

RESONANCE EFFECTS IN THE FIELD OF AN INTENSE LASER BEAM

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Submitted to JETP editor November 22, 1966

J. Exptl. Theoret. Phys. (U.S.S.R.) 52, 1049-1067 (April, 1967)

The scattering of an electron by an electron with exchange of a virtual photon (Moller scattering), in the presence of the field of a monochromatic laser beam, is considered. It is shown that the effective cross section of the Moller scattering contains resonances connected with the discrete nature of the energy spectrum of the electron + plane electromagnetic wave system. The resonance infinities are eliminated with the aid of the total Green's function of the photon $D(k_1, k|A)$, calculated in the vicinity of the point $k^2 = 0$, and the magnitude of the resonant Moller scattering is estimated. It is indicated that the effective interaction potential of two electrons in the field of a plane electromagnetic wave becomes an electron-attraction potential if certain relations are satisfied between the electron momenta, the electric-component intensity, and the electromagnetic-wave frequency. The polarization operator is calculated in the Appendix in second order of perturbation theory with respect to the interaction with the quantized electromagnetic field. The results are gauge-invariant.

1. INTRODUCTION

It is presently possible to obtain with the aid of lasers electromagnetic fields in which the electric-component intensity reaches 10^8 V/cm. The action of such fields on matter gives rise to new physical effects, which depend nonlinearly on the intensity of the laser beam, for example ionization of atoms at frequencies lower than I_0/\hbar (I_0 —ionization energy),^[1] multiquantum dissociation of molecules,^[2] multiquantum interband transitions in solids,^[3] decrease of the coefficient of light absorption by conduction electrons in semiconductors with increasing laser-beam intensity,^[4] and others. Most papers are devoted to a theoretical investigation of the elementary processes which occur in the presence of an intense monochromatic laser beam, such as photon emission, annihilation and creation of electron-positron pairs,^[5] pair production in the field of a nucleus,^[6] bremsstrahlung and absorption in a Coulomb field,^[7] Compton scattering,^[8,9] and coalescence of two laser-beam quanta into one of double frequency in a Coulomb field.^[10]

All the foregoing papers consider processes of not higher than first order of perturbation theory in the interaction with the quantized electromagnetic field. Analysis of higher orders is made difficult by the ultraviolet divergences which are characteristic of quantum field theory. At the same time, an investigation of processes of second and higher orders in the interaction with the quantized

electromagnetic field, which occur in the presence of a laser beam, is apparently of greatest physical interest in connection with the resonant character of the probabilities corresponding to them. The appearance of resonances in the probabilities of processes occurring in the field of a laser beam is connected with the discrete nature of the energy spectrum of the electron + plane electromagnetic wave system. In spite of the fact that the potential of the electromagnetic field depends on the time (and consequently there are no stationary states in the usual sense in the system under consideration), we can speak of energy levels of a quasiparticle corresponding to the electron + electromagnetic wave system. The energy $\epsilon_p - r\omega$ and the momentum $\mathbf{p} - r\omega\mathbf{n}$ ($r = 0, \pm 1, \dots$) of this quasiparticle are connected by relation (15). It is this change of the electron spectrum in the electromagnetic field which leads to the resonant singularities in the probabilities of the processes such as Moller scattering, Compton scattering (with participation of two photons of the quantized electromagnetic field), and others occurring in an external electromagnetic field.

We consider in this paper Moller scattering of electrons in the presence of an intense laser beam.

In Sec. 2 we derive a gauge-invariant formula for the probability of the Moller scattering. An investigation of the formula shows that under certain relations between the energy, momentum, and the scattering angle of the electron and the parameters

characterizing the electromagnetic wave, resonances due to the discrete spectrum of the electron in the laser-beam field appear in the scattering probability. At the resonance points, the scattering probability becomes infinite. To eliminate the resonance divergence, the free Green's function of the photon is replaced by the total Green's function of the photon, which takes into account the action of both the quantized and the external electromagnetic fields.

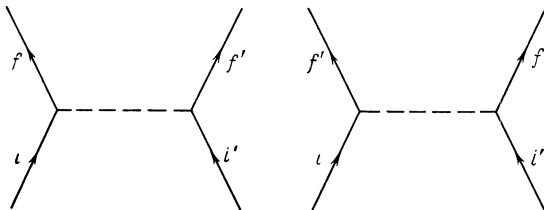
In Secs. 3 and 4 we calculate the total Green's function of the photon and investigate with its aid the resonances in the Moller-scattering cross section. The presented numerical estimate shows that the Moller-scattering cross section may turn out to be quite large as a result of the resonances.

In Sec. 4 we indicate also that under certain conditions the effective potential of interaction of two electrons in the presence of a strong electromagnetic field acquires terms corresponding to electron attraction, and at electron momentum values near the resonance points these terms can exceed the Coulomb-repulsion potential. This gives rise to a possibility of pairing of electrons in a strong electromagnetic field.

In the Appendix we calculate the polarization operator (in second order of perturbation theory with respect to the interaction with the quantized electromagnetic field) in the presence of an external electromagnetic field, and renormalize this operator.

2. SCATTERING OF AN ELECTRON BY AN ELECTRON IN THE FIELD OF A PLANE ELECTROMAGNETIC WAVE

The collision of two electrons with exchange of a virtual photon (Moller scattering) in the presence of an external electromagnetic field is described in second-order perturbation theory with respect to interaction with the quantized electromagnetic field by means of the diagrams shown in the figure. The corresponding matrix element is written as follows:



The dashed lines denote the free Green's function of the photon, and the solid lines the wave function of the electron in the field A.

$$M = \frac{ie^2}{(2\pi)^4} \sum_{\mu} g^{\mu\nu} \int d^4z_1 d^4z_2 d^4k \frac{\exp\{-i(k, z_1 - z_2)\}}{k^2 + i\epsilon} \times \{[\bar{\Psi}_{f'}(1) \gamma_{\mu} \Psi_{i'}(1)] [\bar{\Psi}_f(2) \gamma_{\mu} \Psi_i(2)] - [\bar{\Psi}_f(1) \gamma_{\mu} \Psi_i(1)] \times [\bar{\Psi}_{f'}(2) \gamma_{\mu} \Psi_{i'}(2)]\}; \quad \Psi_{\alpha}(1) \equiv \Psi_{p_{\alpha}}(z_1), \quad (\alpha = i, i', f, f'),$$

$$\bar{\Psi}_{\alpha}(1) = \Psi_{\alpha}^+(1) \gamma_0, \quad \epsilon \rightarrow +0, \quad z = (t, r), \quad (kz) = k_0 t - \mathbf{k}r, \quad g^{00} = 1, \quad g^{ii} = -1 \quad (i = 1, 2, 3). \quad (1)$$

Here $\psi_p(z)$ —wave functions of the electron in the external electromagnetic field with potential $A(A_0, \mathbf{A})$; the normalization volume is $V = 1$; $p_i, p_{i'},$ and $p_f, p_{f'}$ are the 4-momenta of the electrons in the initial and final states, respectively.

We shall henceforth use for the potential A

$$A(z) = a \cos(k'z), \quad a = (a_0, \mathbf{a}), \quad k' = (k'_0, \mathbf{k}') = \omega n, \quad n = (1, \mathbf{n}); \quad (2)$$

a, n, and ω are respectively the 4-amplitude, 4-vector of propagation direction, and frequency of the electromagnetic wave. For a field with potential (2), the wave functions $\psi_p(z)$ of the electron take the form¹⁾[11,9]

$$\psi_p(z) = \left(1 - \frac{e}{2} \frac{\hat{a}\hat{n}}{(np)} \cos \omega(nz)\right) u_p \exp\{-iS_p(z) - i(pz)\},$$

$$S_p(z) = \kappa_{1p} \sin \varphi + \kappa_{2p} (\sin 2\varphi + 2\omega(nz)),$$

$$\varphi = \omega(nz), \quad \kappa_{1p} = \frac{e(ap)}{\omega(np)}, \quad \kappa_{2p} = -\frac{e^2 a^2}{8\omega(np)}, \quad (3)$$

u_p —Dirac bispinor.

The integrations in (1), using the wave functions (3), yield

$$M = ie^2 (2\pi)^4 \sum_{\mu=1}^4 g^{\mu\nu} \sum_{s, s_1} \delta^4 [p_f + p_{f'} - p_i - p_{i'} + sk' + 2k'(\kappa_{2f} + \kappa_{2f'} - \kappa_{2i} - \kappa_{2i'})] \times \left\{ \frac{[\bar{u}_{f'} R_{s-s_1}^{\mu}(f', i') u_{i'}] [\bar{u}_f R_{s_1}^{\mu}(f, i) u_i]}{[p_i - p_{f'} - s_1 k' - 2k'(\kappa_{2f} - \kappa_{2i})]^2} - \frac{[\bar{u}_f R_{s-s_1}^{\mu}(f, i') u_i] [\bar{u}_{f'} R_{s_1}^{\mu}(f', i) u_{i'}]}{[p_i - p_{f'} - s_1 k' - 2k'(\kappa_{2f'} - \kappa_{2i})]^2} \right\}, \quad (4)$$

$$R_s^{\mu}(f, i) = L_s^{(0)}(f, i) \gamma_{\mu} + L_s^{(1)}(f, i) \frac{1}{2} \left(\frac{1}{(np_f)} - \frac{1}{(np_i)} \right) \hat{e}\hat{a}\hat{n}\gamma_{\mu} - \frac{e}{(np_i)} (a_{\mu}\hat{n} - n_{\mu}\hat{a}) L_s^{(1)}(f, i) - n_{\mu}\hat{n} \frac{1}{2} \frac{e^2 a^2}{(np_i)(np_f)} L_s^{(2)}(f, i),$$

¹⁾We use a system of units in which $c = \hbar = 1$. The following notation is used: $(ab) = a_0 b_0 - \mathbf{a} \cdot \mathbf{b}$ — scalar product of the 4-vectors $a = (a_0, \mathbf{a})$ and $b = (b_0, \mathbf{b})$; $a = a_0 \gamma_0 - \mathbf{a} \cdot \boldsymbol{\gamma}$, γ_{μ} — Dirac matrices.

$$L_s^{(j)}(f, i) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} d\varphi S^{(j)}(\varphi) \exp \{i[\sin \varphi (\kappa_{1f} - \kappa_{1i}) + \sin 2\varphi (\kappa_{2f} - \kappa_{2i}) - s\varphi]\}, \quad S^{(0)} = 1, \quad S^{(1)} = \cos \varphi,$$

$$S^{(2)} = \cos^2 \varphi, \quad \kappa_{1i} \equiv \kappa_{1p_i}, \quad \kappa_{2i} \equiv \kappa_{2p_i}, \quad u_i \equiv u_{p_i}. \quad (4a)$$

The square of the matrix element, averaged over the initial and summed over the final spin states of the electrons, is calculated in the usual fashion (see, for example, [12]). The final result is:

$$\frac{1}{4} \sum \frac{|M|^2}{TV} = \frac{1}{2} \frac{e^4 (2\pi)^4}{\varepsilon_i \varepsilon_i' \varepsilon_f \varepsilon_f'} \sum_{s, s_1, s_2} \delta^4 [p_f + p_{f'} - p_i - p_{i'} + sk' + 2k'(\kappa_{2f} + \kappa_{2f'} - \kappa_{2i} - \kappa_{2i'})]$$

$$\times \left\{ \frac{\Phi_{s, s_1, s_2}^{(1)} Y_1 + C_1}{[p_i - p_f - s_2 k' - 2k'(\kappa_{2f} - \kappa_{2i})]^2 [p_i - p_{f'} - s_1 k' - 2k'(\kappa_{2f'} - \kappa_{2i'})]^2} + \frac{\Phi_{s, s_1, s_2}^{(2)} Y_2 + C_2}{[p_i - p_{f'} - s_2 k' - 2k'(\kappa_{2f'} - \kappa_{2i'})]^2 [p_i - p_f - s_1 k' - 2k'(\kappa_{2f} - \kappa_{2i})]^2} - \frac{\Phi_{s, s_1, s_2}^{(3)} Y_3 + C_3}{[p_i - p_f - s_2 k' - 2k'(\kappa_{2f} - \kappa_{2i})]^2 [p_i - p_{f'} - s_1 k' - 2k'(\kappa_{2f'} - \kappa_{2i'})]^2} \right\}. \quad (5)$$

We have introduced here the notation

$$\Phi_{s, s_1, s_2}^{(1)} = L_{s_1}^{(0)}(f, i) L_{s_2}^{(0)}(f, i) L_{s-s_1}^{(0)}(f', i') L_{s-s_2}^{(0)}(f', i'),$$

$$\Phi_{s, s_1, s_2}^{(2)} = L_{s_1}^{(0)}(f', i) L_{s_2}^{(0)}(f', i) L_{s-s_1}^{(0)}(f, i') L_{s-s_2}^{(0)}(f, i'),$$

$$\Phi_{s, s_1, s_2}^{(3)} = L_{s_1}^{(0)}(f', i) L_{s_2}^{(0)}(f, i) L_{s-s_1}^{(0)}(f, i') L_{s-s_2}^{(0)}(f', i'),$$

$$Y_1 = 2m^4 - m^2((p_i p_f) + (p_i p_{f'})) + (p_i p_{i'}) (p_f p_{f'}) + (p_i p_{f'}) (p_{i'} p_f),$$

$$Y_2 = 2m^4 - m^2((p_i p_{f'}) + (p_{i'} p_f)) + (p_i p_{i'}) (p_f p_{f'}) + (p_i p_{f'}) (p_{i'} p_{f'}), \quad Y_3 = -2m^4 - 2(p_i p_{i'}) (p_f p_{f'}) + m^2[(p_i p_f) + (p_i p_{f'}) + (p_i + p_f, p_{i'} + p_{f'})],$$

$$\varepsilon_\alpha = p_{\alpha 0} = \sqrt{p_\alpha^2 + m^2} \quad (\alpha = i, i', f, f'), \quad (6)$$

C_i ($i = 1, 2, 3$) are certain functions of order ea/m compared with the functions $\Phi^{(i)} Y_i$ (at $s = s_1 = s_2 = 0$).

It is easy to verify that expression (5) is invariant against the gauge transformation (λ is an arbitrary constant)

$$a \rightarrow a' = a + \lambda k'. \quad (7)$$

For the differential effective scattering cross section we obtain (J—particle flux density)

$$d\sigma = \frac{2e^4}{(2\pi)^2} \frac{1}{\varepsilon_i \varepsilon_i'} \sum_{s, s_1, s_2} \frac{1}{J} \int d^4 p_f d^4 p_{f'} \Theta(p_{f0}) \Theta(p_{f'0}) \delta(p_f^2 - m^2) \times |\delta(p_{f'}^2 - m^2) \delta^4 [p_f + p_{f'} - p_i - p_{i'} + sk' + 2k'(\kappa_{2f} + \kappa_{2f'} - \kappa_{2i} - \kappa_{2i'})] \{ \dots \},$$

$$\Theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0. \end{cases} \quad (8)$$

In the last expression we make the following change of integration variables (the Jacobian of this transformation is equal to unity):

$$p_f + 2k' \kappa_{2f} = \tilde{p}_f, \quad p_{f'} + 2k' \kappa_{2f'} + sk' = \tilde{p}_{f'}, \quad (9a)$$

and integrate with respect to \tilde{p}_{f_0} and $\tilde{p}_{f'_0}$:

$$d\sigma = \frac{e^4}{(2\pi)^2} \frac{1}{2\varepsilon_i \varepsilon_i'} \sum_{s, s_1, s_2} \int d^3 \tilde{p}_f d^3 \tilde{p}_{f'} \frac{1}{J} \frac{1}{\varepsilon_f \varepsilon_{f'}} \times \delta^4 (\tilde{p}_f + \tilde{p}_{f'} - \tilde{p}_i - \tilde{p}_{i'}) \times \left\{ \frac{\tilde{\Phi}_{s, s_1, s_2}^{(1)} \tilde{Y}_1 + \tilde{C}_1}{(\tilde{p}_i - \tilde{p}_f - s_2 k')^2 (\tilde{p}_i - \tilde{p}_{f'} - s_1 k')^2} + \frac{\tilde{\Phi}_{s, s_1, s_2}^{(2)} \tilde{Y}_2 + \tilde{C}_2}{(\tilde{p}_i - \tilde{p}_{f'} - s_2 k')^2 (\tilde{p}_i - \tilde{p}_f - s_1 k')^2} - \frac{\tilde{\Phi}_{s, s_1, s_2}^{(3)} \tilde{Y}_3 + \tilde{C}_3}{(\tilde{p}_i - \tilde{p}_f - s_2 k')^2 (\tilde{p}_i - \tilde{p}_{f'} - s_1 k')^2} \right\}; \quad (10)$$

$$p_\alpha + 2k' \kappa_{2\alpha} = \tilde{p}_\alpha \quad (\alpha = i, i'), \quad \tilde{\varepsilon}_\alpha = (\tilde{p}_\alpha^2 + m^{*2})^{1/2},$$

$$(\alpha = i, i', f), \quad \tilde{\varepsilon}_{f'} = sk'_0 + ((\tilde{p}_{f'} - sk')^2 + m^{*2})^{1/2},$$

$$m^{*2} = m^2 - e^2 a^2 / 2. \quad (9b)$$

The symbols $\tilde{\Phi}^{(i)}$, \tilde{Y}_i , and \tilde{C}_i denote respectively the functions $\Phi^{(i)}$, Y_i , and C_i in terms of the variables (9a) and (9b).

From (9a) and (9b) we get

$$p_\alpha^2 - m^2 = \tilde{p}_\alpha^2 - m^{*2} = 0 \quad (\alpha = i, i', f),$$

$$p_{f'}^2 - m^2 = (\tilde{p}_{f'} - sk')^2 - m^{*2} = 0 \quad (s = 0, \pm 1, \dots). \quad (9c)$$

To simplify the derivation we change over, in the analysis of (10), to an auxiliary coordinate system, in which the following relations are satisfied (all the quantities pertaining to this system of coordinates will be designated throughout by a unity superscript, for example $\tilde{\mathbf{p}}_1^1$)

$$\tilde{\mathbf{p}}_i^1 + \tilde{\mathbf{p}}_{i'}^1 = 0, \quad \tilde{\mathbf{p}}_f^1 + \tilde{\mathbf{p}}_{f'}^1 = 0,$$

$$2\tilde{\varepsilon}_i^1 = \tilde{\varepsilon}_f^1 + \tilde{\varepsilon}_{f'}^1, \quad |\tilde{\mathbf{p}}_i^1| \neq |\tilde{\mathbf{p}}_f^1| \quad \text{for } s \neq 0. \quad (11)$$

The system defined by (11) goes over in the ab-

sence of an external field into the ordinary c.m.s. of two electrons, so that we shall denote it the \tilde{c} -system. Carrying out in (10) integration in the \tilde{c} -system with the aid of a δ function, we arrive at the expression

$$d\sigma = \frac{e^4}{(2\pi)^2} \frac{1}{2\varepsilon_i\varepsilon_{i'}} \sum_{s, s_1, s_2} \sum_{|\tilde{\mathbf{p}}_f|} \frac{1}{J} \frac{d\tilde{\Omega}_f^1 |\tilde{\mathbf{p}}_f^1|^2}{\tilde{\varepsilon}_f^1 \tilde{\varepsilon}_{f'}^1} \times \left| \frac{|\tilde{\mathbf{p}}_f^1|}{\tilde{\varepsilon}_f^1} + \frac{|\tilde{\mathbf{p}}_f^1| + sk_0^1 \cos \tilde{\varphi}_f^1}{\tilde{\varepsilon}_f^1 - sk_0^1} \right|^{-1} \{ \dots \},$$

$$k'^1 \tilde{\mathbf{p}}_f^1 = |\mathbf{k}'^1| |\tilde{\mathbf{p}}_f^1| \cos \tilde{\varphi}_f^1. \quad (12)$$

Here the sum over $|\tilde{\mathbf{p}}_f^1|$ denotes summation over the roots of the equation $2\tilde{\varepsilon}_f^1 = \tilde{\varepsilon}_f^1 + \tilde{\varepsilon}_{f'}^1$. The connection between $|\tilde{\mathbf{p}}_f^1|$ and the laboratory-frame quantities is given by formulas (20); $d\tilde{\Omega}_f^1$ is the solid-angle element in the direction of the vector $\tilde{\mathbf{p}}_f^1$. In the nonrelativistic case, in the absence of an external field ($ea = 0$) this expression leads to Rutherford's formulas with allowance for exchange^[12,13] (v and θ are the velocity and scattering angle of the electron in the c.m.s.):

$$d\sigma = \frac{1}{16} \left(\frac{e^2}{4\pi m} \right)^2 \frac{1}{v^4} \left[\frac{1}{\sin^4(\theta/2)} + \frac{1}{\cos^4(\theta/2)} - \frac{1}{\sin^2(\theta/2) \cos^2(\theta/2)} \right] d\Omega. \quad (13)$$

Let us examine the denominator of one of the terms in the curly brackets in expression (10) in the \tilde{c} -system:

$$B_{s_2} \equiv (\tilde{p}_i^1 - \tilde{p}_f^1 - s_2 k'^1)^2 = 2[m^{*2} - \tilde{\varepsilon}_i^1 \tilde{\varepsilon}_f^1 + |\tilde{\mathbf{p}}_i^1| |\tilde{\mathbf{p}}_f^1| \cos \tilde{\theta}^1 - s_2 k'^1 (\tilde{p}_i^1 - \tilde{p}_f^1)],$$

$$\cos \tilde{\theta}^1 = \mathbf{p}_i \mathbf{p}_f / |\tilde{\mathbf{p}}_i| |\tilde{\mathbf{p}}_f|. \quad (14)$$

Using (9b), we can show that B_{s_2} vanishes when $s = 0$ only if $\tilde{\theta}^1 = 0$. We note that the angle $\tilde{\theta}^1$ goes over in the limit when $ea = 0$ into the electron scattering angle in the c.m.s. Thus, the case $B_{s_2=0} = 0$ corresponds to the Rutherford scattering of an electron at zero angle. We can therefore state that the infinite value of the Moller-scattering cross section at $\tilde{\theta}^1 = 0$ is due to the properties of the Coulomb field and has no bearing on the action of the field of the plane electromagnetic wave.

We shall consider henceforth the case $\tilde{\theta}^1 \neq 0$. Here, as can be shown by using formulas (19) and (20), the function B_{s_2} vanishes for certain relations between the momentum $\tilde{\mathbf{p}}_i$, the angles $\tilde{\theta}$ and $\tilde{\varphi}_f$, and the parameters s and s_2 ($s_2 \neq 0$); consequently, the effective Moller-scattering cross section becomes infinite.

The resonant behavior of the Moller-scattering

cross section in the presence of the field of a plane electromagnetic wave is explained in the following manner. The Fourier transform of the Green's function of an electron in an electromagnetic field described by the potential (2) contains poles at which the following relations is satisfied (see the Appendix):

$$\varepsilon_p - rk_0' = \pm \sqrt{(\mathbf{p} - r\mathbf{k}')^2 + m^{*2}} \quad (r = 0, \pm 1, \pm 2, \dots). \quad (15)$$

The plus and minus signs pertain respectively to the electron and positron states. According to the usual treatment of Green's-function poles,^[14] the quantities $E_{pr} = \varepsilon_p - rk_0'$ and $\mathbf{P}_r = \mathbf{p} - r\mathbf{k}'$ can be regarded as the energy and momentum of a certain quasiparticle corresponding to a electron + plane electromagnetic wave system²⁾. Thus, in spite of the fact that the potential (2) depends on the time, one can speak of a discrete energy spectrum of the system under consideration, consisting of an infinitely large number of levels (15). Naturally, the discrete structure of the spectrum of the electron + plane electromagnetic wave system causes the probabilities of the processes occurring in such a system to acquire a resonant character under certain conditions. We note that the nature of the resonant infinities in the Moller-scattering probability in the presence of an electromagnetic field is the same as in the scattering of a photon by a bound electron (see, for example,^[12])—in the latter case the resonances are connected with the discrete spectrum of the electron in the Coulomb field and appear whenever the energy of the scattering or scattered photon coincides with the difference between two atomic levels.

The infinite value of the probability at the resonance points is connected with the use in (1) of the free photon Green's function $D_0(\mathbf{k})$. The resonant infinities are eliminated by replacing $D_0(\mathbf{k})$ by the total Green's function of the photon $D(\mathbf{k}_1, \mathbf{k}|A)$, which takes into account the action of the external and quantized electromagnetic fields. Unfortunately, the calculation of the total Green's function of the photon for all values of the arguments is a hopeless task. One can, however, sum approximately the entire perturbation-theory series for the total Green's function $D(\mathbf{k}_1, \mathbf{k}|A)$ in the vicinity of the point $\mathbf{k}^2 = 0$ and use the result $(D(\mathbf{k}_1, \mathbf{k}|A)|_{\mathbf{k}^2=0})$ to eliminate the infinite at the resonant point. This is done in Secs. 3 and 4.

²⁾A similar interpretation of the poles of the Green's function of an electron in the field (2) is contained in a paper by Yakovlev^[10].

However, the \tilde{c} -system introduced by us, which is convenient for the investigation of resonances, is not physical; indeed, it represents an infinite set (with respect to the number s) of coordinates. Therefore, to estimate the Moller-scattering cross section we change over to the laboratory frame (l.s.) in which the i' electron is at rest, and the vectors \mathbf{a} and \mathbf{k}' take the form $\mathbf{a} = (0, \mathbf{a})$ and $\mathbf{k}' = \omega \mathbf{n}$, $\mathbf{n} = (1, \mathbf{n})$. In this system, the following relations are satisfied:

$$\begin{aligned} \tilde{\mathbf{p}}_{i'} &= -\mathbf{n} \frac{e^2 a^2}{4m}, & \tilde{\mathbf{e}}_{i'} &= m - \frac{e^2 a^2}{4m}, & (k' \tilde{p}_{i'}) &= \omega m, \\ (k' \tilde{p}_{f'}) &= (k' \tilde{p}_i) - (k' \tilde{p}_f) + \omega m, & (\mathbf{a} \tilde{\mathbf{p}}_{i'}) &= \mathbf{0}. \end{aligned} \quad (16)$$

Using the invariance of the scalar products of 4-vectors in different Lorentz coordinate systems, we can establish the connection between the vectors and angles of the \tilde{c} -system and of the l.s. and write down an expression for the effective scattering cross section (12) in the l.s. It is simpler, however, to go over the l.s. directly in (10). The formula for the effective scattering cross section in the l.s. will be derived in the nonrelativistic approximation

$$\frac{|\mathbf{p}_\alpha|}{m} \ll 1, \quad \frac{e|\mathbf{a}|}{m} \ll 1, \quad \frac{\omega}{m} \ll 1 \quad (\alpha = i, i', f, f'); \quad (17a)$$

in addition, we shall assume that the most essential values of the parameter s (i.e., those values at which resonances take place) are small, so that

$$|s\omega|/m \ll 1. \quad (17b)$$

We retain in the curly brackets of (10) only the terms that make the contribution of main magnitude at the resonance points. The final result is

$$\begin{aligned} d\sigma &= \frac{e^4}{(2\pi)^2} \sum_{s_i, s_1} \sum_{\tilde{\mathbf{p}}_f} \frac{m^2}{v_i \tilde{\mathbf{e}}_f \tilde{\mathbf{e}}_{f'}} \frac{d\tilde{\Omega}_f}{|\tilde{\mathbf{p}}_f|^2} \\ &\times \left| \frac{|\tilde{\mathbf{p}}_f|}{\tilde{\mathbf{e}}_f} + \frac{|\tilde{\mathbf{p}}_f| - |\tilde{\mathbf{p}}_i| \cos \tilde{\theta} + (s\omega + e^2 a^2/4m) \cos \tilde{\varphi}_f}{\tilde{\mathbf{e}}_{f'} - s\omega} \right|^{-1} \\ &\times \left\{ \frac{\tilde{\Phi}_{s_i, s_1, s_1}^{(1)}}{4[m^{*2} - \tilde{\mathbf{e}}_i^1 \tilde{\mathbf{e}}_{f'}^1 + \tilde{\mathbf{p}}_i^1 \tilde{\mathbf{p}}_{f'}^1 - s_1(k', \tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_{f'})]^2} \right. \\ &\left. + \frac{\tilde{\Phi}_{s_i, s_1+s, s_1+s}^{(2)}}{4[m^{*2} - 2(\tilde{\mathbf{e}}_i^1)^2 + \tilde{\mathbf{e}}_i^1 \tilde{\mathbf{e}}_{f'}^1 - \tilde{\mathbf{p}}_i^1 \tilde{\mathbf{p}}_{f'}^1 - s_1(k', \tilde{\mathbf{p}}_i - \tilde{\mathbf{p}}_{f'})]^2} \right\}, \\ \cos \tilde{\varphi}_\alpha &= \frac{\mathbf{n} \tilde{\mathbf{p}}_\alpha}{|\tilde{\mathbf{p}}_\alpha|} \quad (\alpha = i, f), \quad v_i = |\mathbf{v}_i| = \frac{|\mathbf{p}_i|}{\varepsilon_i}. \end{aligned} \quad (18)$$

All the quantities refer here to the l.s., with the exception of $\tilde{\varepsilon}_i^1$, $\tilde{\varepsilon}_{f'}^1$, $\tilde{\mathbf{p}}_i^1$, $\tilde{\mathbf{p}}_{f'}^1$, and $\tilde{\theta}^1$; the sum over $|\tilde{\mathbf{p}}_f|$ denotes summation over all the roots of the equation $\tilde{\varepsilon}_i + \tilde{\varepsilon}_{i'} - \tilde{\varepsilon}_f - \tilde{\varepsilon}_{f'} = 0$, which in the approximations (17a) and (17b) have the form

$$\begin{aligned} |\tilde{\mathbf{p}}_f| &= \frac{1}{2} \left[|\tilde{\mathbf{p}}_i| \cos \tilde{\theta} - \left(s\omega + \frac{e^2 a^2}{4m} \right) \cos \tilde{\varphi}_f \right] \\ &\pm \frac{1}{2} \left\{ \left[|\tilde{\mathbf{p}}_i| \cos \tilde{\theta} - \left(s\omega + \frac{e^2 a^2}{4m} \right) \cos \tilde{\varphi}_f \right]^2 \right. \\ &\left. - 4s\omega m^* + 4\mathbf{n} \tilde{\mathbf{p}}_i \left(s\omega + \frac{e^2 a^2}{4m} \right) - 2 \left(s\omega + \frac{e^2 a^2}{4m} \right)^2 \right\}^{1/2}. \end{aligned} \quad (19)$$

The quantities $\tilde{\varepsilon}_i^1$, $\tilde{\varepsilon}_{f'}^1$, and $\cos \tilde{\theta}^1$ are connected with the corresponding quantities in the l.s. by the relations

$$\begin{aligned} \varepsilon_{i'} &= m \left\{ \frac{1}{2} \left[\frac{\tilde{\varepsilon}_i}{m} \left(1 - \frac{e^2 a^2}{4m^2} \right) + \frac{e^2 a^2 \mathbf{n} \tilde{\mathbf{p}}_i}{4m^2 m} + \frac{m^{*2}}{m^2} \right] \right\}^{1/2}, \\ \tilde{\varepsilon}_{f'} &= \frac{1}{2\tilde{\varepsilon}_i^1} \left[\tilde{\varepsilon}_f \left(\tilde{\varepsilon}_i + m - \frac{e^2 a^2}{4m} \right) - |\tilde{\mathbf{p}}_i| |\tilde{\mathbf{p}}_f| \cos \tilde{\theta} \right. \\ &\left. + \mathbf{n} \tilde{\mathbf{p}}_f \frac{e^2 a^2}{4m} \right], \\ \cos \tilde{\theta}^1 &= -\frac{1}{2|\tilde{\mathbf{p}}_i^1| |\tilde{\mathbf{p}}_{f'}^1|} \left\{ \tilde{\varepsilon}_f \left(\tilde{\varepsilon}_i - m + \frac{e^2 a^2}{4m} \right) \right. \\ &\left. - |\tilde{\mathbf{p}}_i| |\tilde{\mathbf{p}}_f| \cos \tilde{\theta} - \mathbf{n} \tilde{\mathbf{p}}_f \frac{e^2 a^2}{4m} \right\}. \end{aligned} \quad (20)$$

When

$$s\omega \geq \alpha' = \frac{\mathbf{p}_i^2 \cos^2 \tilde{\theta}}{4m} + \frac{e^2 a^2}{16m^2} (-2|\tilde{\mathbf{p}}_i| \cos \tilde{\theta} \cos \tilde{\varphi}_f + 4\mathbf{n} \tilde{\mathbf{p}}_i) \quad (21)$$

expression (19) becomes complex. Inasmuch as the positive values of the parameter s correspond to emission of light by an incident electron, the quantity $S_{\max} \approx \alpha'/\omega$ gives the maximum number of photons of energy ω , which can be emitted by a system of two interacting electrons, one of which had been at rest prior to the interaction, and the second had a momentum \mathbf{p}_i . The summation over s in the expression (18) is thus carried out from $-\infty$ to S_{\max} . The connection between the electron scattering angle θ in the l.s. and the angle $\tilde{\theta}$ is given by

$$\begin{aligned} \cos \tilde{\theta} &= \left\{ |\mathbf{p}_i| |\mathbf{p}_f| \cos \theta - \mathbf{n} \tilde{\mathbf{p}}_f \frac{e^2 a^2}{4np_i} - \mathbf{n} \tilde{\mathbf{p}}_i \frac{e^2 a^2}{4(np_f)} \right. \\ &\left. + \frac{1}{16} \frac{e^2 a^2}{(np_i)(np_f)} \right\} \left\{ \left[\mathbf{p}_i^2 - \frac{e^2 a^2}{2} \frac{\mathbf{n} \tilde{\mathbf{p}}_i}{(np_i)} + \left(\frac{e^2 a^2}{4(np_i)} \right)^2 \right]^{1/2} \right. \\ &\left. \times \left[\mathbf{p}_f^2 - \frac{e^2 a^2}{2} \frac{\mathbf{n} \tilde{\mathbf{p}}_f}{(np_f)} + \left(\frac{e^2 a^2}{4(np_f)} \right)^2 \right]^{1/2} \right\}^{-1}. \end{aligned} \quad (22)$$

3. TOTAL GREEN'S FUNCTION OF THE PHOTON

The total Green's function of the photon in an external electromagnetic field described by a potential (2) can be represented in the form

$$D^{\mu\nu}(z_2, z_1|A) = \frac{1}{(2\pi)^4} \int d^4 k e^{-i(k, z_2 - z_1)} \sum_{r=-\infty}^{+\infty} D_r^{\mu\nu}(k|A) e^{-ir\omega(nz_2)}. \quad (23)$$

The Fourier transformation of the Green's function of the photon is defined by the formula

$$D^{\mu\nu}(z_2, z_1|A) = \frac{1}{(2\pi)^8} \int d^4k_1 d^4k_2 e^{-i(k_2 z_2) + i(k_1 z_1)} D^{\mu\nu}(k_2, k_1|A). \quad (24)$$

From a comparison of the right sides of (23) and (24) we have

$$D^{\mu\nu}(k_2, k_1|A) = (2\pi)^4 \sum_{r=-\infty}^{+\infty} D_r^{\mu\nu}(k_1|A) \delta(k_2 - k_1 - r\omega n). \quad (25)$$

Relations analogous to (23)–(25) hold also for the polarization operator $\Pi^{\mu\nu}(z_2, z_1|A)$ in the presence of an external electromagnetic field (2).

Using the last relations and Schwinger's equation for the Green's function of the photon^[15] (repeated indices imply summation, for example

$$D^{mn}\Pi_{nk} \equiv \sum_{n=1}^4 D^{mn}\Pi_{nk}$$

we can reduce

$$D^{\mu\nu}(x, y|A) = D^{(0)\mu\nu}(x - y) + \int d^4z_1 d^4z_2 D^{(0)\mu\alpha}(x - z_2) \times \Pi_{\alpha\beta}(z_2, z_1|A) D^{\beta\nu}(z_1, y|A) \quad (26)$$

to the form

$$D_r^{\mu\nu}(k|A) = D^{(0)\mu\nu}(k) \delta_{r0} + D^{(0)\mu\alpha}(k + r\omega n) \times \sum_{r_1} \Pi_{r-r_1, \alpha\beta}(k + r_1\omega n|A) D_{r_1}^{\beta\nu}(k|A) \quad (r = 0, \pm 1, \dots). \quad (27a)$$

We write the solution of (27a) in the form of an infinite perturbation-theory series

$$\begin{aligned} D_r^{\mu\nu}(k|A) &= D^{(0)\mu\nu}(k) \delta_{r0} + D^{(0)\mu\alpha}(k + r\omega n) \\ &\times \Pi_{r, \alpha\beta}(k|A) D^{(0)\beta\nu}(k) \\ &+ D^{(0)\mu\alpha}(k + r\omega n) \sum_{r_1} \Pi_{r-r_1, \alpha\beta}(k + r_1\omega n|A) \\ &\times D^{(0)\beta\nu}(k + r_1\omega n) \Pi_{r_1, \gamma\delta}(k|A) D^{(0)\delta\nu}(k) \\ &+ D^{(0)\mu\alpha}(k + r\omega n) \sum_{r_1, r_2} \Pi_{r-r_1, \alpha\beta}(k + r_1\omega n|A) \\ &\times D^{(0)\beta\gamma}(k + r_1\omega n) \Pi_{r_1-r_2, \gamma\alpha'}(k + r_2\omega n|A) D^{(0)\alpha'\beta'}(k + r_2\omega n) \\ &\times \Pi_{r_2, \beta\gamma'}(k|A) D^{(0)\gamma'\nu}(k) + \dots \end{aligned} \quad (28)$$

For our purposes it is most important to clarify the behavior of the function $D_r^{\mu\nu}(k|A)$ in the vicinity of the point $k^2 = 0$. In this region, the principal role in the sums over r_1, r_2, \dots in the right side of (28) is played by the terms with $r_1 = r_2 = \dots = 0$, since $D^{(0)\mu\nu}(k + r_1\omega n) \rightarrow \infty$ as $k^2 \rightarrow 0$ and $r_1 = 0$, while the function $\Pi_{r-r_1}^{\alpha\beta}(k + r_1\omega n|A)|_{k^2=0}$ ($i = 1, 2, \dots$) is

finite, as can be verified by using the results of the Appendix. If we retain only these terms in the series (28), then the resultant expression will satisfy the equation

$$D_r^{\mu\nu}(k|A) = D^{(0)\mu\nu}(k) \delta_{r0} + D^{(0)\mu\alpha}(k + r\omega n) \times \Pi_{r, \alpha\beta}(k|A) D_0^{\beta\nu}(k|A). \quad (27b)$$

From gauge-invariance considerations (see, for example, ^[15]) we get

$$\begin{aligned} g^{\mu\mu} k_{2\mu} D_{\mu\nu}(k_2, k_1|A) &= g^{\nu\nu} D_{\mu\nu}(k_2, k_1|A) k_{1\nu} = 0, \\ g^{\mu\mu} k_{2\mu} \Pi_{\mu\nu}(k_2, k_1|A) &= g^{\nu\nu} \Pi_{\mu\nu}(k_2, k_1|A) k_{1\nu} = 0, \end{aligned} \quad (29a)$$

which can be written in the form of relations of the type

$$(k_1 + r\omega n)^\mu D_{r, \mu\nu}(k_1|A) = D_{r, \mu\nu}(k_1|A) k_{1\nu} = 0. \quad (29b)$$

It follows therefore that the functions $\Pi_{r, \mu\nu}(k|A)$ and $D_{r, \mu\nu}(k|A)$ should be of the form

$$\begin{aligned} D_{r, \mu\nu}(k|A) &= J_{\mu\alpha}(k + r\omega n) D_r^{\prime\alpha\beta}(k|A) J_{\beta\nu}(k), \\ J_{\alpha\beta}(k) &= g_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}, \quad J_{\mu\alpha}(k + r\omega n) J^{\alpha\mu}(k) \\ &= 2 + \frac{[k(k + r\omega n)]^2}{k^2(k + r\omega n)^2}. \end{aligned} \quad (30)$$

The function $D_r^{\prime\alpha\beta}(k|A)$ will be sought in the form

$$D_r^{\prime\alpha\beta}(k|A) = g^{\alpha\beta} D_r(k|A). \quad (31)$$

To determine $D_r(k|A)$ we obtain the following equation (in the vicinity of the point $k^2 = 0$):

$$D_r(k|A) = D^{(0)}(k) \delta_{r0} + D^{(0)}(k + r\omega n) \Pi_r(k|A) D_0(k|A), \quad (32)$$

$$\begin{aligned} \Pi_r(k|A) &\equiv \left\{ 2 + \frac{[k(k + r\omega n)]^2}{k^2(k + r\omega n)^2} \right\}^{-1} J_{\mu\alpha}(k + r\omega n) \Pi_r^{\prime\alpha\beta}(k|A) \\ &\times J_{\beta\nu}(k) = \left\{ 2 + \frac{[k(k + r\omega n)]^2}{k^2(k + r\omega n)^2} \right\}^{-1} g^{\mu\nu} \Pi_{r, \mu\nu}(k|A). \end{aligned} \quad (33)$$

Hence $(D^{(0)}(k) = -1/k^2)$

$$D_r(k|A) = \begin{cases} \frac{1}{[D^{(0)}(k)]^{-1} - \Pi_0(k|A)} & \text{for } r = 0 \\ \frac{D^{(0)}(k + r\omega n) \Pi_r(k|A)}{[D^{(0)}(k)]^{-1} - \Pi_0(k|A)} & \text{for } r \neq 0 \end{cases}. \quad (34)$$

Using the results of the Appendix, we can show that the function $g_{\mu\nu} \Pi_{R,r}^{\mu\nu}(k|A)$ ($r = 0, \pm 1, \pm 2, \dots$) at the point $k^2 = 0$ are finite. Therefore, as follows from (33), $\Pi_r(k|A)|_{k^2=0} = 0$ when $r \neq 0$. Thus, it is sufficient to take into account only $D_{r=0}(k|A)$ in the expression for the Fourier transform of the complete Green's function of the photon (25). From formula (A.19) we get $(g_{\mu\nu} \Pi_{R,r}^{\mu\nu}(k)|_{k^2=0} = 0)$:

$$\Pi_{\neq 0}(k|A)|_{k^2=0} = -c_1 e^2 a^2 \left(\frac{\omega(nk)}{m^2} \right)^2, \quad c_1 = \frac{\alpha}{3\pi} \cdot \frac{11}{90}, \quad (35)$$

α is the fine-structure constant.

4. DISCUSSION OF RESULTS

We obtain the resonant infinities in the Moller-scattering cross section with the aid of the complete photon Green's function $D(k_k, k|A)$, which we calculated in the preceding section in the vicinity of the point $k^2 = 0$.

Let us consider the case when the denominator of one of the terms in the curly brackets of (18) vanishes for certain values of the parameters \tilde{p}_i , $\tilde{\theta}$, s , or s_1 ($\tilde{\theta}^1 \neq 0$, $s_1 \neq 0$). Let, for example,

$$m^{*2} - \tilde{\varepsilon}_i^1 \tilde{\varepsilon}_f^1 + \tilde{\mathbf{p}}_i^1 \tilde{\mathbf{p}}_f^1 - s_1(k', \tilde{p}_i - \tilde{p}_f) = 0. \quad (36a)$$

As already discussed above, to eliminate the divergence at the resonance point it is necessary to make in (18) the substitution

$$2[m^{*2} - \tilde{\varepsilon}_i^1 \tilde{\varepsilon}_f^1 + \tilde{\mathbf{p}}_i^1 \tilde{\mathbf{p}}_f^1 - s_1(k', \tilde{p}_i - \tilde{p}_f)] \rightarrow -c_1 e^2 a^2 \times \frac{[k'(\tilde{p}_i - \tilde{p}_f)]^2}{m^4} = c_1 \frac{e^2 a^2}{m^4 s_1^2} [m^{*2} - \tilde{\varepsilon}_i^1 \tilde{\varepsilon}_f^1 + \tilde{\mathbf{p}}_i^1 \tilde{\mathbf{p}}_f^1]. \quad (37)$$

If the resonance corresponds to $s = 0$, then Eq. (36a) can be rewritten in the form

$$-|\tilde{\mathbf{p}}_i^1|^2 (1 - \cos \tilde{\theta}^1) = -s_1 |\tilde{\mathbf{p}}_i^1| k_0'^1 (\cos \tilde{\varphi}_i^1 - \cos \tilde{\theta}^1 \cos \tilde{\varphi}_f^1). \quad (36b)$$

Assuming that $1 - \cos \tilde{\theta}^1 \approx \cos \tilde{\varphi}_i^1 - \cos \tilde{\theta}^1 \cos \tilde{\varphi}_f^1$, we obtain the resonance condition in the form

$$|\tilde{\mathbf{p}}_i^1| \approx s_1 k_0'^1, \quad k_0'^1 = \frac{\omega}{2\varepsilon_i^1} (\varepsilon_i + m - |\tilde{\mathbf{p}}_i| \cos \tilde{\varphi}_i). \quad (38a)$$

For a numerical estimate of the resonance scattering cross section we put

$$|\mathbf{E}_0| = 10^5 \text{ V/cm}, \quad \omega = 2.7 \cdot 10^{15} \text{ sec}^{-1} (\mathbf{E} = \mathbf{E}_0 \cos \omega(nx)). \quad (39)$$

\mathbf{E} and ω are the intensity of the electric component and the frequency of the laser beam. At these values of $|\mathbf{E}_0|$ and ω we get

$$\frac{e^2 a^2}{m^2} = 4.7 \cdot 10^{-12}, \quad \frac{m}{\omega} = 0.29 \cdot 10^6,$$

$$\kappa_{1f} - \kappa_{1i} \approx -(\kappa_{1f'} - \kappa_{1i'}) \sim -\frac{e|\mathbf{a}|}{m} \frac{|\tilde{\mathbf{p}}_i|}{\omega},$$

$$\kappa_{1f} - \kappa_{1i} \gg \kappa_{2f} - \kappa_{2i},$$

$$\tilde{\Phi}_{s, s_1, s_1}^{(1)} \approx J_{s_1}^2(\kappa_{1f} - \kappa_{1i}) J_{s-s_1}^2(\kappa_{1f'} - \kappa_{1i'}), \quad (40)$$

$J_s(x)$ is a Bessel function. We consider here the

range of angles for which

$$\cos \tilde{\theta} \cos \tilde{\theta}_f - \cos \tilde{\theta}_i \approx 1,$$

$$\cos \tilde{\theta}_\alpha = \tilde{\mathbf{a}} \mathbf{p}_\alpha / |\mathbf{a}| |\tilde{\mathbf{p}}_\alpha| \quad (\alpha = i, f). \quad (41)$$

Recognizing that $|\tilde{\mathbf{p}}_i^1| \approx |\tilde{\mathbf{p}}_i|/2$, $k_0'^1 \approx \omega$, and $\tilde{\theta}^1 \approx 2\tilde{\theta}$ ($s = 0$), we rewrite the resonance condition (38a) in the form

$$|\tilde{\mathbf{p}}_i| \approx 2s_1 \omega. \quad (38b)$$

We obtain for the scattering cross section $d\sigma$ the formula ($s = 0$, $s_1 \neq 0$)

$$d\sigma_{s_1} = r_0^2 \frac{\cos \tilde{\theta}}{\sin^8 \tilde{\theta}} P_{s_1} d\tilde{\Omega}_f, \quad P_{s_1} = 2^6 \left(\frac{m^2}{e^2 a^2} \right)^2 \frac{s_1^4}{c^2} \frac{J_{s_1}^4(\kappa_{1f} - \kappa_{1i})}{|\mathbf{v}|^8}. \quad (42)$$

In this formula $\sin \tilde{\theta} \sim 1$ (see condition (41)) and r_0 is the classical radius of the electron. If the velocity $|\mathbf{v}|$ of the incident electron is such that the resonance condition (38b) is satisfied for $s_1 = 2$ and 3 ($|\mathbf{v}|/c \approx 2 \times 10^{-5}$), then the numerical values of the functions P_{S_1} are

$$P_{s_1=2} \sim 10^{26}, \quad P_{s_1=3} \sim 10^3. \quad (43)$$

At the chosen values of \mathbf{E}_0 , ω , and \mathbf{p}_i , we get the inequality $d\sigma_{s_1}|_{s \neq 0} \ll d\sigma_{s_1}|_{s=0}$. Thus, the Moller-scattering cross section can be very large if the resonance condition is satisfied at small values of s_1 .

In conclusion let us consider the effective interaction potential of two electrons in the presence of an external electromagnetic field.

As is well known,^[16] the matrix element for exchange of a virtual photon by two free electrons, summed over the photon polarizations, can be represented in the form of a sum of two terms, one of which corresponds to instantaneous Coulomb interaction, and the other to an interaction due to the transverse waves (the second term is known as the Breit interaction). A similar analysis can be carried out also in our case. In fact, from gauge-invariance considerations we get the equality

$$\left(\bar{u}_f \sum_{s_1=-\infty}^{+\infty} \sum_{\mu=1}^4 g_{\mu\mu} q_{s_1}^\mu R_{s_1}^\mu(f, i) u_i \right) = 0,$$

$$q_{s_1} = (q_{s_1}^0, \mathbf{q}_{s_1}) = p_i - p_f - s_1 k' - 2k'(\kappa_{2f} - \kappa_{2i}), \quad (44)$$

which can readily be proved also directly by using (4a).

Let us consider a coordinate frame in which the photon momentum vector $\mathbf{q}_{s_1=0}$ is directed along one of the axes. Let $R_{S_1}^{Q_0}$ and $R_{S_1}^{T_0}$ be the projections

of the matrices $R_{s_1}^\mu$ on the direction of the vector \mathbf{q}_0 and the directions perpendicular to it. We can then write

$$\sum_{s_1=-\infty}^{+\infty} \sum_{\mu=1}^4 g^{\mu\mu} q_{s_1}^\mu R_{s_1}^\mu = \sum_{s_1=-\infty}^{+\infty} (q_0^0 R_{s_1}^0 - |\mathbf{q}_0| R_{s_1}^{\mathbf{q}_0}) + \beta,$$

$$\beta = -\omega \hat{n} \sum_{s_1=-\infty}^{+\infty} s_1 L_{s_1}^{(0)}(f, i). \quad (45)$$

We rewrite similarly the matrix element (4) (we write out only the first term of the curly bracket and omit the δ function and the inessential numerical factors):

$$M \sim \sum_{s, s_1} \frac{(\bar{u}_{f'} R_{s-s_1}^0(f', i') u_i') (\bar{u}_f R_{s_1}^0(f, i) u_i)}{q_{s_1}^2}$$

$$- \sum_{s, s_1} \frac{(\bar{u}_{f'} R_{s-s_1}^{\mathbf{q}_0}(f', i') u_i') (\bar{u}_f R_{s_1}^{\mathbf{q}_0}(f, i) u_i)}{q_{s_1}^2}$$

$$- \sum_{s, s_1} \frac{(\bar{u}_{f'} R_{s-s_1}^{tr}(f', i') u_i') (\bar{u}_f R_{s_1}^{tr}(f, i) u_i)}{q_{s_1}^2}. \quad (46)$$

It follows from (44) and (45) that we can make in the foregoing relation the substitution

$$R_{s_1}^{\mathbf{q}_0} \rightarrow \frac{q_0^0 R_{s_1}^0}{|\mathbf{q}_0|} + \frac{1}{|\mathbf{q}_0|} \left(\sum_{m(\neq s_1)} \alpha_m + \beta \right),$$

$$\alpha_m = q_0^0 R_m^0 - |\mathbf{q}_0| R_m^{\mathbf{q}_0}$$

As a result we get

$$M \sim M^C + M^{tr} + M'';$$

$$M^C = -\frac{1}{|\mathbf{q}_0|^2} \sum_s (\bar{u}_{f'} R_s^0(f', i') u_i') (\bar{u}_f R_0^0(f, i) u_i),$$

$$M^{tr} = -\sum_{s, s_1} \frac{(\bar{u}_{f'} R_{s-s_1}^{tr}(f', i') u_i') (\bar{u}_f R_{s_1}^{tr}(f, i) u_i)}{q_{s_1}^2},$$

$$M'' = -\frac{q_0^2}{\mathbf{q}_0^2} \sum_{\substack{s, s_1 \\ (s_1 \neq 0)}} \frac{(\bar{u}_{f'} R_{s-s_1}^0(f', i') u_i') (\bar{u}_f R_{s_1}^0(f, i) u_i)}{q_{s_1}^2}$$

$$- \frac{1}{\mathbf{q}_0^2} \sum_{s, s_1} \frac{1}{q_{s_1}^2} \left\{ q_0^0 (\bar{u}_{f'} R_{s-s_1}^0(f', i') u_i') \right.$$

$$\times \left[\bar{u}_f \left(\sum_{m(\neq s_1)} \alpha_m + \beta \right) u_i \right] + q_0^0 \left[\bar{u}_f \left(\sum_{m(\neq s-s_1)} \alpha_m' + \beta' \right) \right.$$

$$\times u_i' \left. \right] (\bar{u}_f R_{s_1}^0(f, i) u_i) \left. \right\} + \left[\bar{u}_{f'} \left(\sum_{m(\neq s-s_1)} \alpha_m' + \beta' \right) u_i' \right]$$

$$\times \left[\bar{u}_f \left(\sum_{n(\neq s_1)} \alpha_n + \beta \right) u_i \right] \left. \right\}, \quad \alpha_m \equiv \alpha_m(f, i),$$

$$\alpha_m' \equiv \alpha_m(f', i'), \quad \beta \equiv \beta(f, i), \quad \beta' \equiv \beta(f', i'). \quad (47)$$

The first term in this expression (M^C) corresponds to the Coulomb interaction, and the second (M^{tr}) yields in the absence of an external field the particle interaction due to the transverse waves, while the third is the interaction induced between the charges and the currents by the external electromagnetic field (this term vanishes when the external field is turned off).

Let us consider relation (47) in the nonrelativistic limit (17a). We separate in the expression for $M^{tr} + M''$ the term with $s_1 = 0$. In a manner similar to that used to derive the Breit formula for the interaction of the free electrons,^[16] we can show that this term results in a small correction to the Coulomb interaction. Assume now that the vectors \mathbf{p}_i , \mathbf{p}_f , \mathbf{a} , and the frequency ω assume values such as to satisfy the inequalities (standard notation)

$$\kappa_{1f} - \kappa_{1i} \gg \kappa_{2f} - \kappa_{2i}, \quad \frac{v_i}{c} \gg \kappa_{1f} - \kappa_{1i} \gg \left(\frac{v_i}{c} \right)^2,$$

$$\frac{c |\mathbf{p}_i|}{\hbar \omega} \gg 1, \quad \frac{v_i}{c} \gg \frac{e |\mathbf{a}|}{m c^2}.$$

In this case

$$L_s^{(0)}(f, i) \approx J_s(\kappa_{1f} - \kappa_{1i})_s R_s^\mu(f, i) \approx \gamma_\mu J_s(\kappa_{1f} - \kappa_{1i}),$$

$$\beta \approx -\omega \hat{n} (\kappa_{1f} - \kappa_{1i}).$$

We assume further that at the chosen values of the system parameters we get the inequality $q_1^2 \ll q_{s_1}^2$ ($s_1 \neq 1$), and that the dominating term in the sum over s in the expression for $M^{tr} + M''$ is term with $s = 0$ (an estimate of the Moller-scattering cross section has shown that it is always possible to satisfy this condition by properly choosing the system parameters). We then have

$$M^{tr} + M'' \sim -\frac{1}{\mathbf{q}_0^2} \frac{1}{\omega^2 - \mathbf{q}^2} [\bar{u}_{f'} (q_0^0 \gamma_0 - |\mathbf{q}_0| \gamma^{\mathbf{q}_0}) u_i']$$

$$\times [\bar{u}_f (\gamma_0 q_0^0 - |\mathbf{q}_0| \gamma^{\mathbf{q}_0}) u_i], \quad q_{s_1}^2 \approx (s_1 \omega)^2 - \mathbf{q}_{s_1}^2. \quad (48)$$

Here $\gamma^{\mathbf{q}_0}$ is the projection of the matrices γ on the direction of the vector \mathbf{q}_0 . Recognizing that in the nonrelativistic approximation

$$u_i = \begin{pmatrix} u_{ia} \\ u_{ib} \end{pmatrix}, \quad u_{ib} \approx \frac{\sigma \mathbf{p}_i}{2m} u_{ia}$$

(u_{ib} are small components of the wave function and σ are Pauli matrices) we can verify that expression (48) can be positive in a certain region of the electron momenta, corresponding to attraction between the electrons. With this, at sufficiently small values of q_1^2 , this expression can exceed the Coulomb repulsion of the electrons. In this case the total effective electron interaction potential will corre-

spond to attraction between the particles. From this follows the possibility of production of paired electrons in the presence of an external electromagnetic field.

The author considers it his pleasant duty to thank M. A. Krivoglaz for a discussion of the results and V. N. Piskovoĭ for useful remarks during the discussion of the article.

APPENDIX

Polarization Operator

The self-energy part of the photon (polarization operator) in the presence of a field A has the following form in the second order of perturbation theory with respect to the interaction with the quantized electromagnetic field:

$$g^{nm}\Pi_{nm}(z_2, z_1|A) = -ie^2 g^{nm} \text{Sp} [\gamma_n G(z_2, z_1|A) \times \gamma_m G(z_1, z_2|A)], \quad (\text{A.1})$$

$G(z_2, z_1|A)$ is the Green's function of the electron in the field A .

We choose as the external electromagnetic field that of a plane electromagnetic wave described by the potential (2). The Green's function of the electron in the field of a plane electromagnetic wave was obtained by Schwinger^[17] and by Brown and Kibble.^[9] In the case of the potential (2), it is convenient to represent the function $G(z_2, z_1|A)$ in the form

$$G(z_2, z_1|A) = \frac{1}{(2\pi)^4} \int d^4p e^{-i(p, z_2 - z_1)} G(p, \xi_2|A), \quad (\text{A.2})$$

$$G(p, \xi_2|A) = \sum_{r,s=-\infty}^{+\infty} B_{r,r+s}(p) \frac{e^{-is\omega(nz_2)}}{(p - r\omega n)^2 - m^{*2} + i\varepsilon}, \quad (\text{A.3})$$

$$\xi_2 = (nz_2), \quad m^{*2} = m^2 - e^2 a^2 / 2, \quad \varepsilon \rightarrow +0,$$

$$B_{\mu\nu}(p) = (\hat{p} + m) B_{\mu\nu}^{(0)}(p) - \frac{e}{2(np)} (\hat{a}\hat{p}\hat{n} + \hat{m}\hat{n}\hat{a}) B_{\mu\nu}^{(2)}(p) - e\hat{a} B_{\mu\nu}^{(4)}(p) + \frac{\hat{n}}{2(np)} e^2 a^2 B_{\mu\nu}^{(3)}(p),$$

$$B_{\mu\nu}^{(i)}(p + s\omega n) \equiv B_{\mu\nu}^{(i)}(p),$$

$$B_{\mu\nu}^{(j)}(p) = \frac{1}{(2\pi)^2} \int_{-\pi}^{+\pi} d\varphi_1 d\varphi_2 b^{(j)}(\varphi_1, \varphi_2)$$

$$\times \exp\{i[T_p(\varphi_1, \varphi_2) - \mu\varphi_1 + \nu\varphi_2]\} \quad (j = 0, 1, 2, 3);$$

$$b^{(0)} = 1, \quad b^{(1)} = \cos \varphi_2, \quad b^{(2)} = \cos \varphi_1 - \cos \varphi_2,$$

$$b^{(3)} = \cos \varphi_1 (\cos \varphi_1 - \cos \varphi_2),$$

$$T_p(\varphi_1, \varphi_2) = \kappa_2 g_2 + \kappa_1 g_1, \quad \kappa_2 = -\frac{e^2 a^2}{8\omega(np)},$$

$$\kappa_1 = \frac{e}{\omega} \frac{(ap)}{(np)}, \quad g_2 = \sin 2\varphi_1 - \sin 2\varphi_2, \quad g_1 = \sin \varphi_1 - \sin \varphi_2. \quad (\text{A.4})$$

In order to eliminate correctly the ultraviolet divergence, we regularize the Green's function $G(z_2, z_1|A)$, following Pauli and Villars, in a form proposed by Bogolyubov et al.^[15] In our case, the regularization of the Green's function consists in making in (A.3) the substitution

$$\frac{1}{(p - r\omega n)^2 - m^{*2} + i\varepsilon} \equiv \frac{1}{i} \int_0^{+\infty} dv \exp\{iv[(p - r\omega n)^2 - m^{*2} + i\varepsilon]\} \rightarrow \frac{1}{i} \int_0^{+\infty} dv \exp\left\{iv\left[(p - r\omega n)^2 + \frac{e^2 a^2}{2} + i\varepsilon\right]\right\} (e^{-ivM^2} - e^{-ivM^2}); \quad (\text{A.5})$$

Here M is the auxiliary mass, which is allowed to go to the limit $M \rightarrow \infty$ after the calculations.

Substituting (A.2) and (A.3) with allowance for the regularization (A.5), into the Fourier transform of the polarization operator, we get (we omit for the time being the terms that depend on the mass M)

$$g^{nm}\Pi_{nm}(k_2, k_1|A) = \int d^4z_1 d^4z_2 e^{i(k_2 z_2) - i(k_1 z_1)} g^{nm}\Pi_{nm}(z_2, z_1|A) = ie^2 \sum_{s,r} \int d^4p \int_0^{+\infty} d\mu dv \sum_{r',s'} g^{nm} \text{Sp} \{\gamma_n B_{r',s-s'}(p + r'\omega n) \gamma_m \times B_{r-s', r+r'}(p - k_1 - s'\omega n)\} \exp\{iv[(p - k_1 - r\omega n)^2 - m^{*2} + i\varepsilon] + i\mu(p^2 - m^{*2} + i\varepsilon)\} \delta(k_2 - k_1 - s\omega n). \quad (\text{A.6})$$

We represent the function $g^{nm}\Pi_{nm}(k_2, k_1|A)$ in the form

$$g^{nm}\Pi_{nm}(k_2, k_1|A) = (2\pi)^4 \sum_{s=-\infty}^{+\infty} g^{nm}\Pi_{s, nm}(k_1|A) \times \delta(k_2 - k_1 - s\omega n). \quad (\text{A.7})$$

From a comparison of the right sides of (A.6) and (A.7) we obtain (we simultaneously make the following change of integration variables: $\mu = \lambda(1 - \xi)$, $\nu = \lambda\xi$, $p - \xi(k_1 + r\omega n) = p'$, and sum over r' and s')

$$g^{nm}\Pi_{s, nm}(k|A) = \frac{ie^2}{(2\pi)^6} \int_{-\pi}^{+\pi} d\varphi_1 d\varphi_2 \int_0^1 d\xi \int_0^{+\infty} d\lambda \sum_{r=-\infty}^{+\infty} e^{ir\varphi_1 - i(r-s)\varphi_2} \times \int d^4p \exp\{i[T_{p+\xi k}(\varphi_1, \varphi_2) - T_{p-(1-\xi)k}(\varphi_1, \varphi_2)]\} \times g^{nm} \text{Sp} \{\gamma_n [F(p + \xi k; \varphi_1, \varphi_2) + \xi r\omega \hat{n}] \times \gamma_m [F(p - (1 - \xi)k; \varphi_2, \varphi_1) - (1 - \xi)r\omega \hat{n}]\} \exp\{i\lambda[p^2$$

$$+ \xi(1 - \xi)(k + \omega n)^2 - m^2 + i\epsilon],$$

$$F(p; \varphi_1, \varphi_2) = \hat{p} + m + \hat{n} \frac{e^2 a^2}{2(np)} b^{(4)}(\varphi_1, \varphi_2) \\ + \cos \varphi_2 \left(-e\hat{a} + \hat{n} \frac{e(ap)}{(np)} \right) - \frac{e}{2(np)} (\hat{p} + m) \hat{n} \hat{a} b^{(2)}(\varphi_1, \varphi_2);$$

$$b^{(4)}(\varphi_1, \varphi_2) = 1/2 - \cos \varphi_1 \cos \varphi_2. \quad (\text{A.8})$$

We integrate with respect to d^4p in the following manner. We introduce the functional

$$\Phi_q[x(p)] = \int d^4p x(p) \exp(i(pq) + i\lambda p^2). \quad (\text{A.9})$$

Using for the function $[\gamma + (np)]^{-1}$ the representation

$$\frac{1}{\gamma + (np) + i\epsilon} = \frac{1}{i} \int_0^{+\infty} d\alpha e^{i\alpha[(np) + \gamma + i\epsilon]}$$

and taking Gaussian quadratures, we obtain the formula

$$\Phi_l \left[\left(\frac{1}{\gamma + (np)} \right)^l \left(\frac{1}{\delta + (np)} \right)^k \right] \\ = \left(\frac{1}{\gamma - (nq)/2\lambda} \right)^l \left(\frac{1}{\delta - (nq)/2\lambda} \right)^k \Phi_q[1] \\ (l, k = 0, 1, \dots), \quad \Phi_q[1] = -i \frac{\pi^2}{\lambda^2} \exp\left(-i \frac{q^2}{4\lambda}\right). \quad (\text{A.10})$$

Here γ and δ are certain parameters which do not depend on p .

With the aid of (A.10) and the translation operator $L_\alpha = \exp\{\alpha \partial / \partial x\}$ (the property of the operator L_α is as follows: $L_\alpha f(x) = f(x + \alpha)$ where α is an arbitrary constant and $f(x)$ is an arbitrary function of x) we can easily obtain the following relation:

$$\Phi_q[\exp\{i[T_{p+\xi k}(\varphi_1, \varphi_2) - T_{p-(1-\xi)k}(\varphi_1, \varphi_2)]\}] \\ = \exp\left\{i \left[\frac{f_2 + f_1(-aq)/2\lambda + \xi(ak)}{\xi(nk) - (nq)/2\lambda} \right. \right. \\ \left. \left. - \frac{f_2 + f_1(-aq)/2\lambda - (1-\xi)(ak)}{-(1-\xi)(nk) - (nq)/2\lambda} \right] \right\} \\ \times \exp\left\{-i \frac{a^2 f_1^2}{4\lambda} \left[\frac{1}{\xi(nk) - (nq)/2\lambda} \right. \right. \\ \left. \left. - \frac{1}{-(1-\xi)(nk) - (nq)/2\lambda} \right]^2 \right\} \Phi_q[1], \\ f_2 = -\frac{e^2 a^2}{8\omega} g_2, \quad f_1 = \frac{e}{\omega} g_1. \quad (\text{A.11})$$

In the derivation of the last expression we used the identity

$$a \frac{\partial}{\partial q} \frac{1}{b - (nq)/2\lambda} = 0, \quad b = \text{const.}$$

The integral with respect to d^4p in the relation

(A.8) is expressed in terms of the function (A.11) and its first and second derivatives with respect to q , with subsequent transition to the limit $q = 0$. Simple manipulations lead to the formula (we calculate simultaneously the trace)

$$g^{nm} \Pi_{s, nm}(k|A) = \frac{2e^2}{(2\pi)^4} \sum_r \int_{-\pi}^{+\pi} d\varphi_1 d\varphi_2 \int_0^1 d\xi \int_0^{+\infty} d\lambda \frac{1}{\lambda} \\ \times \exp\left\{i\lambda \left[\xi(1 - \xi) \frac{k^2}{m^2} + \frac{e^2 a^2}{2m^2} + i\epsilon \right] \right\} \\ \times \exp\left\{ir \left[\varphi_1 - \varphi_2 + 2\lambda \xi(1 - \xi) \frac{(k'k)}{m^2} \right] \right\} \\ \times \exp\left(i \left\{ \frac{e^2 a^2}{8(k'k)} \frac{g_2}{\xi(\xi - 1)} - \frac{e^2 a^2}{(k'k)^2} \frac{m^2}{4\lambda} \frac{g_1^2}{[\xi(\xi - 1)]^2} \right\} \right) \\ \times \left[-2i \frac{m^2}{\lambda} + 2m^2 - \xi(\xi - 1)k^2 - R \right] \{e^{-i\lambda} + e^{-i\lambda M^2/m^2} \\ - e^{-i\lambda[\xi + (1-\xi)M^2/m^2]} - e^{-i\lambda[(1-\xi) + \xi M^2/m^2]}\}, \quad k' = \omega n. \quad (\text{A.12})$$

We have restored here the terms containing the auxiliary mass M , and made the change of integration variable $\lambda \rightarrow \lambda/m^2$. The function R is of the form

$$R = \frac{e^2 a^2}{2} [2 \sin^2 \varphi_1 + (\cos \varphi_1 - \cos \varphi_2)^2] \\ + \frac{e^2 a^2}{(k'k)} \frac{m^2}{\lambda} \frac{1}{\xi(\xi - 1)} g_1 \cos \varphi_1 - 3 \frac{e^2 a^2}{(k'k)^2} \\ \times \left(\frac{m^2}{2\lambda} \right)^2 \frac{1}{[\xi(\xi - 1)]^2} g_1^2 + \frac{1}{2} \frac{e^2 a^2}{(k'k)} \frac{m^2}{2\lambda} \frac{1}{\xi(\xi - 1)} g_2 \\ + \frac{e^2 a^2}{4} \frac{1}{\xi(\xi - 1)} (1 - 2 \cos \varphi_1 \cos \varphi_2) \\ - \frac{e^2 a^2}{(k'k)^2} \left(\frac{m^2}{2\lambda} \right)^2 \frac{1}{[\xi(\xi - 1)]^3} g_1^2 + \frac{e^2 a^2}{(k'k)} \frac{m^2}{2\lambda} \frac{1}{[\xi(\xi - 1)]^2} \\ \times \left[\frac{1}{8} g_2 + \frac{1}{2} g_1 (\cos \varphi_1 + \cos \varphi_2) \right]. \quad (\text{A.13})$$

We note that the expression (A.12) is invariant against the gauge-transformation (7), and also satisfies the Furry theorem (see, for example, [15]). In fact, the expansion of the function $g^{nm} \Pi_{nm}$ in powers of ea contains only even powers of ea .

We renormalize the polarization operator in accordance with the formula [18]

$$\Pi_R^{nm}(k_2, k_1|A) = \Pi_R^{nm}(k_1) \delta(k_2 - k_1) (2\pi)^4 + \Pi^{nm}(k_2, k_1|A) \\ - \Pi^{nm}(k_2, k_1|A) \Big|_{A=0} - \sum_\mu g^{\mu\mu} \\ \times \int d^4s \frac{\delta \Pi^{nm}(k_2, k_1|A)}{\delta A_\mu(s)} \Big|_{A=0} A_\mu(s). \quad (\text{A.14})$$

Here $\Pi_R^{nm}(k)$ is the renormalized polarization

operator in the absence of the external field A.

The summation over r in (A.12) can be carried out with the aid of one of the formulas

$$\sum_{r=-\infty}^{+\infty} \exp \left\{ ir \left[\varphi_1 - \varphi_2 + 2\lambda\xi(1-\xi) \frac{(k'/k)}{m^2} \right] \right\} = \begin{cases} 2\pi\delta \left[\varphi_1 - \varphi_2 + 2\lambda\xi(1-\xi) \frac{(k'/k)}{m^2} \right], \\ 2\pi \sum_{n=0}^{\infty} \frac{[2\lambda\xi(1-\xi)(k'/k)/m^2]^n}{n!} \frac{\partial^n}{\partial\varphi_1^n} \delta(\varphi_1 - \varphi_2). \end{cases} \quad (\text{A.15a})$$

$$= \begin{cases} 2\pi\delta \left[\varphi_1 - \varphi_2 + 2\lambda\xi(1-\xi) \frac{(k'/k)}{m^2} \right], \\ 2\pi \sum_{n=0}^{\infty} \frac{[2\lambda\xi(1-\xi)(k'/k)/m^2]^n}{n!} \frac{\partial^n}{\partial\varphi_1^n} \delta(\varphi_1 - \varphi_2). \end{cases} \quad (\text{A.15b})$$

When taking the limit $M \rightarrow \infty$ it is convenient to use relation (A.15b) and also the following integrals:

$$\int_0^{+\infty} d\lambda \frac{1}{\lambda^n} e^{i\lambda(a+i\epsilon)} = \begin{cases} -\ln(a+i\epsilon) + c_1 & \text{for } n=1 \\ -ia \ln(a+i\epsilon) + c_2 a + c_3 & \text{for } n=2, \\ \frac{a^2}{2} \ln(a+i\epsilon) + c_4 a^2 + c_5 a + c_6 & \text{for } n=3 \end{cases} \quad (\text{A.16})$$

$a < 0$, $\epsilon \rightarrow +0$. The constants c_i are related by

$$c_1 = -1 - ic_2, \quad c_2 = -i(1/2 + 2c_4), \quad c_3 = -ic_5. \quad (\text{A.17})$$

Calculation shows that the constants c_i do not enter in the final expression for the renormalized polarization operator, since their multipliers vanish.

Let us calculate the polarization operator in the limiting case:

$$\frac{k^2}{m^2} \ll 1, \quad \frac{e^2 a^2}{m^2} \ll 1, \quad \frac{\omega}{m} \ll 1. \quad (\text{A.18})$$

To take the limit $M \rightarrow \infty$ correctly, it is necessary to replace φ_2 in the expression for R by $\varphi_1 + 2\lambda\xi(1-\xi)(k'/k)/m^2$. In the limiting case (A.18) it is sufficient to take into account only the first seven terms of the series (A.15b). The final result at $a = 0$ will be

$$g_{nm}\Pi_{R,s=0}^{nm}(k|A) = g_{nm}\Pi_{R}^{nm}(k) + \frac{e^2}{2\pi^2} e^2 a^2 \left(\frac{k'/k}{m^2} \right)^2 \times \left\{ \frac{1}{2} \int_0^1 d\xi \frac{\xi^2(1-\xi)^2}{(d-1)^2} + \frac{1}{3} \int_0^1 d\xi \left[2 - \xi(\xi-1) \frac{k^2}{m^2} \right] \times \frac{\xi^2(1-\xi)^2}{(d-1)^3} - \frac{1}{3} \int_0^1 d\xi \frac{\xi(1-\xi)}{(d-1)^2} \right\}, \quad (\text{A.19})$$

$$d = \xi(1-\xi) \frac{k^2}{m^2} + \frac{e^2 a^2}{2m^2}.$$

We can calculate in similar fashion $g_{nm}\Pi_{R,s}^{nm}$ with $s \neq 0$.

It is easy to verify that (A.19) satisfies the following condition, which follows from the gauge-invariance of the polarization operator:

$$g_{nm}\Pi_{R,s=0}^{nm}(k|A)|_{k=0} = 0.$$

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Translated by J. G. Adashko
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