ESTIMATE OF T-INVARIANT POLARIZATION OF RECOIL DEUTERONS

It is known[7] that when T-invariance is conserved the polarization of the recoil deuterons is proportional to the interference between the first and second Born amplitudes M_2 and M_4 and is given by the formula

$$P_T = 2i\frac{\mathrm{Sp}\,M_2^+(\mathbf{js})\,\mathrm{Im}\,M_4}{\mathrm{Sp}\,M_2^+M_2}, \tag{12}$$

where s is the direction of the recoil-deuteron spin and is normal to the scattering plane.

An upper-bound estimate of the T-invariant polarization P_T in large-angle scattering, with allowance for the weak q^2-dependence of the charge and dipole-magnetic form factors f_{10} and f_{20} at large ϑ[4] shows that

$$P_T \approx P_{\text{calc}} = j(j+1)\sqrt{\xi} f_{10}^2([p-k]^2) f_{20}(0)\,\mathrm{tg}^3\frac{\vartheta}{2} \times \ln\frac{1-\cos\vartheta}{2}\left\{24\cdot 137 f_{10}(0)\left[A + B\,\mathrm{tg}^2\frac{\vartheta}{2}\right]\sin\frac{\vartheta}{2}\right\}^{-1}. \tag{13}$$

In this expression ϑ is the c.m.s. scattering angle, but for small ξ it coincides with the l.s. scattering angle θ.

The maximum value of the T-noninvariant polarization occurs at $q^2 = 12\ \mathrm{F}^{-2}$ and $\theta = 127°$. The T-invariant polarization T_P calculated for these values of the scattering angle and momentum transfer by means of formula (13) is equal to 0.02%.

An estimate of the approximations made in the derivation of (13) shows that the true value of the polarization lies in the range $1.5P_{calc} \geq P_T \geq \geq \frac{1}{3}P_{calc}$. It must be noted that when q^2 increases the errors will increase linearly with $\xi^{1/2}$. However, since $P_0^{max} \gg P_T$, the crudeness of the approximations and of the assumptions[2)] made in the derivation of (13) is fully justified.

5. CONCLUSIONS

The main conclusion of the foregoing calculations is that when electrons are scattered by a target polarized normally to the scattering plane the T-noninvariant polarization of the recoil deuterons can exceed by many times (by more than two orders of magnitude) the polarization obtained from the second approximation (T-invariant). Thus, the process under consideration can be used to observe CP-nonconservation in electromagnetic interactions of hadrons. It is possible to dispense here with the positron experiments proposed for the compensation of the T-invariant polarization.[8]

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APPENDIX

We present for further reference the cross section, in the first Born approximation, for the scattering of relativistic electrons by an aligned vector target (deuteron), with allowance for possible violation of T-invariance in electromagnetic interactions.

The spin state of the aligned particles of the target is determined by the density matrix (7), where the polarization vector $\mathbf{P} = 0$. Then

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{align}}_{\text{lab}} = \left(\frac{d\sigma}{d\Omega}\right)^{\text{unpol}}_{\text{lab}} + \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} C_{\text{align}};$$

$$C_{\text{align}} = \frac{2}{3}\xi\,\{(QN_\perp N_\perp) + (QNN) + (QRR) + (QNR)\},$$

$$\mathbf{N} = \mathbf{p}'/|\mathbf{p}'|, \quad \mathbf{R} = [\mathbf{N}\mathbf{N}_\perp],$$

$$(QN_\perp N_\perp) = \left\{\frac{16}{3}\xi f_{12}^2 + 8f_{10}f_{12} + \left(2 + \eta\,\mathrm{tg}^2\frac{\theta}{2}\right)f_{20}^2 - 8\xi^2\,\mathrm{tg}^2\frac{\theta}{2}f_{31}^2\right\}Q_{ij}N_{\perp i}N_{\perp j};$$

$$(QNN) = \left\{-\frac{32}{3}\xi f_{12}^2 - 16f_{10}f_{12} - \left(1 + 2\eta\,\mathrm{tg}^2\frac{\theta}{2}\right)f_{20}^2 - 4\xi^2\left(\frac{3}{\eta} - 4\,\mathrm{tg}^2\frac{\theta}{2}\right)f_{31}^2\right\}Q_{ij}N_iN_j;$$

$$(QRR) = \left\{\frac{16}{3}\xi f_{12}^2 + 8f_{10}f_{12} - \left(1 - \eta\,\mathrm{tg}^2\frac{\theta}{2}\right)f_{20}^2 + 4\xi^2\left(\frac{3}{\eta} - 2\,\mathrm{tg}^2\frac{\theta}{2}\right)f_{31}^2\right\}Q_{ij}R_iR_j;$$

$$(QNN) = -3\,\mathrm{tg}\frac{\theta}{2}\xi^{1/2}\left(\eta + \mathrm{ctg}^2\frac{\theta}{2}\right)^{1/2} f_{12}f_{20}Q_{ij}N_iR_j.$$

The T-invariant part of the cross section for scattering by an aligned deuteron target agrees apart for form-factor normalization, with the calculations of Schildknecht[9], who discusses in[10] the influence of this part of the cross section on the T-invariant scattering asymmetry.

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2)Thus, for example, no account was taken of the diagram in which the deuteron in the intermediate state dissociate into a proton and neutron.