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MODE INTERACTION IN A GAS LASER WITH SPHERICAL MIRROR RESONATORS

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The interaction of off-axis modes in a helium-neon gas laser is investigated experimentally and theoretically. The laser resonator consists of spherical mirrors and can be represented by an equivalent confocal resonator. It is shown that in such a case the laser emits modes with large transverse subscripts. The locking of off-axis modes with close frequencies has been observed and is given a theoretical explanation.

INTRODUCTION

THE most widely used at this time are gas lasers with confocal resonators or their equivalents, since such lasers do not require the precise alignment necessary for plane mirrors. The theory of "hollow" confocal resonators and their equivalents is quite well developed[1-3]. On the other hand, insufficient attention has been paid to lasers with such resonators, in which the situation reduces to interaction between the off-axis modes via the active medium.

The present paper deals with the theoretical and experimental investigation of mode interaction in a gas laser with a near-confocal resonator.

THE EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 1. We used a helium-neon gas laser 1 having a length $L \sim 100$ cm and a plane-sphere resonator. The spherical mirrors had radii R equal to 200 and 400 cm. Spherical mirror Z_1 had angular alignment pivots operating with an accuracy of ~0.2 angular sec, and mirror Z_2 (plane) had a longitudinal movement with an accuracy of $\sim 2 \times 10^{-4}$ cm.

The laser beam was split by semitransparent plate 2. The following data were recorded simultaneously:

a. The emission field structure at the laser output mirror, using camera 3. The distances from the camera lens to the output mirror and to the film were double the focal distance.

b. The beats spectrum of the off-axis modes, using FÉU-12A photomultiplier 4, amplifier 5 having a bandwidth from 25 Hz to 12 MHz and a gain of 10^3. ASChKh spectrum analyzer 6 (frequency band from 20 Hz to 20 kHz) and SCh-8 spectrum analyzer 7 (frequency band from 18 kHz to 30 MHz).

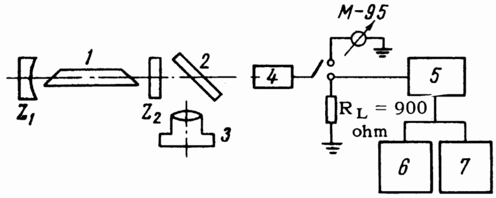

FIG. 1. Experimental setup.

c. Relative emission power, using an FÉU-12A photomultiplier and an M-95 microammeter.

EXPERIMENTAL RESULTS

Beats were not observed within the frequency range up to 20 kHz under any experimental conditions.

We plotted the laser emission power as a function of the angle of inclination of the spherical mirror (Fig. 2). The numbers in the plot indicate the places where we photographed the emission field structure and the beats spectrum in the range above 20 kHz (using the SCh-8 spectrum analyzer)

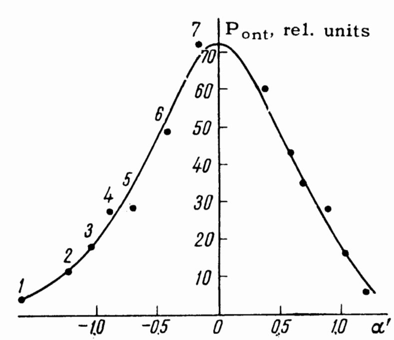

FIG. 2. Emission power as a function of spherical mirror angle for a laser with a plane + sphere resonator (R = 2m).

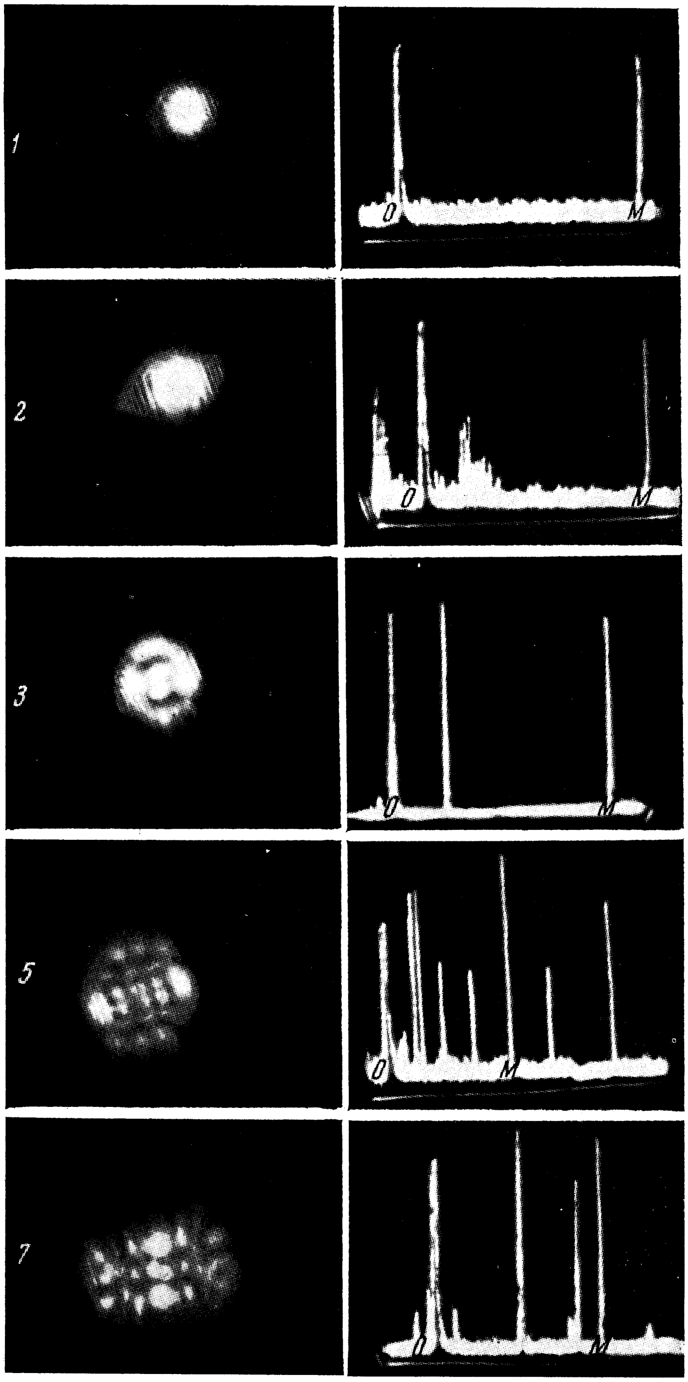

FIG. 3. Variation in emission field structure and beats spectrum due to spherical-mirror (R = 2 m) rotation. 0 – zero frequency, M – marker for 2 MHz.

(Fig. 3). Fig. 3 shows that the laser generates the TEM_{00q} mode when the inclination of the mirror is large. When the mirror is turning toward its optimum position the TEM_{00q} mode vanishes, to be replaced by a more complex mode that, in turn, becomes still more complex until the optimum position of the mirror is reached. At the same time beats appear whose number reaches maximum at the optimal alignment.

As we steadily observe the emission pattern grow more complex (unfortunately this paper can be illustrated only with individual still photographs) we can see that the development consists mainly of the disappearance of the preceding modes and their conversion into modes with larger transverse subscripts. This is particularly noticeable at the beginning of mirror rotation, i.e., in the case of low-order modes. When a fairly complex pattern is reached (the mirror is close to its optimum position) the previous modes do not disappear and new modes with larger transverse subscripts are added.

The above results were obtained with a laser equipped with a near-confocal resonator (radius R of the sphere was 200 cm), but not precisely equivalent to the confocal type. The deviation from confocality in length was ~0.1—0.2 mm. Such a deviation removes the degeneracy in the mode frequency and the beats spectrum between transverse modes should begin (according to the hollow resonator theory[1]) with ~10 kHz. Similar results were obtained for non-confocal resonators (R = 400 cm) that have (according to[2]) a confocal equivalent (Figs. 4 and 5). Lasers with resonators that do not have a "confocal" equivalent (plane type and the concentric type conjugate to the plane resonator[3]) emit a prevailing mode with the smallest transverse subscripts (TEM_{00q})[4-7].

It should be noted that observations were made also with ideally aligned mirrors and with a diaphragm inserted into the resonator (near the spherical mirror); the initial diameter of the diaphragm was such as to generate the TEM_{00q} mode. Gradual opening of the diaphragm produced a field-pattern development that was fully analogous to the above. The results can in no way be brought into agreement with the "hollow" resonator theory since the latter fails to predict mode interaction.

We now consider the theory of interaction of off-axis modes in a laser with a resonator close to the confocal type.

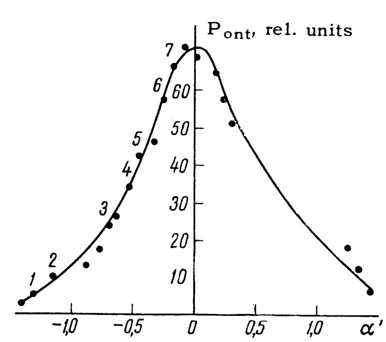

FIG. 4. Emission power as a function of spherical mirror angle (R = 4 m).

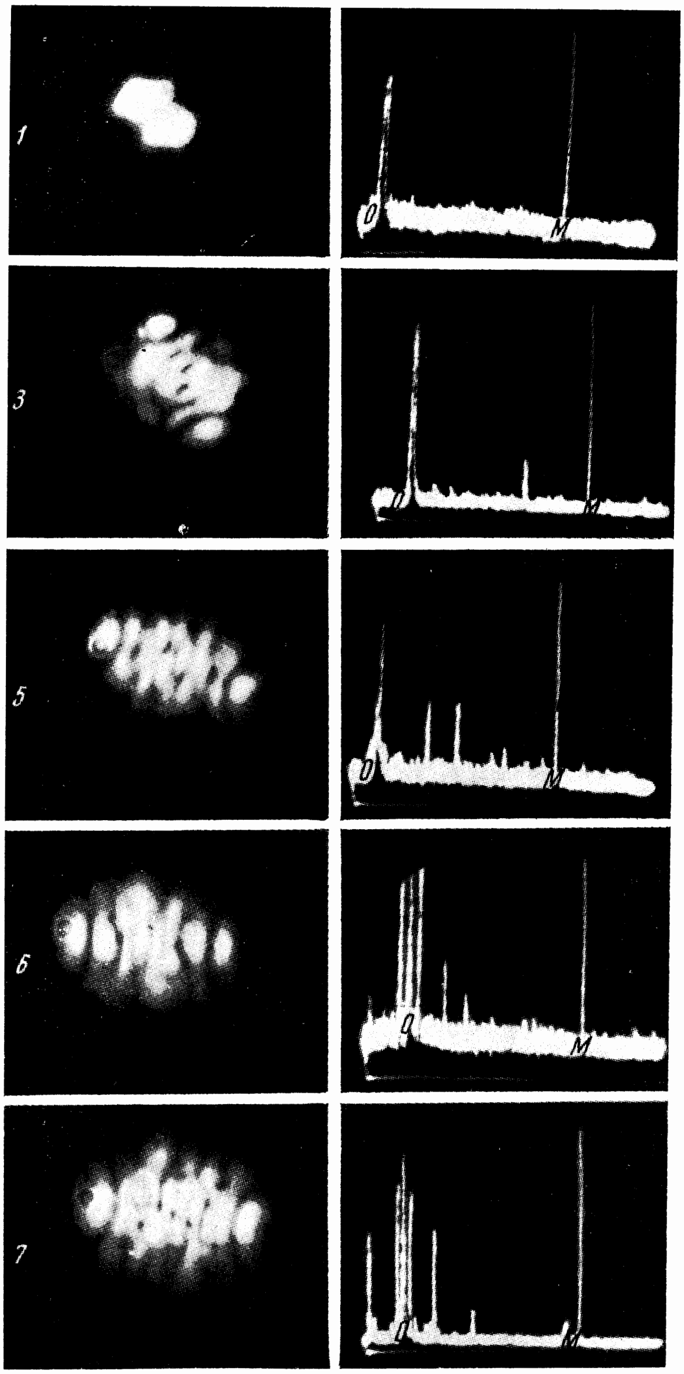

FIG. 5. Variation in emission field structure and beats spectrum due to spherical-mirror (R = 4 m) rotation. 0 – zero frequency, M – marker for 2 MHz.

THEORY

Lamb[8] investigated the interaction of axial modes in a laser with plane mirrors. Following Lamb's method, we consider simultaneous generation of two general modes in a resonator close to confocal. We assume that the light is linearly polarized. We describe the modes after Boyd and Gordon[1] with an accuracy up to the normalization:

$$U_{qmn}(\mathbf{r}) = X_m Y_n Z_q,$$
$$X_m = 2^{-m/2}(m!)^{-1/2} H_m(x/\rho_{qmn}) \exp[-x^2/2\rho^2_{qmn}],$$
$$Y_n = 2^{-n/2}(n!)^{-1/2} H_n(y/\rho_{qmn}) \exp[-y^2/2\rho^2_{qmn}],$$
$$Z_q = \sin\{\xi[(\pi/4)(2q+m+n+1) + {}^1/_2(x^2+y^2)/\rho^2_{qmn}] - (m+n+1)(\pi/2-\psi) + (\pi/4)(2q+m+n+1)\},$$
$$\rho_{qmn} = [b(1+\xi^2)/2K_{qmn}]^{1/2}, \quad \xi = 2z/b,$$
$$K_{qmn} = (\pi/2b)(2q+m+n+1),$$
$$\psi = \mathrm{arctg}[(1-\xi)/(1+\xi)]$$

(b is the distance between the mirrors and H_m and H_n are Hermite polynomials). The origin of coordinates lies in the center of the resonator and the Z axis coincides with the optical axis of the resonator. The functions $U_{qmn}(\mathbf{r})$ are normalized as follows:

$$\int U^2_{qmn}(\mathbf{r})\,d\mathbf{r} = b^3/3[q + {}^1/_2(m+n+1)].$$

When the field and polarization of the medium are expanded into a series in terms of the natural modes of the hollow resonator

$$E(\mathbf{r}, t) = \sum_l E_l(t)\cos[\nu_l t + \varphi_l(t)] U_{q_l m_l n_l}(\mathbf{r}),$$
$$P(\mathbf{r}, t) = \sum_l P_l(t) U_{q_l m_l n_l}(\mathbf{r}),$$
$$P_l(t) = 3[q_l + {}^1/_2(m_l+n_l+1)]\, b^{-3} \int P(\mathbf{r}, t) U_{q_l m_l n_l}(\mathbf{r})\,d\mathbf{r},$$
$$P_l(t) = C_l(t)\cos[\nu_l t + \varphi_l(t)] + S_l(t)\sin[\nu_l t + \varphi_l(t)],$$

Maxwell's equations in the approximation of a slowly-varying amplitude are reduced to a system of ordinary differential equations[8]:

$$(\nu_l - \Omega_l + \dot{\varphi}_l)E_l = -{}^1/_2(\nu/\varepsilon_0)C_l,$$
$$\dot{E}_l + {}^1/_2(\nu/Q_l)E_l = -{}^1/_2(\nu/\varepsilon_0)S_l, \tag{1}$$

where Ω_l and Q_l are the natural frequency and Q of the l-th mode of the resonator respectively; E_l, ν_l, and φ_l are the unknown amplitude, frequency, and phase respectively of the generated l-th mode; ν is the mean frequency of laser emission, and ϵ_0 is the dielectric permittivity of vacuum.

At low pump powers perturbation theory can be used to represent coefficients S_l and C_l as a series in powers of the mode amplitudes E_μ. The problem is thus broken down into two parts: computation of the polarization of the medium and solution of the system of nonlinear equations (1). We depart from Lamb's theory[8] in the first part by taking the transverse mode structure into account and in the second by considering the interaction of modes with close frequencies, a phenomenon significant in spherical mirror resonators.

In the computation of the polarization of the medium we assume that the latter consists of moving two-level atoms having a transition frequency ω, decay constants γ_a and γ_b, and a dipole moment d. According to Lamb, the projection of the polarization on the l-th mode is written in the first order of perturbation theory as follows:

$$P_l^{(1)}(t) = -{}^3/_2 i [d^2/\hbar^2 b^3][q_l + {}^1/_2(m_l + n_l + 1)] \cdot \times \sum_\mu E_\mu(t) \exp[-i(\nu_\mu t + \varphi_\mu(t))] \int d\mathbf{r}\, d\mathbf{v}\, N(\mathbf{r}, t) W(\mathbf{v}) \cdot \times U_l(\mathbf{r}) U_\mu(\mathbf{r} - \mathbf{v}\tau') \exp[-\gamma_{ab} + i(\omega - \nu_\mu)]\tau' + \text{c.c.}, \tag{2}$$

where

$$\gamma_{ab} = (\gamma_a + \gamma_b)/2, \quad N(\mathbf{r}, t) = \Lambda_a/\gamma_a - \Lambda_b/\gamma_b,$$

$\Lambda_\alpha(\mathbf{r}, t)$ is the number of atoms excited to the level α (α = a, b) in unit time at time t in unit volume,

$$W(\mathbf{v}) = \pi^{-3/2} u^{-3} \exp[-(v_x^2 + v_y^2 + v_z^2)/u^2]$$

is the Maxwellian velocity distribution of the atoms, and u is a velocity parameter ($\frac{1}{2}mu^2 = kT$).

We now make some simplifications. The quantities ρ_{qmn} can be assumed equal in typical optical resonators, since the maximum difference between the mode subscript sums $2q_\mu + m_\mu + n_\mu + 1$ is of the order of several times ten, while the average magnitude of these sums is $\sim 10^6$.

Introducing the mean wave number for the modes under consideration,

$$K = (\pi/2b)\overline{(2q + m + n + 1)}, \rho_{qmn} \approx \rho = [b(1 + \xi^2)/2K]^{1/2},$$

and, in addition, neglecting the differences between $\rho(\xi)$ and $\rho(\xi - 2v_z\tau'/b)$ and between $\varphi(\xi)$ and $\varphi(\xi - 2v_z\tau'/b)$, the function products $Z_{ql}(\mathbf{r}) Z_{q\mu} \times (\mathbf{r} - \mathbf{v}\tau')$ in (2) can be written in the form

$$\tfrac{1}{2}\cos A \cos K_\mu' v_z \tau' - \tfrac{1}{2}\sin A \sin K_\mu' v_z \tau' \tag{3}$$

+ a term rapidly oscillating in z,

where

$$K_\mu' = K_\mu\{1 + 2[(x - v_x\tau')^2 + (y - v_y\tau')^2]/b^2(1 + \xi^2)\},$$

$$A = (b/2)(\xi + 1)(K_l - K_\mu) + K_\mu u\tau'[v_x(2x - v_x\tau') + v_y(2y - v_y\tau')] \cdot [bu(1 + \xi^2)]^{-1} - [\pi/2 - \varphi(\xi)] \times (m_l + n_l - m_\mu - n_\mu). \tag{4}$$

Integrating (2) with respect to v_z, we can readily show that the major contribution from further integration with respect to τ' is due to an area in which the parameter $K_\mu' u\tau'$ is of the order of several units. In view of the above and of the fact that

$$[v_x(2x - v_x\tau') + v_y(2y - v_y\tau')]/ub(1 + \xi^2) \ll 1$$

within the region of integration with respect to τ', x, y, v_x, and v_y, we neglect the second term in (4), considering that the portion of (3) slowly varying in z is independent of transverse coordinates and velocities. If we further assume that the pumping is homogeneous, it is easy to integrate with respect to x, v_x, y, and v_y. As a result, (2) is transformed into

$$P_l^{(1)}(t) = -{}^1/_2[d^2/\hbar K u] \sum_\mu E_\mu(t) \exp[-i(\nu_\mu t + \varphi_\mu)] \times \Big\{ N_{q_l - q_\mu\ m_l + n_l - m_\mu - n_\mu} \delta_{m_l, m_\mu} \delta_{n_l\ \mu} Z(\xi) + N_{q_l - q_\mu\ m_l + n_l - m_\mu - n_\mu} \frac{d^2}{d\xi^2} Z(\xi_\mu) \times [\delta_{m_l, m_\mu} \delta_{n_l, n_\mu}(m_l + n_l + 1) + {}^1/_2 \delta_{m_l, m_\mu}(\delta_{n_l, n_\mu - 2}[n_\mu(n_\mu - 1)]^{1/2} + \delta_{n_l, n_\mu + 2}[n_l(n_l - 1)]^{1/2}) + {}^1/_2 \delta_{n_l, n_\mu}(\delta_{m, m_\mu - 2}[m_\mu(m_\mu - 1)]^{1/2} + \delta_{m_l, m_\mu + 2} \times [m_l(m_l - 1)]^{1/2})] \Big\} + \text{c.c.} \tag{5}$$

where

$$N_{q_l - q_\mu, m_l + n_l - m_\mu - n_\mu} = \frac{3}{8} \int_{-1}^{+1} \cos\Big\{\frac{b}{2}(\xi + 1)(K_l - K_\mu) - \Big(\frac{\pi}{2} - \varphi(\xi)\Big)(m_l + n_l - m_\mu - n_\mu)\Big\}(1 + \xi^2) N(\xi)\, d\xi,$$

$$N_{q_l - q_\mu\ m_l + n_l - m_\mu - n_\mu} = \frac{3}{16Kb} \int_{-1}^{+1} N(\xi) \cos\Big[\frac{b}{2}(\xi + 1)(K_l - K_\mu) - \Big(\frac{\pi}{2} - \varphi(\xi)\Big)(m_l + n_l - m_\mu - n_\mu)\Big] d\xi,$$

$$Z(\xi) = 2i \int_{-\infty}^{i\xi} \exp[-(t^2 + \xi^2)]\, dt, \quad \xi = \frac{\nu - \omega}{Ku} + \frac{i\gamma_{ab}}{Ku}.$$

$$\frac{d^2}{d\xi^2} Z(\xi) = 2(2\xi - 1) Z(\xi) + 4\xi.$$

The second term in the braces in (5) is small in comparison with the first, since it contains a large parameter $Kb \sim 10^6$ in the denominator. Furthermore, this term turns to zero if the sums $(m_l + n_l)$ and $(m_\mu + n_\mu)$ are neither both odd nor both even. Therefore only the modes of the same transverse structure as the l-th mode make a significant contribution to the polarization $P_l^{(1)}$. The natural frequencies of such modes are well separated from one another in comparison to the resonator line width: thus the basic contribution to the slowly varying part of $P_l^{(1)}$ is derived from the l-th mode alone.

We use the same assumptions for the computation of the polarization projection on the l-th mode in the case of modes with small transverse subscripts in the third order of perturbation theory. Omitting intermediate computations and neglecting terms with transverse components of atomic velocities (which is allowable in the case of modes with small transverse subscripts), we write the result as

$$P_l^{(3)} = \frac{i\pi^{1/2}}{8}\frac{d^4}{\hbar K u}\sum_{\mu\rho\sigma} E_\mu E_\rho E_\sigma \times \exp\{-i[(\nu_\mu - \nu_\rho + \nu_\sigma)t + (\varphi_\mu - \varphi_\rho + \varphi_\sigma)]\}$$
$$\times [D_a(\nu_\rho - \nu_\sigma) + D_b(\nu_\rho - \nu_\sigma)]\{H_{l\mu\rho\sigma} D[\omega - {}^1/_2(\nu_\mu - \nu_\rho) - \nu_\sigma] + H_{l\mu\sigma\rho} D(-{}^1/_2\nu_\mu - {}^1/_2\nu_\sigma + \nu_\rho)\} + \text{c.c.}, \tag{6}$$

where

$$H_{l\mu\rho\sigma} = {}^1/_4 N_{q_l - q_\mu - q_\rho + q_\sigma,\ m_l + n_l - m_\mu - n_\mu - m_\rho - n_\rho + m_\sigma + n_\sigma} T_{l\mu\rho\sigma},$$

$$T_{l\mu\rho\sigma} = T^x_{l\mu\rho\sigma} T^y_{l\mu\rho\sigma},$$

$$T^x_{l\mu\rho\sigma} = \pi^{-3/2}(m_l!m_\mu!/m_\rho!m_\sigma!)^{1/2} \sum_{q=0}^{(m_\rho+m_\sigma-|m_\rho-m_\sigma|)/2} \{\Gamma[(-m_l + m_\mu + |m_\rho - m_\sigma| + 2q + 1)/2]/[(-m_\rho + m_\sigma + |m_\rho - m_\sigma| + 2q)/2]!\}$$
$$\times\{\Gamma[(+m_l - m_\mu + |m_\rho - m_\sigma| + 2q + 1)/2]/[(m_\rho - m_\sigma + |m_\rho - m_\sigma| + 2q)/2]!\}$$
$$\times\{\Gamma[(m_l + m_\mu - |m_\rho - m_\sigma| - 2q + 1)/2]/[(m_\rho + m_\sigma - |m_\rho - m_\sigma| - 2q)/2]!\}; \tag{7}$$

$T^y_{l\mu\rho\sigma}$ is obtained by substituting n for m in (7):

$$D(\omega) = (\gamma_{ab} + i\omega)^{-1}, \quad D_\alpha(\omega) = (\gamma_\alpha + i\omega)^{-1}\ (\alpha = a, b).$$

The coefficients $C_l(t)$ and $S_l(t)$ can be readily separated from Eqs. (5) and (6) in the first and third orders of perturbation theory. In the general case these coefficients depend on the amplitudes of all the modes under consideration.

We now consider the generation of two modes. There are two cases characteristic of a laser with a resonator close to the confocal type: a) the separation between modes (in terms of frequency) is greater than the resonator line width, and (b) the mode separation is considerably smaller than the line width.

1. Frequency pulling and repulsion can be neglected in the case of modes greatly separated in frequency. The mode amplitudes can then be determined by the following system of equations:

$$\dot{E}_1 = \alpha_1 E_1 - \beta_1 E_1^3 - \theta_{12} E_1 E_2^2,$$
$$\dot{E}_2 = \alpha_2 E_2 - \beta_2 E_2^3 - \theta_{21} E_1^2 E_2, \tag{8}$$

where

$$\alpha_l = {}^1/_2(\nu/Q)\{[Z_i(\nu_l - \omega)/Z_i(0)\tilde{N} - 1\},$$
$$\beta_l = \varkappa T_{llll}[1 + \gamma_{ab}^2 L(\nu_l - \omega)] \quad (l = 1, 2),$$

$$\theta_{12} = {}^1/_2 T_{1122}\varkappa\{2\gamma_{ab}^2[L(\omega - (\nu_1 + \nu_2)/2) + L((\nu_2 - \nu_1)/2)] + \gamma_{ab}[\gamma_a L_a(\nu_2 - \nu_1) + \gamma_b L_b(\nu_2 - \nu_1)][N_{0,0}^{-1} N_{2(q_1 - q_2),\ 2(m_1 + n_1 - m_2 - n_2)} \times L(\omega - \nu_1 + L(\nu_1 - \nu_2)]$$
$$+ (\nu_1 - \nu_2)[L_a(\nu_2 - \nu_1) + L_b(\nu_2 - \nu_1)] \times [N_{0,0}^{-1} N_{2(q_1 - q_2),\ 2(m_1 + n_1 - m_2 - n_2)} L(\omega - \nu_1)(\omega - \nu_1) + L(\nu_1 - \nu_2)(\nu_1 - \nu_2)]\}, \tag{9}$$

$$\varkappa = 2^{-4}\pi^{1/2}(\nu/Q)[d^2\tilde{N}/\hbar^2\gamma_a\gamma_b Z_i(0)],\ \tilde{N} = N_{0,0}[d^2 Q Z_i(0)/\varepsilon_0\hbar K u],$$

$$L(\omega) = (\gamma_{ab}^2 + \omega^2)^{-1}, \quad L_\alpha = (\gamma_\alpha^2 + \omega^2)^{-1} \quad (\alpha = a, b),$$

θ_{21} is obtained by interchanging indices $1 \rightleftarrows 2$ in the expression for θ_{12}. The Q factors of the modes have been assumed to be equal.

The system (8) has been analyzed by Lamb. Our coefficients α, β, and θ differ from Lamb's coefficients only by the presence of the coefficients $T_{l\mu\rho\sigma}$ and $N_{p,q}$, which reflect the difference in the transverse structure of the modes. With symmetrical tuning we have $\theta_{12} = \theta_{21} = \theta$, $\alpha_1 = \alpha_2$, and $\beta_1/\beta_2 = T_{1111}/T_{2222}$.

According to computations based on (7) modes with large transverse subscripts have lower coefficients T_{llll} and consequently, other conditions being equal, lower coefficients β. When the coupling is weak ($\beta_1\beta_2 > \theta^2$) a stability check of the solutions of (8) shows that the mode having a larger coefficient β is either completely attenuated or is weaker than the second mode: when $\beta_1 > \beta_2 > \theta$,

$$(E_1/E_2)^2 = (\beta_2 - \theta)/(\beta_1 - \theta) < 1,$$

and when $\beta_2 < \theta < \beta_1$

$$E_1 = 0.$$

When the coupling is strong ($\theta^2 > \beta_1\beta_2$) either one or the other mode is excited in steady-state operation, depending on the initial conditions.

The coefficients T^x_{llll} were found for the first eight TEM_{m0} modes. The results are as follows:

TEM_{m0}	TEM_{00}	TEM_{10}	TEM_{20}	TEM_{30}	TEM_{40}	TEM_{50}	TEM_{60}	TEM_{70}
T^x_{llll}:	1	0,750	0,641	0,574	0,528	0.500	0.465	0,443

We see that, as the mode numbers increase the coefficients T_{llll}, and consequently β, tend toward "saturation," i.e., modes with sufficiently large transverse subscripts are not more capable of competing than modes with smaller transverse subscripts. The damping of one mode by another is less probable in this case, leading to the simultaneous generation of many modes with large transverse subscripts. These conclusions are confirmed by experiment. Figures 3 and 5 show that as the mirror begins to turn (or as the diaphragm diameter in the laser increases) the change in the emission field structure consists of increasing transverse subscripts of the modes. Beyond a certain point (photo 6 of Fig. 5, for example) the emission field structure becomes more complex by simple addition of new more complex modes to the previous pattern.

2. In the case of modes whose natural frequencies are close to one another, we are mainly interested in the width of the "locking" region. "Locking" can be due to the scattering of modes by inhomogeneities and stem from the linear polarization terms obtained when molecular motion is taken into account, and also from the nonlinearity of the medium. We confine ourselves to locking due to the latter cause. We look for a single-frequency case of two modes with close natural frequencies and with transverse subscripts of opposite parity, such as $TEM_{0,0,q}$ and $TEM_{1,1,q-1}$, $TEM_{0,3,q}$ and $TEM_{1,0,q+1}$, etc. According to (5), there are no linear terms in system (1) to cause locking in such pairs; therefore locking is due only to the nonlinear properties of the medium (we neglect scattering).

We assume that we have symmetrical tuning, that the mode Q factors are equal, and that pumping is homogeneous along the axis and over the cross section of the laser. The system (1) then assumes the following form in single frequency operation for the case of two modes whose transverse subscripts have opposite parity, recognizing that the coefficients T_{1222} and T_{2221} vanish for such a pair:

$$(\dot{\varphi}_1 + \delta\nu_1 - \sigma)E_1 = E_1E_2^2\varkappa T_{1122}(1+\mu)\sin 2\Delta\varphi,$$
$$(\dot{\varphi}_2 + \delta\nu_1 - 2\Delta\Omega - \sigma)E_2 = -E_2E_1^2\varkappa T_{1122}(1+\mu)\sin 2\Delta\varphi,$$
$$\dot{E}_1 = \alpha E_1 - 2\varkappa T_{1111}E_1^3 - E_1E_2^2\varkappa T_{1122}[3 + (1+\mu)\cos 2\Delta\varphi],$$
$$\dot{E}_2 = \alpha E_2 - 2\varkappa T_{2222}E_2^3 - E_2E_1^2\varkappa T_{1122}[3 + (1+\mu)\cos 2\Delta\varphi], \tag{10}$$

where

$$\delta\nu_1 = \nu_1 - \Omega_1,\quad \Delta\varphi = \varphi_2 - \varphi_1,\quad \nu_1 = \nu_2,$$
$$\Delta\Omega = (\Omega_2 - \Omega_1)/2,\quad \mu = H_{2(q_1-q_2),\,2(m_1-m_2+n_1-n_2)}/H_{00},$$

$$\sigma = 2^{-1}\frac{\nu}{Q}\bar{N}\left\{Z_r\left[\frac{\delta\nu_1 - \Delta\Omega}{Ku}\right]\Big/ Z_i(0)\right\} \approx \pi^{-1/2}\frac{\nu}{Q}\bar{N}\left[\frac{\delta\nu_1 - \Delta\Omega}{Ku}\right]. \tag{11}$$

Representing the coefficients T_{1111} and T_{2222} as

$$T_{1111} = T + \Delta T,\quad T_{2222} = T - \Delta T$$

and taking (11) into account, we can readily obtain from (10) in the zeroth order in ΔT the following relations for the amplitudes, frequencies, and phases in the stationary case:

$$E_1^2 = E_2^2 = \Delta\Omega/\varkappa T_{1122}(1+\mu)\sin 2\Delta\varphi_0,\quad \Delta\nu_1 = \Delta\Omega,$$
$$(\alpha/\Delta\Omega)\sin 2\Delta\varphi_0 + \cos 2\Delta\varphi_0 = -(2T + 3T_{1122})/(1+\mu)T_{1122}. \tag{12}$$

According to the last equation of (12), mode locking is possible only if

$$\Delta\Omega < \alpha[(2T + 3T_{1122})^2/T_{1122}^2(1+\mu^2) - 1]^{-1/2},$$

or, if the explicit form of α (9) is taken into account when $\delta\nu_1 = \Delta\Omega$,

$$\Delta\Omega < 2^{-1}(\nu/Q)(\bar{N} - 1)[(2T + 3T_{1122})^2/T_{1122}^2(1+\mu^2) - 1]^{-1/2}. \tag{13}$$

When $T_{1111} = T_{2222} = T$, the stable solution is the one with equal mode amplitudes and with phase differences

$$\varphi_0 = \pi/2 - 2^{-1}\arcsin[1 + (\alpha/\Delta\Omega)^2]^{-1/2}$$
$$+ 2^{-1}\arcsin\{(2T + 3T_{1122})T_{1122}(1+\mu)[1 + (\alpha/\Delta\Omega)^2]^{1/2}\}.$$

When the difference between T_{1111} and T_{2222} is taken into account in the first order of ΔT, we see that the phase difference $\Delta\varphi_0$ does not change and the common frequency of generation shifts toward the mode with the lower coefficient T_{llll}, its amplitude becoming larger.

A numerical estimate of the width of the locking range, based on (13), at twice the threshold pump power gives $\Delta\Omega \sim 20-30$ kHz for modes with moderate transverse subscripts; this is in good agreement with experimental data.

CONCLUSION

1. The experiments showed that modes with large transverse subscripts are excited in a laser with a resonator consisting of spherical mirrors and equivalent to a confocal resonator.

A theoretical analysis of mode interaction in a laser with such a resonator showed that modes with large transverse subscripts are more competitive than modes with smaller subscripts. The number of the maximum transverse mode appears to be determined by the tube diameter (if the volume distribution of the excited atoms is uniform).

2. The experiments show that beats are absent if the radio-frequency spectrum is low enough (up to 20 kHz). A theoretical analysis of the interaction of modes with close resonator frequencies ($\Delta\Omega \ll \nu/Q$) showed that there exists a "locking" region whose width is in a good agreement with experiment.

[1] G. D. Boyd and J. P. Gordon, Bell System Techn. J. **40**, 489 (1961).

[2] G. D. Boyd and H. Kogelnik, Bell System Techn. J. **41**, 1347 (1962).

[3] L. A. Vaĭnshteĭn, JETP **45**, 684 (1963), Soviet Phys. JETP **18**, 471 (1964).

[4] I. M. Belousova, O. B. Danilov, and B. A. Ermakov, JETP **47**, 2013 (1964), Soviet Phys. JETP **20**, 1351 (1965).

[5] D. R. Herriot, JOSA **52**, 31 (1962).

[6] A. L. Mikaelyan, A. B. Korovitsyn, and L. V. Naumova, JETP Letters **2**, 37 (1965), transl. p. 22.

[7] I. M. Belousova and O. B. Danilov, ZhTF, in press.

[8] W. E. Lamb, Phys. Rev. **134**, A1429 (1964).

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