INDUCED SPLITTING OF PHOTONS IN AN ELECTROMAGNETIC FIELD

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We consider the interaction of two beams of photons of different frequency in the presence of a constant external electric field. We obtain the cross section of induced splitting of the higher-frequency photon as a result of fourth-order interaction. Numerical estimates are presented of the cross section for splitting in the Coulomb field of a charged sphere of finite radius.

RECENT progress in the production of intense laser beams makes timely the question of direct observation of nonlinear quantum-electrodynamics effects (due to "polarization" of vacuum) in the optical frequency range. The author has shown in an earlier paper[1] that the cross section for elastic scattering of photons can be greatly enhanced by inducing the process by a third beam of photons having the same frequency (by a factor 10^{10}—10^{15}).

The present paper considers the possibility of intensifying the splitting of the photons in an external constant field. To increase the reaction yield, another beam (of lower frequency) is used. As a result of the interaction of the two beams with the field, the higher-frequency photon can be converted into two photons, one of which is indistinguishable from the photons of the second beam, while the frequency of the second is equal to the difference of the frequencies of the photons of the primary beams.

The matrix element of the process under consideration can be readily obtained from the general expressions for the fourth-order interaction matrix (see[2,3]). Leaving out the straightforward but cumbersome derivations, we present explicit expressions for the cross sections of photon splitting in a constant electric field produced by a sphere of finite radius r_0 charged to a certain potential φ_0. (The case of the Coulomb field of the nucleus corresponds to the values $r_0 = 0$ and $\varphi_0 r_0 = Ze$, where $Ze/(4\pi)^{1/2}$ is the charge of the nucleus.)

1. The beam of photons of frequency ω_1 and the beam of photons of frequency $\omega_3 < \omega_1$ represent linearly polarized plane waves propagating in the same direction. The external scattering field is produced by a non-transparent sphere of radius $r_0 \gg 2\pi/\omega_1$, so that the beams interact before they encounter the sphere.

The differential yield of the splitting reaction, summed over the polarizations of the photon produced in the reaction and normalized to unity flux of photons of beam 1 (differential cross section) turns out in this case to be ($\hbar = c = 1$)

$$\frac{d\sigma}{dO} = \frac{1}{2\cdot(45)^2}\left(\frac{e^2}{4\pi}\right)^4 \frac{\nu_3}{m^5\Omega}\frac{\omega_1\omega_3(\omega_1-\omega_3)}{m^3}\left(\frac{\varphi_0 \sin qr_0}{q}\right)^2 \times \sin^2\theta B(\varphi), \qquad (1)$$

where m is the electron mass, $e^2/4\pi = 1/127$, Ω is the normalization volume in which the energy $\nu_3\omega_3$ of beam 3 is concentrated, θ is the angle of emission of the quantum of frequency $\omega_1 - \omega_3$ relative to the direction of beam 1, $q = 2(\omega_1 - \omega_3)\sin(\theta/2)$ is the momentum transferred by the field, φ is the azimuthal angle between the scattering plane and the primary-beam polarization plane, and $B(\varphi) = 9$ or $B(\varphi) = 16 + 33\sin^2\varphi$, depending on whether the photon polarization planes are in the same direction prior to the reaction or mutually perpendicular.

The total splitting cross section, summed over the polarizations of the produced photon and averaged over the polarizations of photons ω_1 and ω_3, is equal to

$$\sigma_t = \frac{83\pi}{8\cdot(45)^2}\left(\frac{e^2}{4\pi}\right)^4 \frac{\nu_3}{m^5\Omega}\frac{\omega_1\omega_3\varphi_0^2}{m^3(\omega_1-\omega_3)}. \qquad (2)$$

2. Two beams of photons with respective frequencies ω_1 and ω_3, in the form of linearly polarized plane waves, are polarized in the same plane and directed opposite to each other.

Calculations of the differential cross section of splitting of the photon ω_1, summed over the polarizations of the produced photon, lead in this case to the expression

$$\frac{d\sigma}{dO} = \frac{2}{(45)^2}\left(\frac{e^2}{4\pi}\right)^4 \frac{\nu_3}{m^5\Omega}\frac{\omega_1\omega_3(\omega_1-\omega_3)}{m^3}\left(\frac{2\varphi_0 \sin qr_0}{q}\right)^2 \times \frac{(\omega_1-\omega_3)^2\sin^2\theta}{q^4}\{(\omega_1+\omega_3)^2 + (\omega_1-\omega_3)^2 \times [\cos^2\theta + 33\sin^2\varphi + 11\sin^2\theta\sin^2\varphi(8-11\sin^2\varphi)] + 4(\omega_1^2-\omega_3^2)(8-11\sin^2\varphi)\cos\theta\}, \qquad (3)$$

where

$$q^2 = 2[\omega_1^2 + \omega_3^2 - (\omega_1^2 - \omega_3^2)\cos\theta].$$

Using the foregoing results, we present some numerical estimates of the yield of the splitting reaction. We specify, for concreteness, the beam and field parameters in the following manner: $2\pi/\omega_1 = 10^{-4}$ cm is the radiation wavelength in beam 1, $2\pi/(\omega_1 - \omega_3) = 10^{-2}$ cm is the wavelength of the photon produced in the reaction, $(4\pi)^{1/2}\varphi_0 = 10^6$ V, $r_0 = 10^{-2}$ cm, $s = 10^{-5}$ cm^2 is the cross-section area of beam 1, $N_1 = 5 \times 10^{21}\, p_1$ is the number of quanta in one pulse of beam 1 with radiation energy p_1 kJ, and respectively $N_3 = 5 \times 10^{21}\, p_3$, $T = 10^{-8}$ sec is the duration of the radiation pulse of beam 1 or 3, which determines the interaction time and the radiation density in the interaction volume. Let the beam 3 have the same direction as beam 1. Then, according to (2), the number of quanta n experiencing splitting, out of the total number of quanta in the beam, is

$$n \approx 0.8 \cdot 10^{-86}\,\mathrm{cm}^5\, Z^2 \frac{N_1}{s}\frac{N_3}{scT} = 3 \cdot 10^{-14} p_1 p_3,$$

where Z is the charge of the sphere in electron units and c is the speed of light.

This result exceeds by several orders of magnitude the yield of the reaction of coalescence of two photons in an external field, obtained by McKenna and Platzman[4] at approximately the same field geometry. The difference is due essentially to the fact that the splitting reaction takes place at lower momentum transfers, at which the Coulomb field is more intense ($q_{max} r_0 \sim 2\pi$). In spite of this, the additional enhancement of the number of the quanta produced in the reaction is very small.

An increase of the reaction yield by increasing the intensity of the constant field entails great technical difficulties. At the chosen values of φ_0 and r_0 the field intensity on the surface of the sphere is $E = 10^8$ V/cm. When the intensity is raised to 10^9 V/cm, surface atoms begin to become ionized even on a tungsten anode (evaporation by the field[5]). This process can apparently be slowed down significantly by applying to the sphere an alternating potential which reaches its maximum value for a time 10^{-7}—10^{-8} sec only. Nonetheless, fields stronger than 10^9 V/cm are hardly feasible.

It is likewise impossible to decrease noticeably the cross section of beam 1, since this leads to an increase of the angular dimensions of the photon beams, and consequently increases the interval of values of q leading to scattering at the given angle, whereas the presence of the factor $\sin qr_0$ in the expression for the potential imposes the limitation $\Delta q r_0 \lesssim 1$.

The principal difficulty in measuring such small scattered-quantum fluxes is the presence of the background from the splitting of photons by real electrons of the medium (field source, residual vacuum, etc.). By proper choice of the medium and of the geometric conditions it is possible to lower this background markedly. But the cross section for scattering by individual atoms is so large (10^{-52}—10^{-55} cm^2) that the background will always exceed the effect in the optical range.

Comparison with the earlier results[1] shows that the yield of the induced-splitting reaction remains considerably lower than the yield of the reaction of induced elastic scattering at the same light-beam intensities; this was already noted earlier.[1] The splitting (coalescence) process can become more probable than scattering only at lower frequencies, owing to the increase in the Coulomb-field intensity when the parameter q is small.

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