SCATTERING OF ELECTROMAGNETIC WAVES BY A SYSTEM OF DIPOLE CENTERS

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The problem of scattering of electromagnetic waves by two fixed dipole centers is solved. A system of algebraic equations is derived, which determines the amplitude of the scattering of electromagnetic waves by a system composed of an arbitrary number of dipole centers. This system of equations is generalized for the case of a macroscopic medium with a random or ordered distribution of scattering centers. Consideration is given to the shift of the resonance frequency and the change of the resonance width in scattering of electromagnetic waves by a system of two oscillators. The problem of the change of resonance parameters in scattering of electromagnetic waves by a macroscopic medium is discussed.

I. INTRODUCTION

It is known that in the case of scattering of S-waves by a set of fixed point centers, the system of integral equations for the multiple scattering goes over to a system of ordinary algebraic equations.[1] The effective amplitude of the scattering of S-waves by a system of N centers is determined by the formula[1-3]

$$A(\mathbf{q})=\sum_{i=1}^{N}A^{(i)}\exp\{-i\mathbf{q}\mathbf{R}^{(i)}\},$$

where

$$A^{(i)}=a^{(i)}+\sum_{k\neq i}\frac{a^{(i)}A^{(k)}}{R^{(ik)}}\exp\{ikR^{(ik)}+i\mathbf{k}\mathbf{R}^{(ki)}\}. \quad (1)$$

Here $\mathbf{q} = \mathbf{k}' - \mathbf{k}$ is the transferred momentum, $\mathbf{k}$ is the momentum of a particle before scattering, $a^{(i)}$ is the amplitude of the S-scattering by the i-th isolated center (independent of angle), and

$$\mathbf{R}^{(ki)}=\mathbf{R}^{(k)}-\mathbf{R}^{(i)},\quad R^{(ki)}=R^{(ik)}=|\mathbf{R}^{(ik)}|,\quad k=|\mathbf{k}|.$$

The problem of the scattering of S-waves by two fixed centers was apparently first considered in a paper of Chew and Wick[4] in connection with a discussion of corrections to the impulse approximation. A little later a detailed solution of the problem was given by Brueckner.[5] Consideration of a finite number of higher partial waves, in particular of P-scattering, can be found in a paper of Amirkhanov et al.[6] Generalization of the system of equations (1) to the case of an infinitely wide layer is included in papers of Baryshevskiĭ et al.[3] and of Kagan and Afanas'ev.[7]

The system of equations (1) is easily generalized to the case of central spin-spin interaction between the incident particles and the scattering centers. Here the quantities $a^{(i)}$ and $A^{(i)}$ must be considered as operators in the spin space of a particle and of the system of centers.[3] It must be emphasized, however, that for particles with nonvanishing spin,, the system of equations (1) can in principle be correct only in the nonrelativistic approximation, when one can correctly separate the spin from the orbital angular momentum and can speak of the scattering just of S-waves, without admixture of other orbital angular momenta.

In the case of particles with rest mass equal to zero (for example, photons), we cannot go over to a nonrelativistic limit, although the momenta may be as small as we please. Because of the transversality of electromagnetic waves, the scattering matrix in principle cannot be diagonal with respect to the orbital moment. The analog of S-scattering is dipole scattering of electromagnetic waves, the photon being in a state with total momentum unity and with negative parity (coherent superposition of S- and D-waves, see [8]).

In the present paper we consider multiple scattering of electromagnetic waves by a system of dipole centers and derive a system of equations analogous to (1). As we shall see below, this system of equations differs significantly from (1). We consider in particular detail the problem of scattering by two dipole centers. For simplicity we shall confine ourselves to a classical treatment. A quantum treatment (scattering of photons) leads to the same results, as can be verified by use of a diagram technique.

2. SCATTERING OF ELECTROMAGNETIC WAVES BY TWO DIPOLE CENTERS

We consider the scattering of electromagnetic waves by two rigidly fixed point dipole scatterers, each of which has zero charge. Let the amplitudes of the scattering of electromagnetic waves by the isolated centers be $a_1(\mathbf{e}^{(1)}\mathbf{e}^{(2)*})$ and $a_2(\mathbf{e}^{(1)}\mathbf{e}^{(2)*})$. Here $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ are the polarization vectors before and after scattering, respectively; a_1 and a_2 are independent of angle.

We shall find the amplitude of scattering by a single system consisting of the point dipole centers 1 and 2, which do not interact with each other in the absence of an electromagnetic wave. The mechanism of dipole scattering, as is well known, is the following.

Under the influence of the electric field of an electromagnetic wave, there arises in the particle an alternating dipole moment

$$D_\alpha = T_{\alpha\beta} e_\beta^{(1)} \exp\{i\mathbf{k}\mathbf{R}^{(1)} - i\omega t\}, \tag{2}$$

where $T_{\alpha\beta}$ is the polarizability tensor. As a result, radiation of electromagnetic waves occurs, and the scattering amplitude is described in the form

$$M = k^2 T_{\alpha\beta} e_\alpha^{*(2)} e_\beta^{(1)}, \tag{3}$$

where $k = \omega/c$ is the wave vector of the electromagnetic wave. For a single symmetrical scatterer, $T_{\alpha\beta} = T\delta_{\alpha\beta}$. It is obvious that $a_1 = k^2 T_1$, $a_2 = k^2 T_2$. In the presence of two centers, each of them is in a total field consisting of the field of the incident electromagnetic wave and of the field of the alternating dipole moment of the other center. The field of the radiation itself, as usual, is taken into account by the presence of an imaginary part in the scattering amplitudes a_1 and a_2. According to the optical theorem for purely elastic dipole scattering,

$$\operatorname{Im} a_{1(2)} = \frac{k}{4\pi}\sigma_{1(2)} = \frac{2}{3}k|a_{1(2)}|^2. \tag{4}$$

The dipole moments of the first and second centers have the form

$$\mathbf{D}_1 = \mathbf{d}_1 \exp\{i\mathbf{k}\mathbf{R}^{(1)} - i\omega t\} = T_1(\mathbf{e}^{(1)} + \mathbf{E}_2)\exp\{i\mathbf{k}\mathbf{R}^{(1)} - i\omega t\},$$
$$\mathbf{D}_2 = \mathbf{d}_2 \exp\{i\mathbf{k}\mathbf{R}^{(2)} - i\omega t\} = T_2(\mathbf{e}^{(1)} + \mathbf{E}_1)\exp\{i\mathbf{k}\mathbf{R}^{(2)} - i\omega t\}. \tag{5}$$

Here $\mathbf{e}^{(1)}$ is the electric field of the incident wave, whereas $\mathbf{E}_2$ ($\mathbf{E}_1$) is the electric field of the second (first) center at the position of the first (second) center. On using the known formula for the intensity of the electric field at a distance R from an alternating point dipole,[9] we can write

$$\mathbf{E}_1 = \left[\mathbf{d}_1\left(\frac{k^2}{R} + \frac{ik}{R^2} - \frac{1}{R^3}\right) + \mathbf{n}(\mathbf{d}_1\mathbf{n})\left(-\frac{k^2}{R} - \frac{3ik}{R^2} + \frac{3}{R^3}\right)\right] e^{ikR - i\mathbf{k}\mathbf{R}}$$
$$= e^{-i\mathbf{k}\mathbf{R}}[\mathbf{d}_1 k^2 + \nabla(\mathbf{d}_1\nabla)]\frac{e^{ikR}}{R},$$

$$\mathbf{E}_2 = \left[\mathbf{d}_2\left(\frac{k^2}{R} + \frac{ik}{R^2} - \frac{1}{R^3}\right) + \mathbf{n}(\mathbf{d}_2\mathbf{n})\left(-\frac{k^2}{R} - \frac{3ik}{R^2} + \frac{3}{R^3}\right)\right] e^{ikR + i\mathbf{k}\mathbf{R}}$$
$$= e^{i\mathbf{k}\mathbf{R}}[\mathbf{d}_2 k^2 + \nabla(\mathbf{d}_2\nabla)]\frac{e^{ikR}}{R}. \tag{6}$$

Here $\mathbf{R} = \mathbf{R}^{(2)} - \mathbf{R}^{(1)}$, $R = |\mathbf{R}^{(2)} - \mathbf{R}^{(1)}|$ is the distance between the centers, and $\mathbf{n}$ is the unit vector in the direction of $\mathbf{R}$.

It is easy to see that when account is taken of the influence of the two dipole centers on each other, the effective polarization tensors cease to be isotropic. Under these conditions the general structure of $\mathbf{d}_1$ and $\mathbf{d}_2$ has the form

$$\mathbf{d}_{1(2)} = \alpha_{1(2)}\mathbf{e}^{(1)} + \beta_{1(2)}\mathbf{n}(\mathbf{e}^{(1)}\mathbf{n}), \tag{7}$$

where α and β are constants.

On substituting (6) and (7) into (5) and then going over to the scattering amplitude in accordance with formula (3), we get

$$M = [A_1(\mathbf{e}^{(1)}\mathbf{e}^{*(2)}) + B_1(\mathbf{e}^{(1)}\mathbf{n})(\mathbf{e}^{*(2)}\mathbf{n})]\exp\{-i\mathbf{q}\mathbf{R}^{(1)}\}$$
$$+ [A_2(\mathbf{e}^{(1)}\mathbf{e}^{*(2)}) + B_2(\mathbf{e}^{(1)}\mathbf{n})(\mathbf{e}^{*(2)}\mathbf{n})]\exp\{-i\mathbf{q}\mathbf{R}^{(2)}\}, \tag{8}$$

where

$$A_{1(2)} = a_{1(2)} + \frac{a_{1(2)}A_{2(1)}}{R}\left(1 + \frac{i}{kR} - \frac{1}{k^2R^2}\right) \times \exp\{ikR + i\mathbf{k}\mathbf{R}_{21(12)}\},$$

$$B_{1(2)} = \left[2a_{1(2)}B_{2(1)}\left(\frac{1}{kR} - i\right)\frac{1}{kR^2} - \frac{a_{1(2)}A_{2(1)}}{R}\left(1 + \frac{3i}{kR} - \frac{3}{k^2R^2}\right)\right]\exp\{ikR + i\mathbf{k}\mathbf{R}_{21(12)}\} \tag{9}$$

($\mathbf{R}_{12} = -\mathbf{R}_{21} = -\mathbf{R}$). Hence

$$A_{1(2)} = \left[a_{1(2)} + \frac{a_1 a_2}{R}\left(1 + \frac{i}{kR} - \frac{1}{k^2R^2}\right)\exp\{ikR \pm i\mathbf{k}\mathbf{R}\}\right] \times\left[1 - \frac{a_1 a_2}{R^2}\left(1 + \frac{i}{kR} - \frac{1}{k^2R^2}\right)^2 e^{2ikR}\right]^{-1}. \tag{10}$$

(The sign in the argument of the exponential function is taken plus for A_1 and minus for A_2.) For B_1 and B_2 we get the equations

$$B_{1(2)} = -\left[\frac{2a_2a_1}{kR^3}A_{1(2)}\left(\frac{1}{kR} - i\right)\left(1 + \frac{3i}{kR} - \frac{3}{k^2R^2}\right)e^{2ikR} + \frac{a_{1(2)}A_{2(1)}}{R}\left(1 + \frac{3i}{kR} - \frac{3}{k^2R^2}\right)\exp\{ikR + i\mathbf{k}\mathbf{R}_{21(12)}\}\right]$$

$$\times\left[1-\frac{4a_1a_2}{k^2R^4}\left(\frac{1}{kR}-i\right)e^{2ikR}\right]^{-1}, \quad (11)$$

where A_1 and A_2 are determined by formulas (10).

We shall consider limiting cases.

When $kR \gg 1$, the parameter of multiple scattering is the quantity a/R. Since $a \lesssim \lambda \ll R$, to within terms of order $1/kR$ we have $A_1 \approx a_1$, $A_2 \approx a_2$, and $B_1 \approx B_2 \approx 0$. After averaging over R (or over the spread of frequencies in the case of strictly fixed centers), we arrive at the known result on incoherent summation of cross sections:

$$\sigma_{\text{total}} = {}^8/_3\pi(|a_1|^2 + |a_2|^2).$$

When $kR \ll 1$, the parameter of multiple scattering is the quantity $a\lambda^2/R^3$. If $a_1 = a_2 \ll k^2R^3$, then $B_1, B_2 \ll a$,

$$\operatorname{Re} A_1 = \operatorname{Re} A_2 = \operatorname{Re} a, \quad \operatorname{Im} A_1 = \operatorname{Im} A_2 = 2\operatorname{Im} a \quad (12)$$

($\operatorname{Im} a/\operatorname{Re} a \approx {}^2/_3 k|a| \ll (kR)^3$), and we arrive at the result on coherent summation of amplitudes.[1)] In the limiting case $a_{1(2)} \gg k^2R^3$, we find

$$\operatorname{Re} A_1 = \operatorname{Re} A_2 = k^2R^3, \quad \operatorname{Re} B_1 = \operatorname{Re} B_2 = -{}^3/_2 k^2R^3 \quad (13)$$

($\operatorname{Im} A/\operatorname{Re} A \sim \operatorname{Im} B/\operatorname{Re} B \ll (kR)^3$).

We remark that as the wavelength becomes infinite, the cross section for scattering on two centers approaches zero like $1/\lambda^4$. This result is in agreement with a theorem of Rayleigh on the scattering of electromagnetic waves on a neutral system of charges in the long-wave limit.

3. SCATTERING OF ELECTROMAGNETIC WAVES BY AN ARBITRARY NUMBER OF DIPOLE CENTERS. INFINITELY WIDE LAYER

A system of algebraic equations for the amplitude of scattering of electromagnetic waves by a set of N fixed dipole centers can be derived without difficulty by the same method as the system (9)–(11) for scattering by two dipole centers. The resultant amplitude of scattering on N dipole centers can be written in the form

$$M = \sum_{i=1}^{N} L_{\alpha\beta}^{(i)} e_\alpha^{*(2)} e_\beta^{(1)} \exp\{-i\mathbf{q}\mathbf{R}^{(i)}\}, \quad (14)$$

where the tensors $L_{\alpha\beta}^{(i)}$ satisfy the following relations:

$$L_{\alpha\beta}^{(i)} = a^{(i)}\delta_{\alpha\beta} + a^{(i)} \sum_{k\neq i} \frac{L_{\alpha\beta}^{(k)}}{R^{(ik)}} \exp\{ikR^{(ki)} + i\mathbf{k}\mathbf{R}^{(ki)}\}$$
$$+ \frac{a^{(i)}}{k^2} \sum_{k\neq i} \exp\{i\mathbf{k}\mathbf{R}^{(ki)}\} (L_{\alpha\gamma}^{(k)} \nabla_\gamma^{(i)}) \nabla_\beta^{(i)} \frac{\exp\{ikR^{(ik)}\}}{R^{(ik)}}$$
$$\nabla^{(i)} = \partial/\partial\mathbf{R}^{(i)}, \quad \mathbf{R}^{(ki)} = \mathbf{R}^{(k)} - \mathbf{R}^{(i)}, \quad R^{(ik)} = |\mathbf{R}^{(ki)}|. \quad (15)$$

We consider the passage of electromagnetic waves through an infinitely wide layer, consisting of dipole centers randomly distributed with density ρ. We shall suppose that the surface of the layer is perpendicular to the wave vector $\mathbf{k}$. By symmetry, the effective scattering amplitude of the i-th center depends only on its distance from the forward edge, z. Thus one can write

$$L_{\alpha\beta}^{(i)} = A(\mathbf{z}^{(i)})\delta_{\alpha\beta}, \quad \mathbf{z} \parallel \mathbf{k}. \quad (16)$$

On applying formula (15), with allowance for (16), to an infinitely wide layer (in practice, the width of the layer must be many times larger than the wavelength), and on taking into account the transversality of electromagnetic waves, we arrive at the integral equation (cf. [3])

$$A(z) = a + \frac{2\pi\rho a}{k} i \int_0^z A(y)\,dy$$
$$+ \frac{2\pi\rho a}{k} i e^{-2ikz} \int_z^b A(y) e^{2iky}\,dy + K, \quad (17)$$

where b is the thickness of the layer, and

$$K = \frac{1}{3}\frac{\rho a}{k^2} \int \nabla^2 \left[A(z') \frac{\exp\{ikR' + i\mathbf{k}\mathbf{R}'\}}{R'}\right] dV'$$
$$= \frac{4\pi}{3}\frac{\rho a}{k^2} A(z), \qquad R' = [x'^2 + y'^2 + (z'-z)^2]^{1/2}. \quad (18)$$

Equation (17) without K agrees with the integral equation derived in paper [3] for the case of S-waves.[2)]

In (17) it is possible to omit the term K, if one replaces a by the quantity

$$a' = a\left(1 - \frac{4\pi}{3}\frac{\rho a}{k^2}\right)^{-1}. \quad (19)$$

The solution of (18) can then be written in the form

$$A(z) = C_1 e^{ik(n-1)z} + C_2 e^{-ik(n+1)z},$$

where[3]

[1)] Formulas (12) for the real and imaginary parts of the amplitudes occur also for S-scattering [2] and are in agreement with an optical theorem (for more details see [10]).

[2)] Analysis shows that a formula of the type (17) is correct not only when $\lambda \gg d$ (d = mean distance between centers), when the legitimacy of going over from summation to integration is obvious, but also when $\lambda \lesssim d$.

$$C_1 = \frac{2a'}{G}, \quad C_2 = \frac{n-1}{n+1}\frac{2ae^{2iknb}}{G},$$

$$G = (n+1) - (n-1)^2 (n+1)^{-1} e^{2iknb},$$

$$n = \left[1 + \frac{4\pi\rho a'}{k^2}\right]^{1/2} = \left[1 + \frac{4\pi\rho a}{k^2}\left(1 - \frac{4\pi}{3}\frac{\rho a}{k^2}\right)^{-1}\right]^{1/2}. \quad (20)$$

It is easy to see that the quantity n has the meaning of index of refraction of the medium. The Lorentz-Lorentz formula

$$\frac{n^2-1}{n^2+2} = \frac{4\pi}{3}\frac{\rho a}{k^2}, \quad (20')$$

holds.

For a space lattice, a special analysis is necessary; it is analogous, however, to the corresponding treatment of S-waves (see [3, 7]). It can be shown that in the case of a cubic crystal (under the condition that the oscillations of the nuclei in the lattice can be neglected), the index of refraction is equal to

$$n = [1 + 4\pi a''/k^2 d^3]^{1/2}, \quad (20'')$$

where

$$a'' = a\left(1 - \frac{4\pi a}{3k^2 d^3} + \xi\frac{a}{d} + \frac{2}{3}ika\right)^{-1}. \quad (21)$$

Here d is the length of the edge of a cell; ξ is a material constant of the order of unity, the explicit form of which we shall not present here.[3] From the relations (20″) and (21) it follows that

$$\frac{n^2-1}{n^2+2} = \frac{4\pi}{3k^2 d^3}\frac{a}{1 + \xi a/d + 2ika/3}. \quad (22)$$

It is easy to see that if there is purely elastic scattering ($\mathrm{Im}\, a = \frac{2}{3} k |a|^2$), the index of refraction n is a real number; that is, an electromagnetic wave without attenuation is propagated in the crystal. Attenuation of an electromagnetic wave passing through a crystal is caused by inelastic (incoherent) processes (see [3]).

4. RESONANCE SCATTERING BY A SYSTEM OF DIPOLE CENTERS

We consider resonance scattering of electromagnetic waves by two identical isotropic oscillators. In order to study the nature of such scattering, it is sufficient, in the general formulas (10) and (11), to set

$$a = a_1 = a_2 = \frac{r_0\omega^2}{\omega_0^2 - \omega^2 - 2ir_0\omega^3/3c}. \quad (23)$$

Here $\omega = kc$ is the frequency of the incident radiation, ω_0 is the resonance frequency of an oscillator, and r_0 is the electromagnetic radius.

Near resonance,

$$a = \frac{3}{4k}\frac{\gamma}{\omega_0 - \omega - i\gamma/2}, \quad \gamma = \frac{2}{3}r_0\frac{\omega_0^2}{c}, \quad (23')$$

where γ is the width of the resonance. We shall suppose that $\gamma \ll \omega_0$. When $a_1 = a_2 = a$, the quantities $A_1 + A_2$ and $B_1 + B_2$ can be put into the form

$$A_1 + A_2 = 2a\left(Q_+ \sin^2\frac{\mathbf{kR}}{2} + Q_- \cos^2\frac{\mathbf{kR}}{2}\right),$$

$$Q_\pm = \left[1 \pm \frac{a}{R}\left(1 + \frac{i}{kR} - \frac{1}{k^2R^2}\right)e^{ikR}\right]^{-1};$$

$$B_1 + B_2 = 2a(F_+ - Q_+)\sin^2\frac{\mathbf{kR}}{2} + 2a(F_- - Q_-)\cos^2\frac{\mathbf{kR}}{2},$$

$$F_\pm = \left[1 \pm \frac{2a}{kR^2}\left(\frac{1}{kR} - i\right)e^{ikR}\right]^{-1}. \quad (24)$$

It is easy to see that each term in formulas (24) has resonance character. When $kR \ll 1$, one can write (near the resonance frequencies)

$$A_1 + A_2 = \frac{3}{4k}\left[\frac{\gamma_1}{\omega_0^{(1)} - \omega - i\gamma_1/2} + \frac{5}{2}\frac{\tilde\gamma_1 \cos^2\psi}{\tilde\omega_0^{(1)} - \omega - i\tilde\gamma_1/2}\right] \quad (25)$$

$$B_1 + B_2 = \frac{3}{4k}\left[\frac{\gamma_2}{\omega_0^{(2)} - \omega - i\gamma_2/2} - \frac{\gamma_1}{\omega_0^{(1)} - \omega - i\gamma_1/2}\right] + \frac{15}{8k}\cos^2\psi\left[\frac{\tilde\gamma_2}{\tilde\omega_0^{(2)} - \omega - i\tilde\gamma_2/2} - \frac{\tilde\gamma_1}{\tilde\omega_0^{(1)} - \omega - i\tilde\gamma_1/2}\right]. \quad (26)$$

Here ψ is the angle between **k** and **R**. To within terms much smaller than the width γ, we have

$$\omega_0^{(1)} = \left[\omega_0^2 + \frac{3}{2}\omega_0\gamma\left(\frac{c}{\omega_0 R}\right)^3 - \frac{3}{4}\gamma\frac{c}{R}\right]^{1/2} \approx \omega_0 + \frac{3}{4}\gamma\left(\frac{c}{\omega_0 R}\right)^3,$$

$$\omega_0^{(2)} = \left[\omega_0^2 - 3\omega_0\gamma\left(\frac{c}{\omega_0 R}\right)^3 - \frac{3}{2}\gamma\frac{c}{R}\right]^{1/2} \approx \omega_0 - \frac{3}{2}\gamma\left(\frac{c}{\omega_0 R}\right)^3,$$

$$\tilde\omega_0^{(1)} = \left[\omega_0^2 - \frac{3}{2}\omega_0\gamma\left(\frac{c}{\omega_0 R}\right)^3 + \frac{3}{4}\gamma\frac{c}{R}\right]^{1/2} \approx \omega_0 - \frac{3}{4}\gamma\left(\frac{c}{\omega_0 R}\right)^3,$$

$$\tilde\omega_0^{(2)} = \left[\omega_0^2 + 3\omega_0\gamma\left(\frac{c}{\omega_0 R}\right)^3 + \frac{3}{2}\gamma\frac{c}{R}\right]^{1/2} \approx \omega_0 + \frac{3}{2}\gamma\left(\frac{c}{\omega_0 R}\right)^3. \quad (27)$$

The corresponding resonance widths are connected with the resonance width for an isolated oscillator by the relations

[3]In the paper [3] it was shown that in the case of scattering of S-waves by a cubic crystal, $a' = a(1 + \xi a/d + ika)^{-1}$, where ξ is the same quantity as in formula (21). The formula for ξ follows immediately from the results of paper [7].

$$\gamma_{1,2} = 2\gamma\left(\frac{\omega_0^{(1,2)}}{\omega_0}\right)^2, \quad \tilde{\gamma}_{1,2} = \frac{\delta_{1,2}}{5}\gamma\left(\frac{\tilde{\omega}_0^{(1,2)}}{\omega_0}\right)^2\left(\frac{\tilde{\omega}_0^{(1,2)}R}{c}\right)^2, \tag{28}$$

where $\delta_1 = 1$ and $\delta_2 = 1/2$. From the expressions (27) it is is immediately evident that although the shift of the resonance frequencies may be small in comparison with ω_0, in the long-wave limit this shift is always considerably larger than the width γ.[4)]

We turn now to the total scattering cross section. The optical theorem for scattering of unpolarized electromagnetic radiation on two dipole centers gives[10]

$$\sigma_{\text{total}} = \frac{4\pi}{k}\,\mathrm{Im}\,\overline{M} = \frac{4\pi}{k}\Big[\mathrm{Im}\,(A_1 + A_2) + \frac{1}{2}\sin^2\psi\,\mathrm{Im}\,(B_1 + B_2)\Big], \tag{29}$$

where ψ is the angle between **k** and **R**. As a result, one can write in the vicinity of the resonance frequencies $\omega_0^{(1)}$, $\tilde{\omega}_0^{(1)}$, $\omega_0^{(2)}$, and $\tilde{\omega}_0^{(2)}$

$$\sigma_1 = \frac{3\pi}{k^2}\frac{\gamma^2}{(\omega_0^{(1)} - \omega)^2 + (2\gamma)^2/4}(1 + \cos^2\psi),$$

$$\sigma_2 = \frac{3\pi}{k^2}\frac{\gamma^2}{(\omega_0^{(2)} - \omega)^2 + (2\gamma)^2/4}\sin^2\psi,$$

$$\sigma_3 = \frac{15}{8k^2}\pi\frac{\tilde{\gamma}_1^2}{(\tilde{\omega}_0^{(1)} - \omega)^2 + \tilde{\gamma}_1^2/4}\cos^2\psi(1 + \cos^2\psi),$$

$$\sigma_4 = \frac{15}{32k^2}\pi\frac{\tilde{\gamma}_2^2}{(\tilde{\omega}_0^{(2)} - \omega)^2 + \tilde{\gamma}_2^2/4}\sin^2 2\psi. \tag{30}$$

Since the widths $\tilde{\gamma}_1$ and $\tilde{\gamma}_2$ in the long-wave limit are vanishingly small, the contribution of σ_3 and σ_4 to the total scattering cross section can be neglected when $\lambda \gg R$, if we omit from consideration narrow bands of frequencies ($\sim\gamma(\omega_0 R/c)^2$) in the neighborhood of $\tilde{\omega}_0^{(1)}$ and $\tilde{\omega}_0^{(2)}$.[5)]

We consider several special cases. If $\psi = 0$ (**k** ∥ **R**), then the quantities B_1 and B_2 make no contribution to the scattering amplitude. In this case there remain two resonances, with frequencies $\omega_0^{(1)}$ and $\tilde{\omega}_0^{(1)}$. The corresponding angular distributions have the form $1 + \cos^2\theta$ for the resonance with frequency $\omega_0^{(1)}$ and $\cos^2\theta\,(1 + \cos^2\theta)$ for the resonance with frequency $\tilde{\omega}_0^{(1)}$. If $\psi = \pi/2$, the contribution of the narrow resonances to the scattering cross section is identically equal to zero. It is interesting to note that in this case, the resonance scattering with frequency $\omega_0^{(1)}$ is absent for a wave polarized in the direction of the vector **R**; the scattering occurs only at $\omega = \omega_0^{(2)}$. On the other hand, resonance scattering occurs only at frequency $\omega_0^{(1)}$ for a wave polarized in the direction of the vector **k** × **R**. In this connection it should be mentioned that an assemblage of identically oriented pairs of oscillators can serve as a good model of a doubly refracting medium.

So far, the topic has been purely elastic scattering. In the presence of absorption, the relations (27) and (28) remain in force; in this case, γ, γ_1, γ_2, $\tilde{\gamma}_1$, and $\tilde{\gamma}_2$ have the meaning of partial widths, corresponding to elastic scattering. As regards the width connected with absorption, it remains the same as for an isolated oscillator. In particular, for $kR \ll 1$ and $\Delta\omega_0 \ll \omega_0$ we have

$$\gamma_{\text{total}}^{(1)} = \gamma_{\text{total}}^{(2)} = 2\gamma + \gamma_{\text{total}}, \quad \gamma_{\text{total}}^{(3)} = \gamma_{\text{total}} + {}^1/_5\gamma(\omega_0 R/c)^2,$$

$$\gamma_{\text{total}}^{(4)} = \gamma_{\text{total}} + {}^1/_{10}\gamma(\omega_0 R/c)^2. \tag{31}$$

We turn now to a macroscopic medium. We have already shown that in the case of an amorphous body, the effective scattering amplitude a' that enters into the index of refraction is determined by formula (19). On substituting into (19) the expression (23′), we get a resonance term of the form

$$a' = \frac{3}{4k}\frac{\gamma}{\omega_0' - \omega - i\gamma/2},$$

where γ is the width of the resonance scattering for an isolated oscillator and

$$\omega_0' = \omega_0 - \pi\gamma\frac{c^3}{\omega_0^3}\rho \tag{32}$$

4)In the case of resonance S-scattering by two centers, it is not difficult to show that when $\lambda \gg R$,

$$A_1 + A_2 = \frac{1}{2k}\left(\frac{\gamma_1}{\omega_0^{(1)} - \omega - i\gamma_1/2} + \frac{\tilde{\gamma}_1\cos^2\psi}{\tilde{\omega}_0^{(1)} - \omega - i\tilde{\gamma}/_1 2}\right),$$

where ψ is the angle between **k** and **R**,

$$\omega_0^{(1)} = \omega_0 - {}^1/_2\gamma c/\omega_0 R, \qquad \tilde{\omega}_0^{(1)} = \omega_0 + {}^1/_2\gamma c/\omega_0 R;$$

$$\gamma_1 = 2\gamma, \; \tilde{\gamma}_1 = {}^1/_6\gamma(\omega_0 c/R)^2.$$

The first resonance, with doubled width, corresponds to S-scattering. The second very narrow resonance is connected with a P-wave. Its contribution to the scattering cross section is extremely small outside a narrow band of frequencies (energies) close to $\tilde{\omega}_0^{(1)}$.

5)It should be remarked that when $c/\omega_0^0 \sim R$, all four widths have the order of magnitude γ. In this case the shift of resonance frequencies is of the same order as the width γ. When $\omega_0 R/c \gg 1$, we have $\Delta\omega = \gamma c/\omega_0 R \ll \gamma$.

(we suppose that $(\pi\gamma/\omega_0)(c/\omega_0)^3\rho \ll 1$). We remark that here the resonance width does not change in comparison with the case of an isolated oscillator.

We emphasize that in scattering of electromagnetic waves, the shift of the resonance differs from zero even in an amorphous body, whereas in scattering of ordinary particles (for example, neutrons) such a shift can occur only in crystalline bodies.[7]

In resonance scattering of electromagnetic radiation by a three-dimensional lattice, in addition to a shift of the type (32), there occurs an additional shift of the resonance frequency, which is of the same nature as in the case of S-waves. Here the width corresponding to purely elastic scattering disappears completely (cf. [7]). In fact, if we take account of formulas (21) and (23) for the total cross section of resonance scattering of electromagnetic waves in a cubic crystal (in the calculation for a single dipole center), we get the following expression:

$$\sigma = \frac{4\pi}{k}\operatorname{Im} a'' = \frac{3\pi}{2k^2}\frac{\gamma(\gamma_{\text{total}}-\gamma)}{(\omega_0''-\omega)^2+{}^1/_4(\gamma_{\text{total}}-\gamma)^2}, \quad (33)$$

where γ_{total} is the total resonance width for an isolated center, γ is the width corresponding to elastic scattering, and ω_0'' is the resonance frequency, which is connected with the frequency ω_0 for an isolated center by the relation

$$\omega_0'' = \omega_0 - \frac{\pi}{d^3}\left(\frac{c}{\omega_0}\right)^3\gamma + \frac{3}{4}\xi\gamma\left(\frac{c}{\omega_0 d}\right). \quad (34)$$

It should be remarked that if the contribution of inelastic processes to the total scattering cross section is not too large, then in the long-wave limit the shift of resonance frequency can be much larger than the resonance width. When $\lambda \ll d$ ($\lambda \ll \rho^{1/3}$), the shift of resonance is many times smaller than the width and can apparently be neglected.

5. CONCLUDING REMARKS

By a completely analogous method, one can study the problem of dipole radiation in an assemblage consisting of two or more radiators. In particular, in agreement with the results of the present paper, it is clear that in radiation of electromagnetic waves by a molecule (nucleus, atom) surrounded by an assemblage of unexcited molecules (nuclei, atoms) of the same kind, there will occur a change of the line width in comparison with the case of an isolated radiator, and also a shift of the frequency of the radiation.[6)] From the quantum point of view, because of interaction with the field of the radiation there occurs a shift of the collective energy levels. In the absence of spherical symmetry (for example, two centers with coordinates $R^{(1)}$ and $R^{(2)}$), there occurs also a splitting of the energy levels. We propose to carry out in a separate paper a more detailed analysis of these and other problems, connected with the radiation of electromagnet waves by a system of dipole centers.

It should be mentioned that the application of the results, which follow from a model of fixed centers, to specific real systems (molecules, crystals, etc.) requires special consideration. This pertains both to dipole scattering of electromagnetic waves and to the case of S-scattering of ordinary particles. Discussion of these problems is outside the scope of the present paper.

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6)In the radio range, these effects are well known and have been recorded in the literature [11] (see also [12]).

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