STATIONARY GENERATION IN AN OPTICAL RESONATOR WITH LENSES

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The mechanism, connected with the shape of the caustic surface, of the damping of relaxation oscillations in a spherical resonator is investigated. It is shown that under certain conditions the damping can be quite strong and can lead to stationary generation. The conclusions of the theory are confirmed by results of experimental investigations conducted with a laser having a ruby rod and lenses (similar to a spherical resonator).

1. INTRODUCTION

REGULAR relaxation oscillations of radiation generated in a spherical resonator were obtained and investigated by many authors.[1-4] Regular damped oscillations were investigated[5] in a plane-parallel resonator equipped with lenses and similar to a spherical resonator. The oscillation damping observed in the cited papers was so weak that no stationary state could set in during the time of the pump pulse. This experimental fact is in good agreement with the results of the theory[6,7].

It is shown in a number of papers, however,[8-11] that in some cases the radiation oscillations in a spherical resonator experience a strong damping which leads to establishment of a steady state during the greater part of the generation time. The mechanism of this damping has not been satisfactorily explained in any of the foregoing papers.

The present paper is devoted to an investigation of a new mechanism of relaxation-oscillation damping, which is much more effective under certain conditions then the presently known damping mechanism and makes it possible to observe stationary generation even in the case of pulsed pumping. This damping mechanism is connected with the singularities of the shape of the caustic surface within which the electromagnetic field is localized in a spherical resonator. The theory of this process is presented as applied to a spherical resonator, and the experiments were carried out on a resonator with lenses, which is similar to a spherical resonator but is much more convenient from the point of view of the experimental technique.

As shown by Vaĭnshteĭn[12], the electromagnetic field in a spherical resonator is concentrated inside a caustic surface whose equation, in cylindrical coordinates, is

$$\frac{r^2}{a^2} - \frac{2z^2}{lR} = 1 - \frac{l}{2R}. \tag{1}$$

Here R is the radius of curvature of the reflectors, and a and l are the radius of the reflectors[1)] and the distances between them. The position of the axes r and z is shown in Fig. 1.

We shall assume for simplicity that the intensity of the generated light is uniformly distributed over the cross section inside the caustic surface and is equal to zero outside this surface. (The question of the validity of this assumption will be discussed in Sec. 4.) Then the intensity of the light I(z, t) is inversely proportional to the cross section S(z) and can be represented in the form

$$I(z, t) = I(0, t)\frac{S(0)}{S(z)} = \frac{\rho^2 I(0, t)}{\rho^2 + 4z^2/l^2}, \tag{2}$$

where

$$\rho^2 = 2R/l - 1. \tag{3}$$

Formula (2) can be extended to include the entire

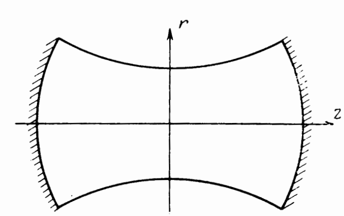

FIG. 1

1)In the case of a resonator with external mirrors, a must be taken to be the radius of the region with negative absorption coefficient.

interval $-l/2 \leq z \leq l/2$, if we neglect the sag of the reflectors compared with l.

In calculating the generation kinetics it is usually assumed that the intensity I depends only on the time t. In our case I depends not only on t but also on z. If $\rho \gtrsim 1$, then allowance for the latter dependence does not lead to essentially new results; but if $\rho \ll 1$, then a sharp dependence of I on z leads to a strong damping of the intensity oscillations.

2. KINETIC EQUATIONS

We denote by τ the lifetime of the photon in the unexcited resonator, and write for the gain, when the light is amplified by an excited active medium,

$$n/n^*\tau v, \tag{4}$$

where n is the number of excited luminescence centers per unit volume[2)], n^* the threshold value of n, and v the speed of light in the medium. The proportionality coefficient for n in (4) is chosen such that when the generation threshold is reached ($n = n^*$) the gain compensates for the light loss, the magnitude of which per unit length is $1/\tau v$.

Taking (2) and (4) into account, we write the balance equation for the number of excited atoms:

$$\hbar\omega_0 \frac{\partial n(z, t)}{\partial t} = N - \frac{n\hbar\omega_0}{T} - \frac{n}{n^*\tau}\frac{\rho^2 I(0, t)}{\rho^2 + 4z^2/l^2}. \tag{5}$$

Here ω_0 is the laser operating frequency, N the pump power absorbed per unit volume, and T the time of spontaneous de-excitation. The first term in the right side describes the excitation of the atoms by the pump, the second the spontaneous de-excitation, and the third the induced de-excitation of the excited atoms (the last term is the power drawn by the light from the excited active medium).

We also write the equation for the light intensity:

$$\frac{\partial I(0, t)}{\partial t} = I(0, t)\left[-\frac{1}{\tau} + \frac{\bar{n}(t)}{n^*\tau}\right], \tag{6}$$

$$\bar{n}(t) = \int_{-l/2}^{l/2} n(z, t)\frac{dz}{l}. \tag{7}$$

Equation (6) has been written out for $I(0, t)$ accurate to within a constant factor equal to the total optical energy in the resonator. This equation contains the gain averaged over the resonator length, since light propagating at small angles to the optical axis traverses equal intervals of z in equal time intervals.

Changing over in the kinetic equations (5) and (6) to more convenient dimensionless variables

$$\nu(z, t) = [n(z, t) - n^*]/n^*, \quad w(t) = \rho^2 I(0, t)T/\tau\hbar\omega_0 n^*, \tag{8}$$

we rewrite these equations in the form

$$T\frac{\partial \nu}{\partial t} = \zeta - \frac{w}{\rho^2 + 4z^2/l^2} - \nu\left(1 + \frac{w}{\rho^2 + 4z^2/l^2}\right), \tag{9}$$

$$\tau dw/dt = w\bar{\nu}(t). \tag{10}$$

Here

$$\bar{\nu}(t) = \int_{-l/2}^{l/2} \nu(z, t)\frac{dz}{l}, \tag{11}$$

ζ is the relative excess pump power above threshold.

We consider first a stationary mode, in which the left sides of (9) and (10) vanish. We denote by w_0 and $\nu_0(z)$ the stationary values of w and ν. From (9) we have

$$\nu_0(z) = \frac{\zeta(\rho^2 + 4z^2/l^2) - w_0}{\rho^2 + 4z^2/l^2 + w_0}. \tag{12}$$

Recognizing that $\bar{\nu}_0 = 0$, we get for the constant w_0 the equation

$$\int_{-l/2}^{l/2} \nu_0(z)\frac{dz}{l} \equiv \zeta - \frac{w_0(\zeta + 1)}{\sqrt{\rho^2 + w_0}}\operatorname{arctg}\sqrt{\rho^2 + w_0} = 0. \tag{13}$$

We now consider small oscillations of the intensity. We put

$$\nu(z, t) = \nu_0(z) + \nu_1(z)e^{i\Omega t}, \quad w(t) = w_0 + w_1 e^{i\Omega t}, \tag{14}$$

where ν_1 and w_1 are small oscillation amplitudes. Substituting this in Eqs. (9) and (10) and linearizing the latter with respect to ν_1 and w_1, we obtain equations that are homogeneous in ν_1 and w_1:

$$-i\Omega T\nu_1 = \frac{\nu_0(z) + 1}{\rho^2 + 4z^2/l^2}w_1 + \left[1 + \frac{w_0}{\rho^2 + 4z^2/l^2}\right]\nu_1, \tag{15}$$

$$i\Omega\tau w_1 = w_0\int_{-l/2}^{l/2} \nu_1(z)\frac{dz}{l}. \tag{16}$$

To find the oscillation frequency, we solve the first of these equations with respect to ν_1 and substitute in the second. We get

$$-i\Omega\tau = w_0\int_{-l/2}^{l/2} \frac{\nu_0(z) + 1}{w_0 + (i\Omega T + 1)(\rho^2 + 4z^2/l^2)}\frac{dz}{l}. \tag{17}$$

[2)]We consider for concreteness a four-level system. In the case of a three-level system, n must be taken to mean $n - n_0/2$, where n_0 is the total volume density of the active centers (excited and unexcited).

Combining (17) and (13), we simplify the integrand of (17), after which the integral can be readily obtained. We transform (17) finally into

$$\tau T\Omega^2 = \zeta - \frac{(1+\zeta)w_0}{\rho[(1+i\Omega T)(1+i\Omega T + w_0/\rho^2)]^{1/2}} \times \operatorname{arctg}\left[\frac{1+i\Omega T}{(1+i\Omega T)\rho^2 + w_0}\right]^{1/2}. \quad (18)$$

Equation (18) must be solved simultaneously with Eq. (13) for w_0.

3. DAMPING OF OSCILLATIONS

Usually the time of spontaneous de-excitation T is much longer than the period of the intensity oscillation, so that

$$\Omega T \gg 1 \quad (19)$$

(for a ruby or neodymium laser $\Omega T \sim 100$).

We shall take $1/\Omega T$ to be the small parameter of the theory. In the zeroth approximation with respect to this parameter we have from (18)

$$\Omega = \sqrt{\zeta/\tau T}. \quad (20)$$

In the zeroth approximation the frequency is real, i.e., there is no damping. The imaginary part of the frequency, which determines the attenuation, can be calculated in the first approximation, by substituting (20) in the right side of (18). To simplify the result, we shall assume that ρ is not too small:

$$\rho \gg 1/\sqrt{\Omega T}. \quad (21)$$

As will be shown later, this region of ρ is of greatest practical interest. Usually $w_0 \sim 1$, so that the conditions (10) and (21) allow us to neglect the quantities 1 and w_0/ρ^2 in (18) compared with $i\Omega T$. Thus we get for the logarithmic damping decrement of the oscillations

$$\delta = 2\pi\frac{\operatorname{Im}\Omega}{\operatorname{Re}\Omega} = \pi(\zeta^{1/2} + \zeta^{-1/2})\sqrt{\frac{\tau}{T}}K \equiv \delta_{\|}K, \quad (22)$$

where

$$K = (w_0/\zeta\rho)\operatorname{arcctg}\rho. \quad (23)$$

Here $\delta_{\|}$ is the logarithmic damping decrement of small oscillations in a plane-parallel resonator and K is the factor by which the damping is increased by the bending of the caustic surface.

The quantity w_0 in (23) is determined from (13). The latter can be solved in explicit form in the limiting cases of large or small relative excess over the threshold ζ. We ultimately get the following asymptotic formulas for the coefficient K:

$$K = \frac{\zeta}{\pi\rho}\left[1 + \sqrt{1 + \left(\frac{\pi\rho}{\zeta}\right)^2}\right] \quad \text{for} \quad \zeta \ll 1, \quad (24)$$

$$K = \left(\frac{1}{3} + \rho^2\right)\frac{\operatorname{arcctg}\rho}{\rho} \quad \text{for} \quad \zeta \gg 1. \quad (25)$$

A plot of (24) and (25) is shown in Fig. 2, which shows that the damping increases monotonically with decreasing ρ. When $\rho \to 0$ the coefficient K increases in accord with the hyperbolic laws[3]:

$$K = \zeta/\pi\rho \quad \text{for} \quad \zeta \ll 1, \quad \rho \ll \zeta/\pi; \quad (26)$$

$$K = \pi/6\rho \quad \text{for} \quad \zeta \gg 1, \quad \rho \ll 1. \quad (27)$$

Let us examine the physical meaning of the result. We recall first that the parameter ρ characterizes the form of the caustic surface shown in Fig. 1: the smaller ρ, the narrower the neck of this surface (when $\rho = 0$ the caustic surface degenerates into the surface of a circular cone).

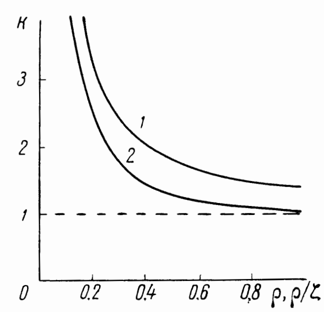

FIG. 2. K vs. ρ for small and large ξ: 1–$K = f(\rho/\xi)$, $\xi \ll 1$; 2–$K = f(\rho)$, $\xi \gg 1$. Dashed–asymptotic behavior as $\rho \to \infty$, corresponding to the transition to the limit of a plane-parallel resonator ($K \to 1$).

Let us examine the process of formation of the intensity peak. With increasing light intensity, the inverted population in the resonator becomes depleted and the gain decreases accordingly. The intensity reaches a maximum at the instant when the gain is comparable with the loss of light per unit length, after which it begins to decrease. During the intensity growth, the inverted population becomes depleted first in the region of the neck of the caustic surface, where the density of the light energy is highest. But the region of the neck makes a noticeable contribution to the average gain (proportional not to the volume but to the length of this region). Therefore the average gain

[3]When $\rho \to 0$ the damping remains finite, since the region of applicability of formulas (24) – (27) is bounded by the inequality (21). When $\rho = 0$ the decrement is of the order of $(\tau/T)^{1/4}$.

decreases to the value of the loss before the inverted population is depleted in the greater part of the volume, which is far from the neck. Consequently, the intensity reaches a maximum before the main reserve of the excited luminescence center becomes exhausted; this decreases the intensity peak compared with the case of a plane-parallel resonator. The unutilized reserve of inverted population is further de-excited during the time that the intensity decreases, smoothing out this decrease and bringing the generation mode closer to stationary.

The greater the amplitude of the intensity oscillations, the greater the difference between the inverted population in the neck region and that in the more remote region, and the more pronounced the described effect and the associated damping should be. Therefore in real cases, when the intensity oscillation amplitude is large, the coefficient K, which characterizes the damping, should be in any case larger than in the case of small oscillations considered above.

The foregoing analysis pertains to a four-level system, but the results can be generalized directly to the case of a three-level laser by substituting for ζ the quantity ζq, where ζ is the relative excess above the pumping threshold power and q is the ratio of the loss of light in the unexcited resonator to that part of the loss which is not connected with the absorption of light by the active medium.

4. EXPERIMENTAL RESULTS AND DISCUSSION

We did not compare quantitatively the foregoing theory with the experimental data because of the difficulty of satisfying in real experiments the conditions used in the calculation. The calculation is based on the assumption that the resonator is spatially homogeneous; this homogeneity is violated in the case of external reflectors, which are usually employed in experiments involving changes in the geometric parameters of the resonator. The calculation was carried out for small oscillations and not for the large-amplitude oscillations observed in customary experiments.

However, these circumstances do not prevent us from comparing the qualitative deductions of the theory with the experimental results, all the more because (as noted at the end of Sec. 3) the investigated effect is more pronounced in the case of strong oscillations than in the case of low intensity oscillations.

The experiments on the generation kinetics were made with a ruby laser (length of ruby rod l = 120 mm, diameter d = 12 mm). As in our earlier investigation [5], the resonator consisted of two plane-parallel mirrors and two short-focus positive lenses (F = 18.5 cm) placed near the reflector coaxially with the active rod and symmetrically with respect to the latter. The emission was recorded with a photocell (F-5) whose signal was fed to the input of an oscilloscope (OK-17M).

The spectral composition was investigated simultaneously with the generation kinetics, using a Fabry-Perot interferometer (IT-28-30) with different intermediate rings.

We investigated the dependence of the generation kinetics on the distance between the foci of the lenses; this distance characterizes the bending of the caustic surface[5)]. At the minimum distance between the lenses, the distance between their foci was about 20 cm, corresponding to a slight bending of the caustic surface ($\rho > 1$). At this lens position we observed the usual picture of regular slowly damped oscillations (Fig. 3a). We see from this figure that no steady-state manages to set in during the course of generation. If the distance between lenses is increased (leaving them symmetrical relative to the rod), then the foci come closer together and the curvature of the surface increases (the parameter ρ decreases); this increases the damping. Figure 3b shows an oscillogram of the oscillations observed at a distance of 5 cm between the lens foci ($\rho < 1$). We see that at this lens position the damping increases so much that a practically steady state is obtained during the greater part of the generation. The experiment thus confirms fully the deductions of the theory.

Bringing the foci closer together should lead to an even greater damping. We were unable, however, to perform the corresponding experiment, since the oscillations become random when the distance between the foci is too small (Fig. 3c).

The irregularity of the oscillations is usually attributed to independent generation of several modes. In a spherical resonator (or in an equivalent resonator with lenses) the spectrum of the

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4)A lens of focal length F placed inside the resonator ahead of the end mirror is practically equivalent to a spherical mirror with curvature radius F (the light passes through the lens twice in one reflection).

5)In the cited papers [9-11] the shape of the caustic surface did not change during the course of the experiment (in [8] the surface of the active rod has the same shape as the caustic surface, so that the latter was fixed).

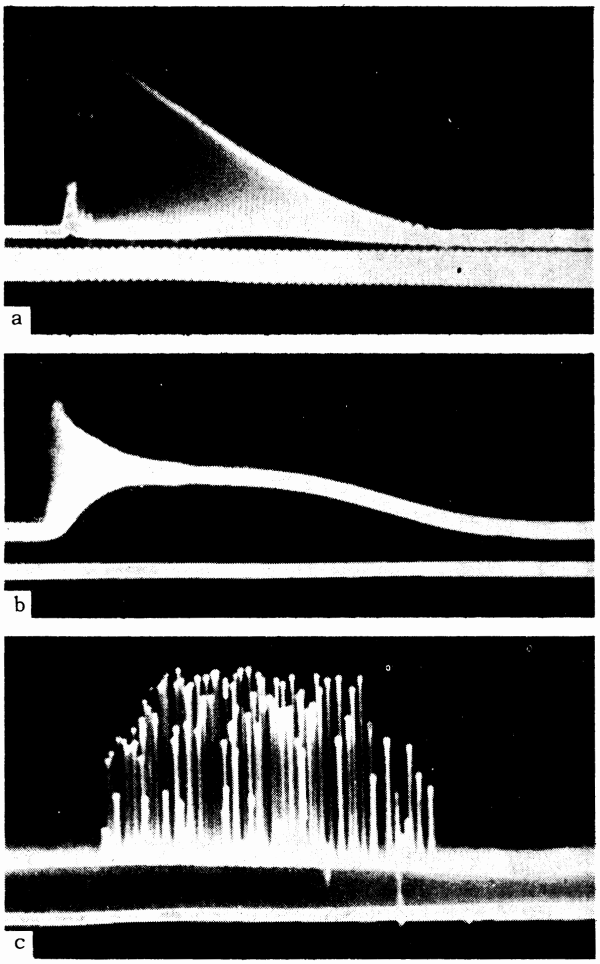

FIG. 3. Oscillograms of the output emission: a) $\rho > 1$, b) $\rho < 1$, c) $\rho \to 0$.

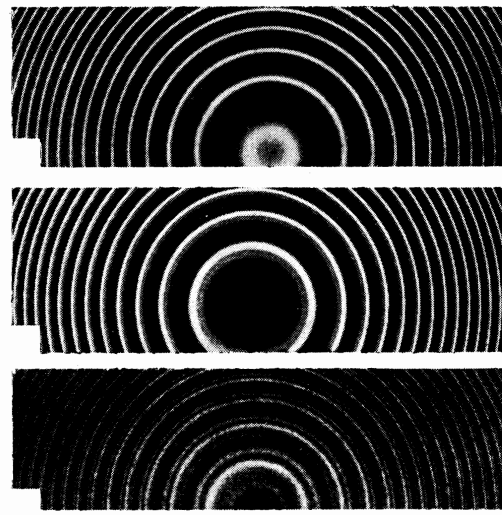

FIG. 4. Fabry-Perot interference pattern: a) $\rho > 1$, b) $\rho < 1$, c) $\rho \to 0$. Thickness of intermediate ring 4 mm, focal distance of lens 800 mm.

natural oscillations is given by the well known Vaĭnshteĭn formula [12]

$$\omega = \frac{c}{l}\left[\pi m_{\parallel} + 2m_{\perp} \arcsin \sqrt{\frac{l}{2R}}\right], \qquad (28)$$

where $m_{\parallel}$ and $m_{\perp}$ are the integer indices of the longitudinal and transverse modes. The regularity of the oscillations observed when $l \sim R$ is apparently due to the fact that the interval between the longitudinal modes is comparable with the interval between the transverse modes. This causes interaction between the systems of the longitudinal and transverse modes, causing the modes to lose their individuality and the radiation to be emitted as one entity[6)].

The regularity of the oscillations becomes disturbed in the case of concentric mirrors or confocal lenses[7)], when $l = R$, and formula (28) takes the form $\omega = c\pi(m_{\parallel} + m_{\perp})/l$. We see therefore that the transverse modes produce no changes in the spectrum, and consequently cannot interact with the longitudinal modes. Naturally, the regularity of the oscillation is disturbed also when the lens arrangement is nearly confocal.

The regularity of the oscillations should be disturbed also in the case of long-focus lenses, when the interaction between the longitudinal and transverse modes is weakened by the smallness of the spectral interval between the transverse modes, compared with the spacing of the longitudinal modes. Experiment confirms this conclusion.

The interaction between the transverse and longitudinal modes affects also the spectral composition of the radiation. In the case of a confocal (or near-confocal) arrangement of the lenses, the weakness of this interaction allows the longitudinal modes to retain their individuality; the emission spectrum consists accordingly of several lines with a total width of about 0.5 Å (Fig. 4c). When the lenses are brought together (i.e., the distance between their foci is increased), the longitudinal modes lose their individually; the multiple lines of the spectrum disappear gradually and the spectral emission width decreases (Figs. 4b and 4a). We note that in case of Fig. 4b a second weak line sometimes appears in the spectrum.

At the optimal lens position it is possible to observe simultaneously stationary generation and a narrow line approximately 0.01 Å wide.

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6)The theory of this question is developed in [5].

7)The position of the foci in a resonator with external mirrors must be determined with allowance for the active medium.

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