THERMOMAGNETIC EFFECTS IN FINITE METAL SAMPLES

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It is shown that the thermal flux in thin metal samples in a strong magnetic field is extremely nonuniform if the Fermi surface of the metal is an open one or if the numbers of the electrons and "holes" compensate each other ($n_1 = n_2$). The distribution of thermal flux within the sample is quite similar to that of the electric current (a static skin effect takes place). The Kelvin-Onsager relations hold for the thermoelectric coefficients in the nonuniform case.

A constant electric current in samples that are thin (in comparison with the length l of the free path of the electron) varies little, in the absence of an external magnetic field, over the cross section of a conducting sample. If the turning on of a strong magnetic field greatly changes the resistance of the conductor,[1)] then, simultaneously with an unrestricted increase in the resistance, the current distribution over the sample is completely changed—the current is attenuated in the depth of the sample, being concentrated near the surface of the conductor in a layer thickness of the order of r (static skin effect).

This phenomenon, predicted by one of the authors,[1] is associated with the special non-statistical character of the collisions of the electrons with the surface of the sample. In a more detailed study of this question,[2] it was made clear that in a thin metallic sample, the strong nonuniformity of the current can be so large, even close to the surface, that the electric current in the conductor flows in opposite directions at different depths.

The effects that have already been pointed out should take place for all transport phenomena in the presence of a strong magnetic field. There is every reason for supposing that under these conditions, when the static skin effect sets in for the electric current, the heat will flow principally close to the surface of the conductor, inasmuch as the directed flow of particles—carriers of charge and energy—is concentrated near the surface of the sample.

1. The study of thermomagnetic phenomena in thin samples is of fundamental interest. Actually, the equations of the problem themselves change in the microscopic approach.

In a bulk sample (the dimensions of which are the largest parameter of the problem), in the macroscopic approach, the connection between the current density $\mathbf{j}$ and the thermal flux $\mathbf{q}$ on one hand and the electric field intensity $\mathbf{E} = -\nabla\varphi$ and the temperature gradient on the other remain the same in the first approximation as in the unbounded medium,[3] and should be regarded as specified:

$$\mathbf{j} = \hat{\sigma}E + \hat{b}\nabla T = -\hat{\sigma}\nabla\varphi + \hat{b}\nabla T,$$
$$\mathbf{q} = \hat{c}E + \hat{d}\nabla T = -\hat{c}\nabla\varphi + \hat{d}\nabla T, \qquad (1)$$

and the Onsager principle of the symmetry of the kinetic coefficients is satisfied:

$$\hat{c} = -T\hat{b}, \qquad (2)$$
$$\hat{\sigma}(-H) = \hat{\sigma}^{\mathrm{T}}(H), \quad \hat{b}(-H) = \hat{b}^{\mathrm{T}}(H), \quad \hat{d}(-H) = \hat{d}^{\mathrm{T}}(H); \qquad (2a)$$

the index T denotes the transposed matrix. (Here and in what follows, we limit ourselves to the linear approximation in $\mathbf{E}$ and ∇T, which is virtually always sufficient.)

The fundamental equations of the problem are the conditions of continuity of particle and energy flux:

$$\operatorname{div}\mathbf{j} = 0, \quad \operatorname{div}\mathbf{q} = 0. \qquad (3)$$

The first equation is a strict equality, while the

[1)] That is, if the numbers of "holes" and electrons in the metal are equal, or if the orbits of the electrons are open in a plane perpendicular to the magnetic field. A strong magnetic field corresponds to a Larmor radius r that is small in comparison with l and with the linear dimension d of the sample.

second is valid only in the linear approximation (in **E** and ∇T) in which we are interested, with accuracy up to the energy dissipation in the specimen (quadratic effect).

Equations (3), with account of the relations (1), (2) and (2a), give differential equations of second order in φ and T. The role of the boundary conditions for these equations is played by the condition of the absence of an electric current through the surface of the sample:

$$j_n^s = 0 \tag{4}$$

(**n** is the inward normal to the surface of the metal, the index s means that the corresponding quantities are calculated on the surface of the specimen) and the heat flow condition on the boundary:

$$(T - T_s) + \alpha \partial T/\partial n = 0. \tag{5}$$

In the adiabatic case,

$$q_n^s = 0, \tag{6a}$$

and in the isothermal case,

$$T_s = \psi(r_s), \tag{6b}$$

where ψ is a given function of the point $\mathbf{r}_s$ on the surface of the sample.

By determining the temperature and potential distribution in the sample through the use of Eq. (3), we can find the charge distribution. The density of uncompensated charge is determined from the Poisson equation

$$\rho' = -(4\pi)^{-1} \operatorname{div} \mathbf{D}, \tag{7}$$

and for metals in the first approximation (see[2]) we have

$$\rho' = 0. \tag{8}$$

In a sample whose dimensions are equal to or less than the microscopic characteristics of the electron motion (free path lengths, radii of Larmor orbits), the situation is fundamentally different. The connection of the charge and energy fluxes **j** and **q** with the electrostatic potential φ and the temperature T cannot be assumed to be given beforehand—it must be determined from the solution of the microscopic problem: here the operators in (1) will be operators with respect to the coordinates also. It is clear that in the microscopic problem, the conservation of charge both in the volume and on the surface (in connection with the condition of reflection, at the boundary of the conductor, of the conduction electrons which do not penetrate the surface) and the conservation of energy (in the linear approximation) in the volume are automatically guaranteed—Eqs. (3) and (4) reduce to identities.

Microscopic theory also determines the connection of ρ' with the potential φ and the temperature T; as a result, Eq. (8), from the definition of the uncompensated charge density, becomes the fundamental equation of the problem. The replacement of Eq. (7) by Eq. (8) is connected with the high density of free electrons in the metal and has theoretical significance, inasmuch as only with such accuracy are the kinetic coefficients determined by the properties of the conductor alone (see[2]). Since the microscopic theory connects the temperature at any point of the sample,[2)] for example, with the specified temperature on the surface (see the boundary condition (6b)), Eq. (8) is an integral equation relative to the only remaining unknown function φ.

Thus, for small samples, the problem of the thermal conductivity divides into two parts:

1) the determination (from the microscopic theory) of the connection of the charge and energy fluxes **j** and **q** and the charge density ρ' with the potential φ and temperature T, and finding the temperature in the sample for given conditions of heat exchange on the surface;

2) solution of Eq. (8).

2. We proceed to the solution of the problem. As is well known, for the determination of the electric current density, the uncompensated charge density and the energy flux, it suffices to know the distribution function $n(\mathbf{r}, \mathbf{p})$ of the electrons:

$$\mathbf{j} = \frac{2e}{h^3} \int \mathbf{v} n(\mathbf{r}, \mathbf{p})\, d\mathbf{p}, \tag{9}$$

$$\mathbf{q} = \frac{2}{h^3} \int \mathbf{v} (\varepsilon - \mu_0) n(\mathbf{r}, \mathbf{p})\, d\mathbf{p}, \tag{10}$$

$$\rho' = \frac{2e}{h^3} \int \left\{ n(\mathbf{r}, \mathbf{p}) - n_0\left(\frac{\varepsilon - \mu_0}{T_0}\right) \right\} d\mathbf{p}; \tag{11}$$

μ_0 is the chemical potential, and $n_0(x) = (e^x + 1)^{-1}$ the equilibrium Fermi function; the integration is carried out over momentum space; $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ is the velocity of the electrons.

The distribution function $n(\mathbf{r}, \mathbf{p})$ can be found from the kinetic equation. However, since one can neglect the volume collisions in the first approximation in $1/l$, it is simpler to start out from physical considerations.

We shall assume the reflection of the electrons from the surface of the sample to be diffuse (this is correct in metals, since the deBroglie wavelength is of the order of the interatomic distance, i.e.,

[2)] It should be noted that the concept of a locally equilibrium temperature at a given point has a somewhat formal character for a small sample, since a locally equilibrium temperature can in turn be introduced only for a subsystem whose dimensions are large in comparison with the free path length.

extremely small even in comparison with the distance at which the boundary condition is established), so that the distribution function for the reflected electrons is an equilibrium one:

$$n(\mathbf{r}_s, \mathbf{p})|_{v_n>0} = n_0\left\{\frac{\varepsilon-\mu(\mathbf{r}_s)}{T(\mathbf{r}_s)}\right\}. \quad (12)$$

The chemical potential $\mu(\mathbf{r}_S)$ is generally not identical with the equilibrium μ_0, since the numbers of electrons moving toward the surface and away from it are not equal, owing to the electric field and the nonuniformity of the temperature (it is this directed flow which ensures the transport of charge and energy).

As they move from the surface, the electrons acquire an energy $\Delta\epsilon = -e\{\varphi(\mathbf{r}) - \varphi(\mathbf{r}_S)\}$; as a result, the density of electrons which have an energy ϵ at the point $\mathbf{r}$ is equal to the density of the electrons which have an energy $\epsilon - \Delta\epsilon$ at the point $\mathbf{r}_S$, i.e.,

$$n(\mathbf{r}, \mathbf{p}) = n_0\left\{\frac{\varepsilon + e[\varphi(\mathbf{r})-\varphi(\mathbf{r}_s)]-\mu(\mathbf{r}_s)}{T(\mathbf{r}_s)}\right\} = n_0\left(\frac{\varepsilon-\mu_0}{T_0}\right) + \frac{\partial n_0}{\partial\varepsilon}\{e\varphi(\mathbf{r}) - eh(\mathbf{r}_s) - (\varepsilon-\mu_0)\tau(\mathbf{r}_s)\}, \quad (13)$$

where the functions $h(\mathbf{r}_S)$ and $\tau(\mathbf{r}_S)$ are defined by the relations

$$eh(\mathbf{r}_s) = \mu(\mathbf{r}_s) - \mu_0 + e\varphi(\mathbf{r}_s),$$
$$T^{-1}(\mathbf{r}_s) = T_0^{-1}(1-\tau(\mathbf{r}_s)). \quad (14)$$

The point $\mathbf{r}_S$, from which come electrons to the point $\mathbf{r}$ with momentum $\mathbf{p}$, is determined by the intersection of the electron trajectory (for $\mathbf{E} = 0$ and $\tau(\mathbf{r}_S) = 0$) with the surface of the sample $G(\mathbf{r}_S) = 0$:

$$\mathbf{r} - \mathbf{r}_s = \mathbf{r}(t) - \mathbf{r}(\lambda) = \int_\lambda^t \mathbf{v}(t')\,dt',$$
$$G(\mathbf{r}_s) = 0, \quad \lambda < t, \quad v_n(\lambda) > 0, \quad (15)$$

where t is the time of revolution of the electron along its trajectory of motion in the magnetic field, λ is the instant of collision with the surface (obviously, the root of Eq. (15) closest to t); ϵ, p_H, and t determine $\mathbf{p}$ uniquely. Thus, the expression

$$\mathbf{r}_s = \mathbf{r} + \mathbf{r}(\lambda) - \mathbf{r}(t)$$

must be substituted in Eq. (13) in place of $\mathbf{r}_S$.

By substituting (13) in the expression for the electric (9) and thermal (10) fluxes and the uncompensated charge density (11), we find

$$\mathbf{j}(\mathbf{r}) = e\langle \mathbf{v}(t)h(\mathbf{r}+\mathbf{r}(\lambda)-\mathbf{r}(t))\rangle + \frac{\pi^2T^2}{3}\frac{d}{d\mu_0}\langle \mathbf{v}(t)\tau(\mathbf{r}+\mathbf{r}(\lambda)-\mathbf{r}(t))\rangle, \quad (16)$$

$$\mathbf{q}(\mathbf{r}) = \frac{\pi^2T^2}{3e}\langle \mathbf{v}(t)\tau(\mathbf{r}+\mathbf{r}(\lambda)-\mathbf{r}(t))\rangle + \frac{\pi^2T^2}{3}\frac{d}{d\mu_0}\langle \mathbf{v}(t)h(\mathbf{r}+\mathbf{r}(\lambda)-\mathbf{r}(t))\rangle, \quad (17)$$

$$\rho' = -\langle e\rangle\varphi(\mathbf{r}) + \langle eh(\mathbf{r}+\mathbf{r}(\lambda)-\mathbf{r}(t))\rangle + \frac{\pi^2T^2}{3}\frac{d}{d\mu_0}\langle \tau(\mathbf{r}+\mathbf{r}(\lambda)-\mathbf{r}(t))\rangle. \quad (18)$$

Here $\langle g\rangle$ denotes the integral of the function eg, computed over the Fermi surface:

$$\langle g\rangle \equiv e\int_{\varepsilon(\mathbf{p})=\mu_0} g\frac{dS_p}{v_\perp}.$$

By making use of Eq. (8) we obtain the potential $\varphi(\mathbf{r})$:

$$\varphi(\mathbf{r}) = \frac{1}{\langle e\rangle}\left\{\langle eh(\mathbf{r}+\mathbf{r}(\lambda)-\mathbf{r}(t))\rangle + \frac{\pi^2T^2}{3}\frac{d}{d\mu_0}\langle \tau(\mathbf{r}+\mathbf{r}(\lambda)-\mathbf{r}(t))\rangle\right\}. \quad (19)$$

It remains to determine the function $h(\mathbf{r}_S)$, requiring that the condition (5) of the continuity of charge flux at the boundary be automatically satisfied:

$$h(\mathbf{r}_s)\langle v_n\rangle_+ + \langle v_n(t)h(\mathbf{r}_s+\mathbf{r}(\lambda_s)-\mathbf{r}(t))\rangle_- = -\frac{\pi^2T^2}{3e}\frac{d}{d\mu_0}\{\langle v_n\rangle_+\tau(\mathbf{r}_s) + \langle v_n(t)\tau(\mathbf{r}_s+\mathbf{r}(\lambda_s)-\mathbf{r}(t))\rangle_-\}. \quad (20)$$

The meaning of the notation $\langle v_n\chi\rangle_+$ is that it is necessary to integrate $v_n\chi$ over the region where $v_n > 0$, while

$$\langle v_n\chi\rangle_- \equiv \langle v_n\chi\rangle - \langle v_n\chi\rangle_+.$$

To determine the connection between the thermal flux and the temperature gradient on the boundary of the conductor, the last component in Eq. (17) can be omitted, since its contribution to the thermal conductivity tensor is proportional to $(T/\mu_0)^2$, and account of this component would be of higher order of accuracy (everywhere only the first approximation in the parameter T/μ_0 has been used). Therefore the thermal conductivity of the sample is determined only by the given function $\tau(\mathbf{r}_S)$:

$$\mathbf{q}(\mathbf{r}) \approx \frac{\pi^2T^2}{3e}\langle \mathbf{v}(t)\tau(\mathbf{r}+\mathbf{r}(\lambda)-\mathbf{r}(t))\rangle.$$

To determine the thermoelectric field, it is necessary to know the function $h(\mathbf{r}_S)$, i.e., to solve Eq. (20). In the case of a plane-parallel plate this problem is solved exactly for $l = \infty$ for any value of an oblique magnetic field $H \neq 0$.

3. We consider a thin ($d \ll l$) plane-parallel plate placed in an oblique magnetic field. In this

case the collisions of the electrons with the surfaces of the plate take place much more frequently than collisions inside the conductor, and it is enough for us to compute the asymptotic expressions for the electric and thermal fluxes, when the free path l of the electrons tends to infinity.

The theory of thermomagnetic phenomena in a plane-parallel plate can be constructed in the general case for an arbitrary temperature distribution over the surfaces of the plate. However, we shall limit ourselves (only to be specific; generalization to the general case is similar to the problem of the electrical conductivity with arbitrary contacts[2] and presents no difficulties) to the case which is "natural" for the experiment, in which the temperatures are given on the faces of the plate and, for example, there is no heat transfer on its surfaces:

$$q_\xi(0) = q_\xi(d) = 0. \tag{21}$$

In the microscopic problem, it is sufficient here to consider the dependence of all the quantities only on ξ, the coordinate normal to the surface of the plate. This means that the equations of continuity are automatically satisfied only in the first approximation:

$$\partial j_\xi/\partial\xi = 0, \quad \partial q_\xi/\partial\xi = 0, \tag{22}$$

while it follows from Eq. (3) (η and ζ are the coordinates in the plane of the plate)

$$\frac{\partial j_\eta}{\partial\eta} + \frac{\partial j_\zeta}{\partial\zeta} = 0, \quad \frac{\partial q_\eta}{\partial\eta} + \frac{\partial q_\zeta}{\partial\zeta} = 0. \tag{23}$$

Equations (23) now again lead, in conjunction with Eqs. (16) and (17), to constancy of the gradients of the potential φ and the temperature T in the plane of the plate, which is natural for the one-dimensional problem. If the temperatures of the upper and lower surfaces at the ends of the plate are the same, then the temperature gradient on the boundary of the conductor is not only uniform but also the same on the lower and upper surfaces of the plate.

The choice of axes in the plane of the plate for metals with closed Fermi surfaces does not play an important role. In metals with open Fermi surfaces, it is convenient to use the line of intersection of the plane of the plate with the plane of all possible drifts of the electrons as the η axis (the yz plane; the z axis is the direction of the magnetic field; for simplicity, we assume that the open plane cross sections p_z = const of the Fermi surface have a common mean direction p_x and the motion is finite in the direction of the x axis).

Because of the constancy of E_η and E_ζ, the function $h(\xi, \eta, \zeta)$ should be sought in the form

$$h(\xi, \eta, \zeta) = h_1(\xi) - E_\eta\eta - E_\zeta\zeta, \tag{24}$$

while the function $\tau(\xi, \eta, \zeta)$ can be represented in the following fashion

$$\tau(\xi, \eta, \zeta) = \tau_1(\xi) + \frac{1}{T_0}\left(\frac{\partial T}{\partial\eta}\eta + \frac{\partial T}{\partial\zeta}\zeta\right). \tag{25}$$

It would appear that the temperature ought to be the same along the transverse cross section of the plate. However, the frequent traveling back and forth of the electron between the two surfaces along the magnetic field leads to the equalization of the temperatures just along the magnetic field. Therefore, $\tau_1(0) \neq \tau_1(d)$. To determine the temperature difference along the ξ axis on the surfaces of the plate, one can use the boundary condition (21) for the thermal flux q_ξ.

Making use of Eqs. (20) and (21), we determine the potential difference and the difference in temperatures on the surfaces of the plate. Omitting simple transformations, and limiting ourselves to the principal terms of the expansion in powers of T/μ_0, we get

$$h_1(0) - h_1(d) = \frac{a_{\xi\alpha}}{\langle\bar{v}_\xi\rangle^-}E_\alpha, \qquad \alpha \equiv (\eta, \zeta), \tag{26}$$

$$\tau_1(0) - \tau_1(d) = \frac{a_{\xi\alpha}}{\langle\bar{v}_\xi\rangle^+}\frac{\partial T}{\partial x_\alpha}, \tag{27}$$

where

$$a_{i\alpha}(\xi) \equiv \{\langle S_- v_i(t)\,(r_\alpha(t) - r_\alpha(\lambda))\rangle^- + \langle(1 - S_-)\,v_i(t)\,(r_\alpha(t) - r_\alpha(\lambda'))\rangle^- + \langle S_+ v_i(t)\,(r_\alpha(t) - r_\alpha(\lambda'))\rangle^+ + \langle(1 - S_+)\,v_i(t)\,(r_\alpha(t) - r_\alpha(\lambda))\rangle^+\}. \tag{28}$$

Here S_- is a step function, equal to unity when the electron traveling toward the surface $\xi = 0$ is reflected from this same surface, and equal to zero if the electron arrives by being reflected from the other surface of the plate ($\xi = d$). The function S_+ has the same meaning, except that the roles of the surfaces have been reversed.

In expressions of the type $\langle\chi\rangle^-$, the integration is carried out over all the states of the electrons on the Fermi surface, for which the value of the velocity averaged over one period is $\bar{v}_\xi < 0$, and $\langle\chi\rangle^+ = \langle\chi\rangle - \langle\chi\rangle^-$; $\lambda(\xi; t)$ and $\lambda'(\xi; t)$ are the instants of reflection of the electrons from the surface $\xi = 0$ and the surface $\xi = d$, i.e., the roots of the equations

$$r_\xi(t) - r_\xi(\lambda) = \xi, \qquad r_\xi(t) - r_\xi(\lambda') = \xi - d.$$

It is easy to note that $a_{\xi\alpha}$ does not depend on ξ, which guarantees the vanishing of j_ξ and q_ξ at any

depth inside the plate:

$$j_\xi(\xi) = q_\xi(\xi) = 0.$$

The fact that we have determined only the temperature difference (27) on the two surfaces of the plate is connected with a specific feature of the one-dimensional problem, namely that the ends—the points of heat supply, where the temperature is fixed—are not considered. Therefore, "instead," it is necessary to fix the temperature at some point of one of the surfaces. (It is clear that there ought to be supplies of heat or current without fail in any case, for otherwise, under adiabatic conditions and in the absence of an outside emf, an equilibrium state with constant temperature and potential would be established throughout the sample.)

By substituting Eqs. (24)—(27) in the expressions (16) and (17) for the electric and thermal fluxes, we obtain relations which connect the electric and thermal fluxes with the uniform electric field and the temperature gradient applied along the plate:

$$j_\alpha(\xi) = \sigma_{\alpha\beta}(\xi)E_\beta - \frac{\pi^2 T}{3e}\frac{d\sigma_{\alpha\beta}(\xi)}{d\mu_0}\frac{\partial T}{\partial x_\beta}, \quad (29)$$

$$q_\alpha(\xi) = \frac{\pi^2 T^2}{3e}\frac{d\sigma_{\alpha\beta}(\xi)}{d\mu_0}E_\beta - \frac{\pi^2 T}{3e^2}\sigma_{\alpha\beta}(\xi)\frac{\partial T}{\partial x_\beta}, \quad (30)$$

where

$$\sigma_{\alpha\beta}(\xi) = a_{\alpha\beta}(\xi) + \frac{\langle S_- v_\alpha\rangle^- + \langle (1-S_+)v_\alpha\rangle^+}{\langle \bar v_\xi\rangle^-}a_{\xi\beta} \quad (31)$$

is the electric conductivity tensor in the plane of the plate, the method of calculation of which is set forth in detail in the work of the authors[2], and $(\alpha, \beta) \equiv (\eta, \zeta)$. It so happens that the tensor $\sigma_{\alpha\beta}(\xi)$ is computed there for plates and wires in the case of closed Fermi surfaces. For brevity, the step function $S_\pm$ has been set equal to zero in all intermediate formulas in[2], inasmuch as the final result did not depend in any essential fashion on consideration of electrons which collided twice with the same wall within the period of motion in the magnetic field.

We note that the relations (29) and (30) correspond to the Onsager principle of the symmetry of the kinetic coefficients (2):

$$c_{\alpha\beta}(\xi) = -Tb_{\alpha\beta}(\xi) = \frac{\pi^2 T^2}{3e}\frac{d}{d\mu_0}\sigma_{\alpha\beta}(\xi).$$

Calculation of the electric conductivity tensor of the plate $\sigma_{\alpha\beta}(\xi)$ in the case of metals with open Fermi surfaces does not present any difficulties. In strong magnetic fields, $\sigma_{\eta\eta}$ does not depend on ξ:

$$\sigma_{\eta\eta} = \delta a_{\eta\eta}{}^0\frac{\langle \bar v_z{}^2\rangle\langle \bar v_y{}^2\rangle - \langle \bar v_y \bar v_z\rangle^2}{\langle \bar v_\xi{}^2\rangle^2},$$

where δ is the thickness of the layer of the open trajectories in momentum space, and $a^0_{\eta\eta}$ is identical in order of magnitude with the electrical conductivity of the plate in the absence of a magnetic field. All the remaining components of the electric conductivity tensor are proportional to r. Therefore the electric current in the direction of the η axis is uniform ($\rho_{\eta\eta} \approx \sigma^{-1}_{\eta\eta}$) and a sharp nonuniformity of the electric current appears when its direction departs from the η axis by an angle $\theta \gg r/d$.

So far as the mean value of the electric conductivity tensor

$$\sigma_{\alpha\beta} = d^{-1}\int_0^d \sigma_{\alpha\beta}(\xi)\,d\xi$$

is concerned, all the Onsager relations are satisfied for it, and $\sigma_{\xi\xi} \propto r^2/d$ and the asymptote of the resistance in strong magnetic fields have the same character as in an unbounded sample.[4,5]

The uniform thermoelectric field E_α is easily determined from the condition $j_\alpha = 0$ (in Eq. (29), we must set $j_\alpha = 0$). For the determination of the nonuniform electric field, it is necessary to use the condition of electrical neutrality of the metal (8). As a result, we have

$$E_\alpha = \frac{\pi^2 T}{3e}\sigma_{\alpha\gamma}{}^{-1}\frac{d\sigma_{\gamma\beta}}{d\mu_0}\frac{\partial T}{\partial x_\beta}; \quad (32)$$

$$E_\xi = -\frac{\partial\varphi(\xi,\eta,\zeta)}{\partial\xi} = \frac{\pi^2 T}{3e\langle 1\rangle}\left\{\sigma_{\alpha\gamma}{}^{-1}\frac{d\sigma_{\gamma\beta}}{d\mu_0}\frac{d}{d\xi}\langle (r_\alpha(\lambda) - r_\alpha(t))\rangle - \frac{d^2}{d\mu_0 d\xi}\langle (r_\beta(\lambda) - r_\beta(t))\rangle\right\}\frac{\partial T}{\partial x_\beta}. \quad (33)$$

In those cases in which the thermal resistance reaches saturation in strong magnetic fields, the electric field E_ξ is practically uniform, since $(r_\alpha(\lambda) - r_\alpha(t)) \sim \xi$. In all other cases, the electric field E_ξ is essentially nonuniform.

The results obtained above are valid if the electrons collide much more frequently with the surfaces of the conductor than inside the volume. Asymptotic expressions for the electric and thermal fluxes and the electric field correspond to the zeroth approximation in the parameter d/l. We note that there are always electrons with $\bar v_\xi \approx 0$, which either collide with the surface within a time shorter than the period of revolution of the electron in the Larmor orbit (the electrons close to the surface), or do not collide with the surface within the free path time (electrons in the interior of the plate). It is not difficult to show that the role of such electrons in the electrical and thermal conductivity is not important if the angle that the magnetic field makes with the surface of the sample is $\vartheta \gg d/l$.

In the opposite case $\vartheta \lesssim d/l$, i.e., in a magnetic field almost parallel to the surface of the sample, an electron with a closed orbit cannot collide with the two surfaces of the plate if the plate thickness exceeds the diameter of orbit of the electron in the magnetic field. Therefore it is not possible to determine the difference in potential on the surfaces of the plate with the help of Eqs. (20) and (21). In this case, it is impossible to regard the mean free path as infinite, since $\overline{v}_\xi \approx 0$ for a significant fraction of the electrons (for all except electrons with open orbits), and only volume collisions of electrons are possible in the interior of the sample, and much more important than surface collisions.

4. Thus the thermoelectric coefficients were shown to be connected with the electric conductivity tensor in simple fashion. We can thus conclude that in thin metallic samples the thermal flux has the same features as the electric current. In those cases in which the electrical and thermal resistances increase quadratically with the strong magnetic field (metals with open Fermi surfaces or metals with equal numbers of electrons and "holes," $n_1 = n_2$), the thermal flux and the electric current are highly nonuniform, being concentrated principally in the region near the surface of the conductor, while their directions can be reversed in the interior of the conductor.

With accuracy up to terms of the order of T/μ_0, the components of the thermal conductivity tensor of a thin plate are equal to

$$\varkappa_{\alpha\beta}(\xi) = \frac{\pi^2 T}{3e^2}\sigma_{\alpha\beta}(\xi), \tag{34}$$

and the law of Wiedemann and Franz holds for any value of ξ.

Moreover, as was pointed out earlier, the Kelvin-Onsager relations are satisfied at any depth in a thin plate, i.e., the thermal diffusion tensor of the electrons, multiplied by the temperature, is equal to the tensor of "diffusion thermal conductivity."

It is not difficult to show that in strong magnetic fields the thermal flux in a thin conductor is also highly nonuniform. In a statement of the problem analogous to that in the case of the plate, it is necessary to make use of the results of Sec. 6 of the previous paper of the authors.[2] There is no necessity of carrying out the calculations here, inasmuch as the problem of the distribution of thermal flux is entirely analogous to the problem of the electrical conductivity of a wire.[2]

Evidently an analogy exists, even in bulk samples, between the distributions of the thermal and electrical fluxes in the sample. For bulk samples, it is convenient to seek a solution of the kinetic equation

$$\frac{\partial}{\partial t}\{n(\mathbf{r},\mathbf{p}) - n_0\} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} n(\mathbf{r},\mathbf{p}) + \hat{W}\{n(\mathbf{r},\mathbf{p}) - n_0\} = -\frac{\partial n_0}{\partial \varepsilon} e\mathbf{E}\mathbf{v} \tag{35}$$

immediately in the following form:

$$n(\mathbf{r},\mathbf{p}) = n_0\left(\frac{\varepsilon - \mu_0}{T_0}\right) - \frac{\partial n_0}{\partial \varepsilon}\left\{e\psi_1(\mathbf{r},\mathbf{p}) + \frac{\varepsilon - \mu_0}{T_0}\psi_2(\mathbf{r},\mathbf{p})\right\}. \tag{36}$$

In the case of a bulky plane-parallel plate, the uniform electric field E_β and constant temperature gradient $\partial T/\partial x_\beta$ can be taken outside the sign of the operator $\partial/\partial t + \mathbf{v}\partial/\partial \mathbf{r} + \hat{W}$, and the kinetic equation (35) can be linearized over a small temperature gradient along the surface of the plate. Then the functions ψ_1 and ψ_2 will satisfy the following equations:

$$\frac{\partial \psi_1}{\partial t} + \mathbf{v}\frac{\partial \psi_1}{\partial \mathbf{r}} + \tilde{W}\{\psi_1\} = \mathbf{v}\mathbf{E},$$

$$\frac{\partial \psi_2}{\partial t} + \mathbf{v}\frac{\partial \psi_2}{\partial \mathbf{r}} + \tilde{W}_1\{\psi_2\} = -\mathbf{v}\frac{\partial T}{\partial \mathbf{r}},$$

where the linear operators $\tilde{W}\{\psi_1\}$ and $\tilde{W}_1\{\psi_2\}$ are connected with the collision integral $\hat{W}$ by the relations

$$\hat{W}\left\{\psi_1\frac{\partial n_0}{\partial \varepsilon}\right\} = \frac{\partial n_0}{\partial \varepsilon}\tilde{W}\{\psi_1\},$$

$$\hat{W}\left\{\psi_2\frac{\varepsilon - \mu_0}{T_0}\frac{\partial n_0}{\partial \varepsilon}\right\} = \frac{\varepsilon - \mu_0}{T_0}\frac{\partial n_0}{\partial \varepsilon}\tilde{W}_1\{\psi_2\}.$$

If the kernels of the integral operators $\tilde{W}\{\psi_1\}$ and $\tilde{W}_1\{\psi_2\}$ are the same (which is the case for temperatures high in comparison with the Debye temperatures and for very low temperatures, when the scattering of the energy flux and momentum flux by the impurities is important[6]), then we find that for the electric and thermal fluxes

$$\mathbf{j} = e^2\langle \mathbf{v}\psi_1(\mathbf{r},\mathbf{p})\rangle + \frac{\pi^2 T e}{3}\frac{d}{d\mu_0}\langle \mathbf{v}\psi_2(\mathbf{r},\mathbf{p})\rangle,$$

$$\mathbf{q} = \frac{\pi^2 T}{3}\langle \mathbf{v}\psi_2(\mathbf{r},\mathbf{p})\rangle + \frac{\pi^2 e T^2}{3}\frac{d}{d\mu_0}\langle \mathbf{v}\psi_1(\mathbf{r},\mathbf{p})\rangle$$

the formulas (29) and (30) are rigorously valid, with $\sigma_{\alpha\beta}(\xi)$ the thermal conductivity tensor of the plate, computed for finite free path length l. Therefore, the Wiedemann-Franz law (34) is also valid in a bulky plate for any value of ξ.

Account of thermal radiation by the surface of the conductor in the medium recalls the problem of the electrical conductivity with asymmetrical current contacts. Therefore some results on the distribution of the electric current in conductors with asymmetric contacts can be transferred to the case

of the thermal conductivity of samples covered by a thermal isolator with apertures. (The role of these apertures, which guarantee removal of the heat, is similar to the role of current contacts.)

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